Hong Kong Institute of Vocational Education
(Chai Wan, Sha Tin & Kwai Chung Campuses)

Department of Computing & Mathematics / Computing

Solution of Sessional Examination (1999/2000)

Course Code: 41111 Subject : Discrete Mathematics & Statistics

Section A: Short Questions. (5% each)

1. $(A \cap \overline{B}) \cup (\overline{A} \cap B) \cup (A \cap B)$

$= (A \cap \overline{B}) \cup (A \cap B) \cup (\overline{A} \cap B)$

$= (A \cap (\overline{B} \cup B)) \cup (\overline{A} \cap B)$

$= (A \cap U) \cup (\overline{A} \cap B)$

$= A \cup (\overline{A} \cap B)$

$= (A \cup \overline{A}) \cap (A \cup B)$

$= U \cap (A \cup B)$

$= A \cup B$

2. (a) "$\exists m \in E$ such that $m^2 = m$" is false.

Note that $m^2 = m$ is not true for any integers m from 5 to 10:

$5^2 = 25 \neq 5$, $6^2 = 36 \neq 6$, $7^2 = 49 \neq 7$

$8^2 = 64 \neq 8$, $9^2 = 81 \neq 9$, $10^2 = 100 \neq 10$

(b) Take $x = 0.5$. Then x in R (since 0.5 is a real number) and $(0.5)^2 = 0.25 \not\geq 0.5$

Hence "$\forall x \in R$, $x^2 = x$" is false.

3. (a) Let n and m be odd integers.

Then $n = 2k + 1$ for some integer k and $m = 2j + 1$ for some integer j

The product $nm = (2k+1)(2j+1) = 2(2kj + k + j) + 1$ which is odd.

(b) The truth set of $P = \{i, e, a, o\}$

4. (a) AQ + BQ = I

$$\begin{bmatrix}-2 & 3\\ 1 & 0\end{bmatrix}Q + \begin{bmatrix}5 & 1\\ -1 & 2\end{bmatrix}Q = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$\left(\begin{bmatrix}-2 & 3\\ 1 & 0\end{bmatrix} + \begin{bmatrix}5 & 1\\ -1 & 2\end{bmatrix}\right)Q = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$\begin{bmatrix}3 & 4\\ 0 & 2\end{bmatrix}Q = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$Q = \frac{1}{6}\begin{bmatrix}2 & -4\\ 0 & 3\end{bmatrix}$$ Note: Q is the inverse of $\begin{bmatrix}3 & 4\\ 0 & 2\end{bmatrix}$

(b) RA = C

$$R\begin{bmatrix}-2 & 3\\ 1 & 0\end{bmatrix} = \begin{bmatrix}-5 & 9\\ 6 & -6\end{bmatrix}$$

$R.A.A^{-1} = C.A^{-1} \Rightarrow R = C.A^{-1}$

$$R = \begin{bmatrix}-5 & 9\\ 6 & -6\end{bmatrix}.\begin{bmatrix}0 & 1\\ 1/3 & 2/3\end{bmatrix} = \begin{bmatrix}3 & 1\\ -2 & 2\end{bmatrix}$$ Note: $A^{-1} = \begin{bmatrix}0 & 1\\ 1/3 & 2/3\end{bmatrix}$

5. In figure 1, n = 5 but deg $v_2 = 2 < n/2$, so Dirac's theorem does not apply. However, deg v_i + deg $v_j \geq 5$ for all pairs of non-adjacent vertices v_i and v_j, so this graph is Hamiltonian by Ore's theorem.

One of the possible cycle is $v_1\ v_5\ v_2\ v_4\ v_3\ v_1$.

6. (a) The required number is:

$$= \left\lfloor \frac{1000}{4} \right\rfloor + \left\lfloor \frac{1000}{6} \right\rfloor - \left\lfloor \frac{1000}{12} \right\rfloor$$

= 250 + 166 – 83

= $333_{\#}$

(b) The required number is:

$$= 1000 - \left\{\left\lfloor \frac{1000}{4} \right\rfloor + \left\lfloor \frac{1000}{6} \right\rfloor + \left\lfloor \frac{1000}{9} \right\rfloor - \left\lfloor \frac{1000}{12} \right\rfloor - \left\lfloor \frac{1000}{36} \right\rfloor - \left\lfloor \frac{1000}{18} \right\rfloor + \left\lfloor \frac{1000}{36} \right\rfloor\right\}$$

= 1000 – 289

= $611_{\#}$

7. (a) P(the committee consists of 3 men and 3 women)

$$= \frac{{}_{20}C_3 \cdot {}_{17}C_3}{{}_{37}C_6} = \frac{775{,}200}{2{,}324{,}784} = 0.333_{\#}$$

(b) P(the committee must have at least 4 men)

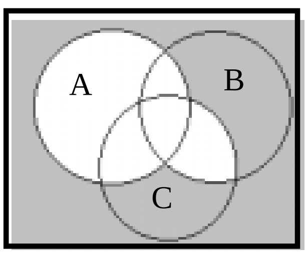
$$= \frac{{}_{20}C_2 \cdot {}_{17}C_4 + {}_{20}C_1 \cdot {}_{17}C_5 + {}_{20}C_0 \cdot {}_{17}C_6}{{}_{37}C_6} = \frac{512{,}176}{2{,}324{,}784} = 0.220_{\#}$$

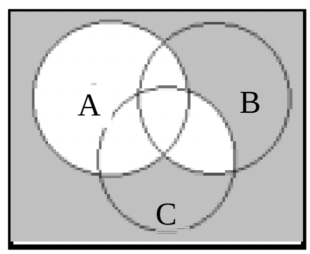
8. (a) P (2) = $e^{-1.5}(1.5^2)/2!$ = 0.251

(b) P(0) = $e^{-1.5}$ = 0.223
Probability of no breakdown during the three consecutive shifts
= P(0) x P(0) x P(0)
= 0.223^3
= 0.0111

Section B: Structured Questions. (12% each)

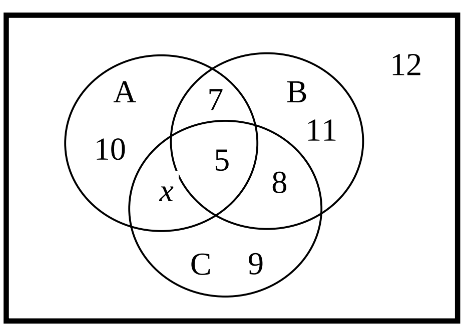
9. (a) $\overline{A \cup (B \cap C)}$ $\quad$ $(\overline{C} \cup \overline{B}) \cap \overline{A}$





Since they have the same Venn diagram, therefore they are equivalent.

(b)



(i) 31+(16-x)+(15-x)+x+12 = 68
x = 6 students
(ii) 9+10+11 = 30 students
(iii) 5+6+7+8 = 26 students
(iv) 8 students

10. (a) (i) p : Tom is on team A

q : Peter is on team B

$\therefore$: Therefore

$\sim p \to q$

$\sim q \to p$

$\therefore \sim p \vee \sim q$

(ii)

p	q	$\sim p$	$\sim q$	$\sim p \to q$	$\sim q \to p$	$\sim p \vee \sim q$	$[(\sim p \to q) \wedge (\sim q \to p)] \to (\sim p \vee \sim q)$
T	T	F	F	T	T	F	F
T	F	F	T	T	T	T	T
T	T	T	F	T	T	T	T
T	F	T	T	F	F	T	T

The statement form $[(\sim p \to q) \wedge (\sim q \to p)] \to (\sim p \vee \sim q)$ is not a tautology, hence the argument is invalid.

(b) (i) The contrapositive of a conditional statement of the form "If p then q" is If $\sim q$ then $\sim p$.

(ii) If Robert cannot swim to the island, then Robert cannot swim across the lake.

(c) The truth value of $(\sim (p \wedge q)) \to q$ can be determined

If $p \to q$ is false, then p is true and q is false.

Hence $p \wedge q$ is false, $\sim (p \wedge q)$ is true, and $(\sim (p \wedge q)) \to q$ is false.

11. (a) i) $R_1 \cup R_2$

$= \{(a, S1), (a, S4), (b, S2), (b, S3), (b, S5), (c, S2), (c, S5), (c, S6), (d, S4), (d, S5), (d, S6)\}$

ii) $R_1 \cap R_2 = \{(a, S1), (b, S2), (d, S4), (d, S6)\}$

iii) $R_1 \setminus R_2 = \{(b, S3), (c, S2), (c, S5), (c, S6)\}$

(b) i) $P(E_1 \mid E_3) = \dfrac{P(E_1 \cap E_3)}{P(E_3)} = \dfrac{C(4, 2)}{C(5, 3)} = \dfrac{6}{10} = \dfrac{3}{5}$ #

ii) $P(E_3 \mid E_2) = \dfrac{P(E_3 \cap E_2)}{P(E_2)} = \dfrac{C(4, 2)}{2^4} = \dfrac{6}{16} = \dfrac{3}{8}$ #

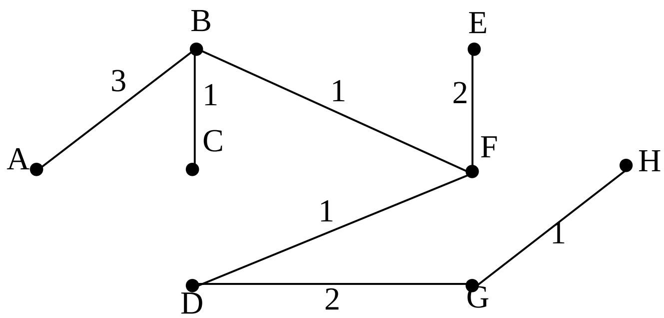
iii) $P(E_2 \mid E_1) = \dfrac{P(E_2 \cap E_1)}{P(E_1)} = \dfrac{2^3}{2^4} = \dfrac{1}{2}$ #

iv) $P(E_3 \mid E_1 \cap E_2) = \dfrac{P(E_1 \cap E_2 \cap E_3)}{P(E_1 \cap E_2)} = \dfrac{C(3, 1)}{2^3} = \dfrac{3}{8}$ #

v) Yes

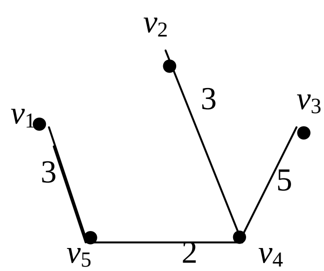
vi) No

12. (a)



Center	Earliest time each receives the news
B	3:03 p.m.
C	3:04 p.m.
D	3:05 p.m.
E	3:06 p.m.
F	3:04 p.m.
G	3:07 p.m.
H	3:08 p.m.

(b)



minimum weight = 3 + 3 + 2 + 5 = 13

13. a. i. $\mu = 4.5$, $\sigma = 1.1$,

$P(X > 6) = P[Z > (6 – 4.5)/1.1] = P(Z > 1.364) = 1 – 0.9131 = 0.0869$

$P(X < 5) = P[Z < (5 – 4.5)/1.1] = P(Z < 0.4545) = 0.6736$

$P(X<5 \text{ or } X>6) = P(X < 5) + P(X > 6) = 0.6736 + 0.0849 = 0.7585$

ii. $P (Z > z_o) = 0.05$

$z_o = 1.645$

$(X – 4.5)/1.1 = 1.645$

$X = 6.31$ minutes

b. i. $P(2) = {}_{15}C_2 \, (0.1)^2 \, (1 – 0.1)^{13} = 0.2669$

ii. $P(X > 0) = 1 – P(0) = 1 – (1 - 0.1)^{15} = 1 – 0.2059 = 0.7941$