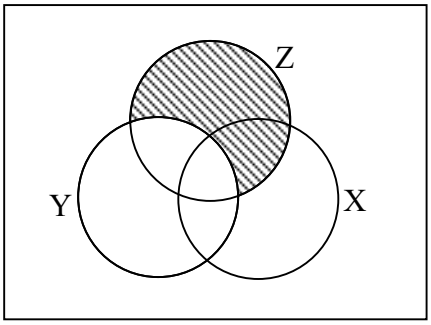
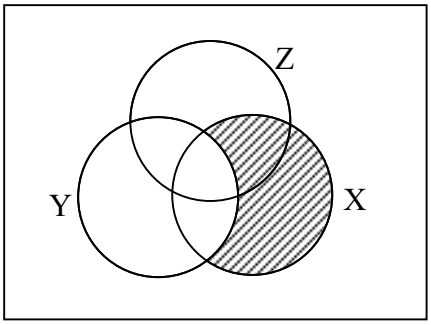
1(a)(i)

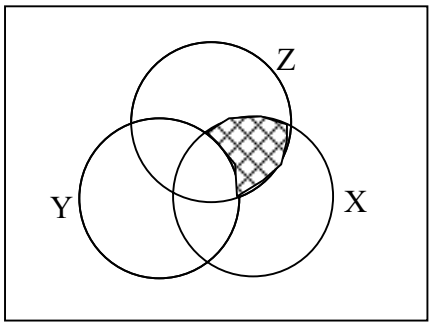
Z\Y



X\Y

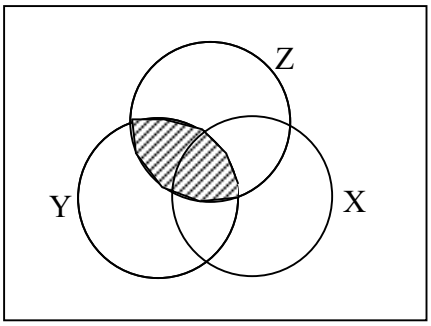


(Z\Y)∩(X\Y)

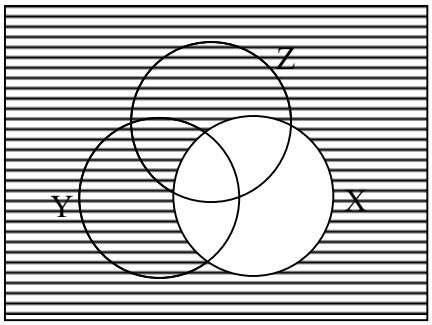


(ii)

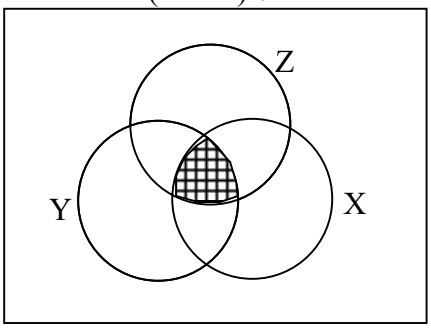
$(Z \cap Y)$



$\overline{X}$

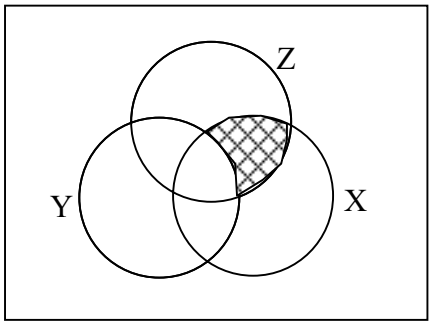


$(Z \cap Y) \setminus \overline{X}$



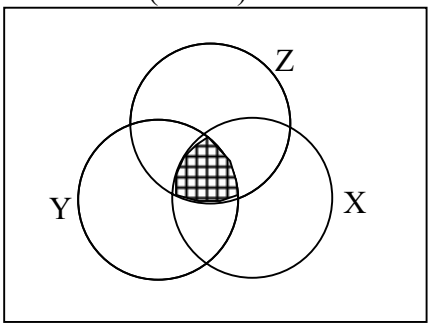
1(b)

$(Z \setminus Y) \cap (X \setminus Y)$



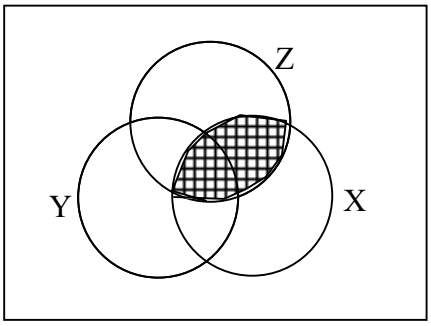
$\cup$

$(Z \cap Y) \setminus \overline{X}$



$[(Z \setminus Y) \cap (X \setminus Y)] \cup [(Z \cap Y) \setminus \overline{X}]$

$=$



$= X \cap Z$

Therefore the expression is true.

2.

Truth Table

p	∨	(q	∧	r)	↔	(p	∨	q)	∧	(p	∨	r)
T	T	T	T	T	T	T	T	T	T	T	T	T
T	T	T	F	F	T	T	T	T	T	T	T	F
T	T	F	F	T	T	T	T	F	T	T	T	T
T	T	F	F	F	T	T	T	F	T	T	T	F
F	T	T	T	T	T	F	T	T	T	F	T	T
F	F	T	F	F	T	F	T	T	F	F	F	F
F	F	F	F	T	T	F	F	F	F	F	T	T
F	F	F	F	F	T	F	F	F	F	F	F	F

Therefore, it is a tautology.

3.(a)

$$A^2 = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 1 \\ 2 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 1 \\ 2 & 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 9 & 6 & 4 \\ 6 & 6 & 5 \\ 4 & 5 & 5 \end{pmatrix}$$

(b) 9

4. Graph A is not Eulerian ($\because$ deg d = 3 = odd)
Graph B is Eulerian ($\because$ all vertices are even degrees).

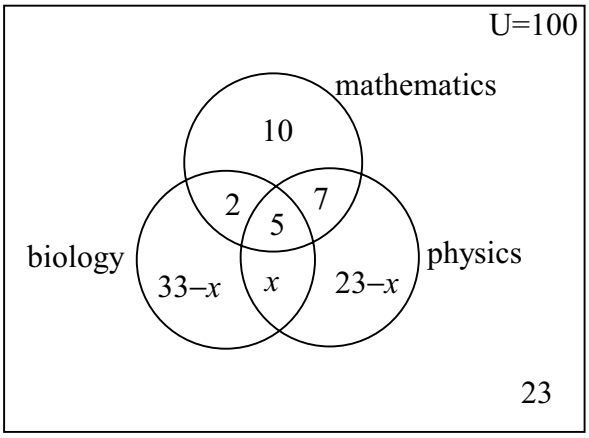
5. Suppose x is a prime, i.e. all its divisors are 1 and x only.
Therefore the sum of all its divisors except the number itself = $1 \neq x$.
Hence x is not a perfect number.
Thus, the statement is proved by indirect proof.

6.(a) ${}_{20}C_{10} \times {}_{10}C_8$
= 8314020

(b) $f(p) = (p-6) \bmod 26$
$f^{-1}(p) = (p+6) \bmod 26$

	A	I	I	X	F	O	W	E
p	0	8	8	23	5	14	22	4
$f^{-1}(p)$	6	14	14	3	11	20	2	10
message	G	O	O	D	L	U	C	K

7.(a)(i)



(ii) $10 + 2 + 5 + 7 + x + 33 - x + 23 - x + 23 = 100$

$x = 3$

(iii) $10 + 30 + 20 = 60$

(b)(i) p: printer is defective; q: the power is on; r: LED is blinking.

Symbolic form:

$q \wedge r \rightarrow p$

$r \rightarrow q$

r

$\therefore p$

(ii)

(	q	$\wedge$	r	$\rightarrow$	p	)	$\wedge$	(	r	$\rightarrow$	q	)	$\wedge$	r	$\rightarrow$	p
	T	T	T	T	T		T		T	T	T		T	T	T	T
	T	F	F	T	T		T		F	T	T		F	F	T	T
	F	F	T	T	T		F		T	F	F		F	T	T	T
	F	F	F	T	T		T		F	T	F		F	F	T	T
	T	T	T	F	F		F		T	T	T		F	T	T	F
	T	F	F	T	F		T		F	T	T		F	F	T	F
	F	F	T	T	F		F		T	F	F		F	T	T	F
	F	F	F	T	F		T		F	T	F		F	F	T	F

The statement form $(q \wedge r \rightarrow p) \wedge (r \rightarrow q) \wedge r \rightarrow p$ is a tautology, hence the argument is valid.

8.(a) R is not reflexive. (Book *a* cost the same and contains the same pages of book *a*.) Therefore (a, a)$\notin$ R.

Since it is impossible for Book *a* cost more than Book *b* cost and Book *b* cost more than Book *a* cost (similar to pages comparison), R is not symmetric. But it is anti-symmetric.

If Book *a* cost more than Book *b* and Book *b* cost more than Book *c*, then Book *a* cost more than Book *c* (similar to pages comparison). Therefore, R is transitive.

(b)(i) $R_1 \cap R_2 = \{(b, HKT), (b, IBM).(c, IBM), (c, Orange), (d, HKT), (d, IBM)\}$

Students attend the interviews and are offered the companies.

(ii)

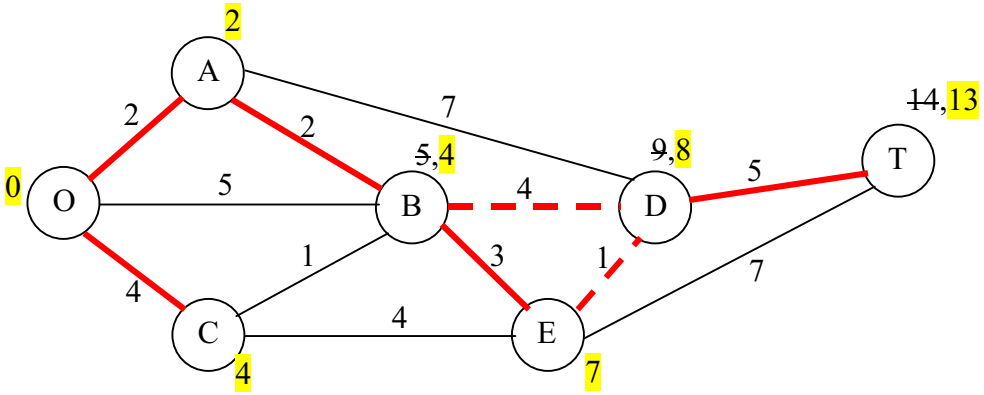
$R_1 \setminus R_2 = \{(a, AT\ \&\ T), (a, 3Com), (a, IBM), (b, AT\ \&\ T), (b, 3Com), (b, Orange), (d, AT\ \&\ T)\}$

Students attend the interviews but are rejected.

(iii) $R_2 \setminus R_1 = \{(c, 3Com), (d, Orange)\}$

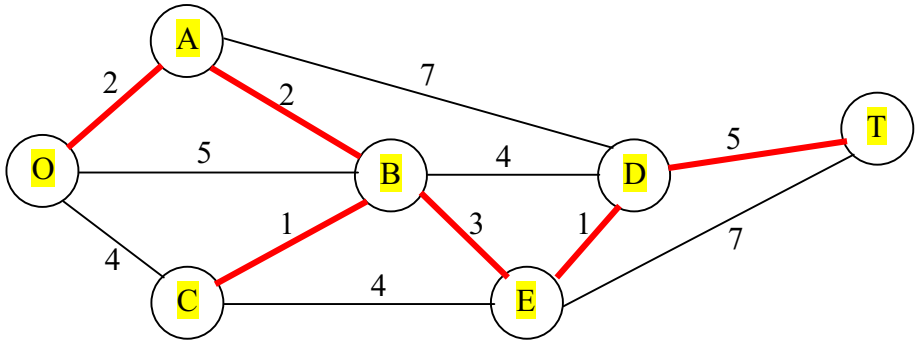
Students are offered by the companies but without needing to attend the interviews.

9.(a)



The smallest distance from O to T is 13.
The routes are OABEDT or OABDT

(b)



Min total number of miles of line installed = 2 + 2 + 1 + 3 + 1 + 5 = 14 miles

10.(i) Let A : Smart cards from factory I; B: Smart cards from factory II; D: Defective smart cards.

Given P(A) = 2/3
P(B) = 1/3
P(D | A) = 0.2
P(D | B) = 0.05

P(D) = P(A)P(D|A) + P(B)P(D|B) = (2/3)(0.2) + (1/3)(0.05) = 0.15
$P(\overline{D}) = 1 - 0.15 = 0.85$

(ii)

$$P(A \mid D) = \frac{P(A)P(D \mid A)}{P(D)}$$
$$= \frac{(2/3)(0.2)}{0.15}$$
$$= 0.889$$

11.(a)(i) $\mu = \lambda T = 2 \times 8 = 16$

Therefore, on average, 16 customers arriving in an 8-hour period.

(ii) P(at least one customer in a 1-hour period)
= 1 – P(no customer in a 1-hour period)
$= 1 - \dfrac{2^0 e^{-2}}{0!}$
= 0.865

(b) X ~ Bin(3, 0.4)

$$E(X) = 0 \times p(0) + 1 \times p(1) + 2 \times p(2) + 3 \times p(3)$$
$$= 0 + 1 \times {}_3C_1 (0.4)(0.6)^2 + 2 \times {}_3C_2 (0.4)^2 (0.6) + 3 \times {}_3C_3 (0.4)^3$$
$$= 1.2$$

12.(a) P(a zero is received correctly)
= P(X < 0.4)
$= P(Z < \dfrac{0.4 - 0}{0.16})$
= P(Z < 2.5)
= 0.9938

(b) P(a one is received correctly)
= P(Y > 0.8)
$= P(Z > \dfrac{0.8 - 1}{0.09})$
= P(Z > –2.22)
= 0.9868

(c) P(a digit is received correctly)
$= 0.5 \times 0.9938 + 0.5 \times 0.9868$
= 0.9903

(d) P(the received signal is interpreted as an error)
$= 0.5 \times P(0.4 < X < 0.8) + 0.5 \times P(0.4 < Y < 0.8)$
$= 0.5 \times P(\dfrac{0.4 - 0}{0.16} < Z < \dfrac{0.8 - 0}{0.16}) + 0.5 \times P(\dfrac{0.4 - 1}{0.09} < Z < \dfrac{0.8 - 1}{0.09})$
$= 0.5 \times P(2.5 < Z < 5) + 0.5 \times P(-6.67 < Z < -2.22)$
$= 0.5 \times (1 - 0.9938) + 0.5 \times (1 - 0.9864)$
= 0.0099