Section A

Q1. Use identities of sets to simplify the following set expression:

$$(A \cap \overline{B}) \cup (\overline{A} \cap B) \cup (A \cap B)$$

Q2. (a) Let $E = \{5,6,7,8,9,10\}$ and consider the statement $\exists m \in E$ such that $m^2 = m$. Determine the truth or falsity of this statement. Explain your answer.

(b) Consider the statement $\forall x \in R,\ x^2 = x$. Find a counter example to show that this statement is false. Note that the set of all real numbers is denoted by R.

Q3. (a) Prove that the product of any two odd integers is odd.

(b) If L is the set of letters in the phrase "science and hypothesis" and P is the propositional function on L defined by $P(x) =$ "x is a vowel", find the truth set of P.

Q4. Given A=$\begin{bmatrix} -2 & 3 \\ 1 & 0 \end{bmatrix}$, B=$\begin{bmatrix} 5 & 1 \\ -1 & 2 \end{bmatrix}$, C=$\begin{bmatrix} -5 & 9 \\ 6 & -6 \end{bmatrix}$ and I = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find matrices Q and R such that

(a) AQ + BQ = I
(b) RA = C

Q5. Check whether the conditions of Dirac's theorem and Ore's theorem hold for the following Hamiltonian graph in Figure 1. Write down one of the possible Hamiltonian cycle.

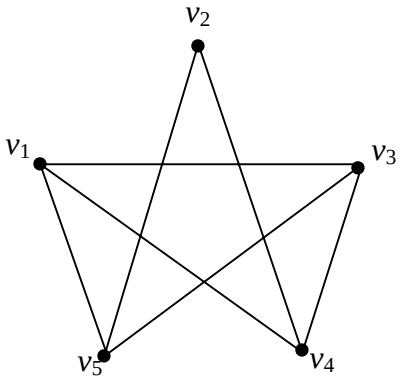

Figure 1

Q6. (a) How many positive integers not exceeding 1000 are divisible by either 4 or 6?

(b) Find the number of positive integers not exceeding 1000 and *not* divisible by 4, 6 or 9.

Q7. A club with 20 women and 17 men needs to form a committee of 6 members. Find the probability that

(a) the committee consists of 3 men and 3 women.

(b) the committee must have at least 4 men.
Give your answers in 3 decimal places.

Q8. A production line of a factory reports that the number of breakdowns per 8-hour shift follows a Poisson distribution with a mean of 1.5. What is the probability of

(a) exactly two breakdowns in an 8-hour shift ?

(b) no breakdown during three consecutive 8-hour shifts ?
(You may assume that the machine operates independently across shifts.)

Q9. (a) Use Venn diagrams to determine whether the following set expressions are equivalent.

$\overline{A \cup (B \cap C)}$ and $(\overline{C} \cup \overline{B}) \cap \overline{A}$

(b) A survey of 68 part-time study students produced the following information:
28 study Accounting,
31 study Business Administration,
28 study Computer Studies,
12 study Accounting and Business Administration,
13 study Business Administration and Computer Studies,
5 study all three subjects, and
12 study none of the three subjects.
Determine how many students

(i) study Accounting and Computer Studies but not Business Administration.

(ii) study only one subject.

(iii) study two or more subjects.

(iv) study Business Administration and Computer Studies but not Accounting?

Q10.(a) If Tom is not in team A, then Peter is in team B.
If Peter is not in team B, then Tom is in team A.
Therefore, Tom is not in team A or Peter is not in team B.

(i) Use symbols to write the logical form of the above argument.
(ii) Use a truth table to test for validity of the above argument.

(b) (i) Define the contrapositive of a conditional statement of the form "If p then q".
(ii) Write the following statement "If Robert can swim across the lake, then Robert can swim to the island" in its equivalent contrapositive form.

(c) If $p \to q$ is false, can you determine the truth value of $\left(\sim\left(p \wedge q\right)\right) \to q$?
Explain your answer.

Q11. (a) Let $A = \{a, b, c, d\}$ be a set of students, and
$B = \{S1, S2, S3, S4, S5, S6\}$ be a set of courses.

R_1 is binary relation from A to B describing the courses that the students are taking,

and

		S1	*S2*	*S3*	*S4*	*S5*	*S6*
$R_1 =$	*a*	1	0	0	0	0	0
	b	0	1	1	0	0	0
	c	0	1	0	0	1	1
	d	0	0	0	1	0	1

R_2 is binary relation from A to B describing the courses that the students are interested in,

and

		S1	*S2*	*S3*	*S4*	*S5*	*S6*
$R_2 =$	*a*	1	0	0	1	0	0
	b	0	1	0	0	1	0
	c	0	0	0	0	0	0
	d	0	0	0	1	1	1

Find the binary relations that represent
(i) $R_1 \cup R_2$.
(ii) $R_1 \cap R_2$.
(iii) $R_1 \setminus R_2$.

(b) In an experiment you pick a bit string of length 5 at random. Consider the following events:

E_1: the bit string chosen begins with 1
E_2: the bit string chosen ends with 1
E_3: the bit string chosen has exactly three 1s

(i) Find $P(E_1 \setminus E_3)$.
(ii) Find $P(E_3 \setminus E_2)$.

(iii) Find $P(E_2 \setminus E_1)$.
(iv) Find $P(E_3 \setminus E_1 \cap E_2)$.
(v) Determine whether E_1 and E_2 are independent.
(vi) Determine whether E_2 and E_3 are independent.

Q12. (a) The graph in Figure 2 shows the communication channels and the communication time delays in the channels among eight communication centers. The centers are represented by vertices, the channels are represented by edges, and the communication time delay (in minutes) in each channel is represented by the weight of the edge. Suppose that at 3:00 p.m. communication center **A** broadcasts the news through all its channels. Other communication centers will then broadcast this news through all their channels as soon as they receive it.

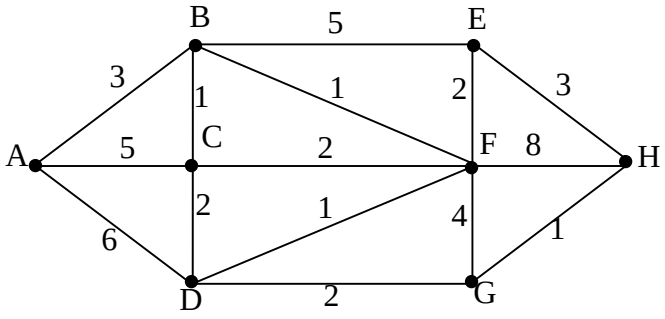

Figure 2

For the communication centers B, C, D, E, F, G, and H, determine the earliest time each receives the news.

(b) Find a minimum spanning tree of the following weighted graph in Figure 3. What is the minimum weight?

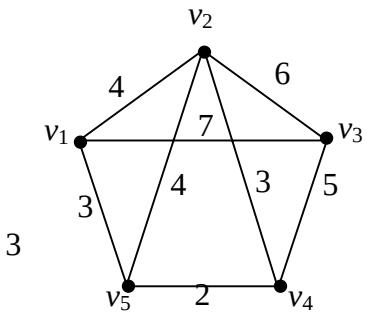

Figure 3

Q13. (a) The length of talking time of customers for each call through a mobile phone service company is normally distributed with a mean of 4.5 minutes and a standard deviation of 1.1 minutes.

(i) What is the probability that a randomly selected call takes more than 6 minutes or under 5 minutes ?

(ii) Top 5 % of the large consumption customers will be granted bonus free talking time. What is the minimum talking time of a call that a bonus will be given ?

(b) The probability that a HD student can enter a university to continue his study is 0.1. For a HD class with 15 students, what is the probability that

(i) exactly two students can have their further studies in a university ?

(ii) at least one student can have their further studies in a university ?

Appendix

Area under the standard normal curve

Note : An entry in the table is the area under the curve to the left of z, i.e. $\Phi(z)$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7703	0.7734	0.7764	0.7793	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8906	0.8925	0.8943	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998