

You are required to answer **ALL** questions in Section A and **ALL** questions from Section B. Section A is worth a total of **FORTY** marks. Each question in Section B is worth **TWELVE** marks.

A Standardised Normal Distribution Table is attached as an appendix to this paper.

Section A

You are required to answer **ALL** questions.

You are advised to spend no more than **SEVENTY** minutes on this section.

A1. The secret message, “ZKBDSKV PBKMDSYX”, was produced by the Caesar’s encryption $f(p) = (p + 10) \bmod 26$. Decrypt this message. (5 marks)

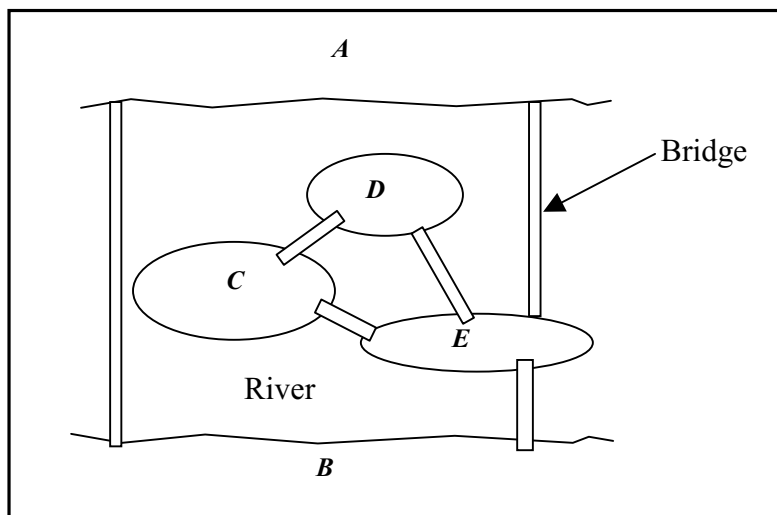
A2. i) Define the truth set of a propositional function. (2 marks)

ii) Determine the set of truth values of the following propositional function

$$Q(x) = "x^3 - 2x^2 - x + 2 = 0". \quad (3 \text{ marks})$$

A3. Determine how many different numbers can be formed from the digits 0, 1, 4 and 7 if they are not allowed to be used more than once. (5 marks)

A4. Determine whether it is possible to take a walk around the city the map of which is shown below, starting and ending at the same point and crossing each bridge exactly once. If so, give the route. (5 marks)



A5. A set consists of ten rods of different lengths. Five of the rods are selected randomly. Find the probabilities that:

- i) the five selected rods include the longest and the shortest of the ten rods in the set; (2 marks)
- ii) at least three of the shortest four rods in the set are in the sample. (3 marks)

A6. Prove that $n^3 + n$ is even if n is an integer. (5 marks)

A7. In a building, it is found that an average of three light bulbs need to be replaced every month. Assume that the number of failures of light bulbs follows a Poisson distribution.

- i) Find the probability that there is no need to replace a light bulb in a month. (2 marks)
- ii) If there are three light bulbs held in stock, determine the probability that there are not enough light bulbs in stock to replace failed light bulbs in a month. (3 marks)

A8. i) Given $A = \begin{bmatrix} 1 & 3 & 0 \\ 2 & 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 0 & 2 \\ 1 & 3 \end{bmatrix}$, calculate the products AB and BA . (2 marks)

ii) Find the matrix A such that $\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \cdot A = \begin{bmatrix} 3 & 0 \\ 1 & 2 \end{bmatrix}$ (3 marks)

Section B

You are required to answer **ALL** questions. Each question is worth a total of **TWELVE** marks.

You are advised to spend no more than **TWENTY** minutes on any one question.

- B1.** i) Each of the 5 members in a project team is asked to borrow three books about discrete mathematics from the library. The coverage of some of the topics in these books is summarized as follows:

1 book covers sets, graphs and relations;
 4 books cover sets and graphs;
 3 books cover sets and relations;
 5 books cover graphs and relations;
 6 books cover sets;
 9 books cover graphs;
 9 books cover relations.

Determine how many of these books:

- (a) cover exactly one of these topics; (2 marks)
- (b) cover at least two topics; (2 marks)
- (c) cover sets and graphs but not relations. (2 marks)
- ii) State De Morgan's laws of algebra for two sets A and B , and verify them by drawing the necessary Venn diagrams. (6 marks)
- B2.** i) Use a truth table to show that the following argument is valid. (8 marks)

$$\begin{array}{l} p \vee (q \vee r) \\ \neg r \\ \therefore p \vee q \end{array}$$

- ii) (a) Convert the following argument to its symbolic form. (3 marks)

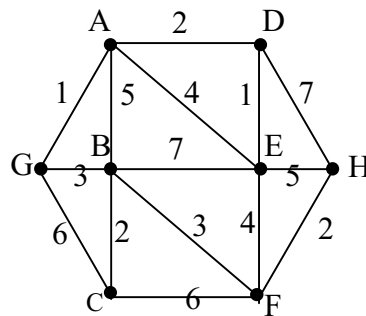
If interest rates go up, then stock market prices will go down.
 Interest rates are not going up.
 Therefore, stock market prices will not go down.

- (b) Determine whether or not the argument in symbolic form you gave as your answer to ii)(a) is valid. (1 mark)

- B3.** i) A pipeline is to be built to link six districts. The cost (in millions of dollars) of constructing each link depends on the distance and the terrain and is shown in the table below.

	Tsuen Wan	Tuen Mun	Yuen Long	Tai Po	Sha Tin	Sai Kung
Tsuen Wan	-	1.5	1.6	1.9	2.5	-
Tuen Mun	1.5	-	0.8	-	-	1.8
Yuen Long	1.6	0.8	-	2.9	2.2	1.7
Tai Po	1.9	-	2.9	-	1.2	-
Sha Tin	2.5	-	2.2	1.2	-	1.1
Sai Kung	-	1.8	1.7	-	1.1	-

- (a) Construct a weighted graph for the pipeline project as specified by the information given in the above table. (3 marks)
- (b) Find a system of pipelines to connect all the districts whilst minimizing the total cost. (4 marks)
- (c) Determine the minimum cost of the pipeline project. (1 mark)
- ii) Show the steps involved in applying the shortest path algorithm to find the shortest distance from vertex G to vertices D, E, F and H in the following weighted graph. (4 marks)



- B4.** i) The intelligence quotient (IQ) of secondary seven students is Normally distributed with a mean of 100 and a standard deviation of 16.
- (a) Calculate the proportion of secondary seven students that have an IQ of above 120. (3 marks)
- (b) A secondary seven student can go to university if he or she is in the top 20% of secondary seven students in terms of IQ. Calculate the minimum IQ which allows entry to university. (3 marks)

- ii) A factory finds that on average 2% of the product produced by a given machine will be defective according to certain specific criteria. If 10 units produced by the machine are selected at random for inspection, find the probability that:
- (a) exactly two will be defective; (3 marks)
- (b) two or more will be defective. (3 marks)
- B5.** i) (a) List all the ordered pairs in the relation $R = \{(a, b) : a = b+1\}$ on the set $A = \{1, 2, 3, 4, 5\}$ where both a and b belong to set A . (2 marks)
- (b) Display the relation, described in i)(a) above, in tabular form. (2 marks)
- (c) Suppose that $A = \{a_1, a_2, a_3, a_4, a_5\}$ and $B = \{b_1, b_2, b_3\}$. Determine the ordered pairs that are represented by the matrix M_R . (2 marks)

$$M_R = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

- ii) In a certain college, 64% of pupils are boys and 36% are girls. The probability that a boy of this college studies discrete mathematics is $\frac{1}{4}$ and the probability that a girl of this college studies discrete mathematics is $\frac{1}{6}$. Find the exact probability that:
- (a) a discrete mathematics pupil selected at random from this college is female; (3 marks)
- (b) a pupil selected at random from this college is not studying discrete mathematics. (3 marks)

****END OF PAPER****

