

Section A: (40 marks)**Answer ALL Questions in this section.**

1. (a) Use Venn diagram to represent
- (i) $(Z \setminus Y) \cap (X \setminus Y)$ [3 marks]
- (ii) $(Z \cap Y) \setminus \bar{X}$ [3 marks]
- (b) Investigate the truth or falsity of the following expression for sets X , Y , and Z .
 $[(Z \setminus Y) \cap (X \setminus Y)] \cup [(Z \cap Y) \setminus \bar{X}] = X \cap Z$ [2 marks]

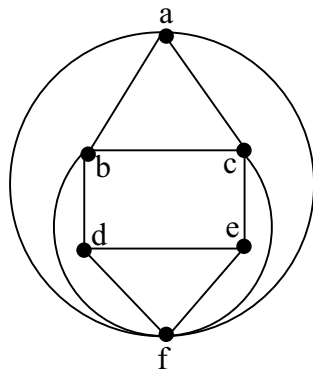
2. Use truth table to determine whether $p \vee (q \wedge r) \leftrightarrow (p \vee q) \wedge (p \vee r)$ is a tautology [6 marks]

3. Given the undirected graph with vertices v_1 , v_2 and v_3 and with

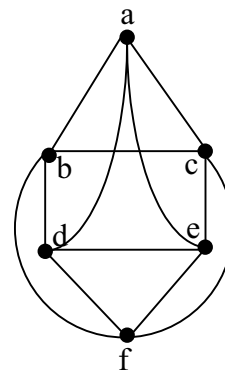
$$A = \begin{matrix} & \begin{matrix} v_1 & v_2 & v_3 \end{matrix} \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \end{matrix} & \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 1 \\ 2 & 1 & 0 \end{pmatrix} \end{matrix} \text{ as its adjacency matrix.}$$

- (a) Find A^2 . [4 marks]
- (b) Find the number of walks of length 2 from v_1 to v_1 . [2 marks]

4. Consider the following two graphs



Graph A



Graph B

Which graph(s) is/are Eulerian? Justify your answers. [6 marks]

5. A perfect number is an integer which is equal to the sum of all its divisors except the number itself.
Thus, 6 is a perfect number, since $6 = 1 + 2 + 3$, and so is 28.
Prove the following theorem by indirect proof.
Theorem:
If x is a perfect number then x is not a prime [6 marks]
6. (a) In a class of 20 students, 10 of them are selected to perform an IQ test. From the remaining students, 8 of them are selected to perform an EQ test. How many total kinds of ways can we select students to perform the two tests? [4 marks]
- (b) The following secret message was produced by the Caesar's encryption
 $f(p) = (p - 6) \bmod 26$.
"A I I X F O W E"
Decrypt the above message. [4 marks]

Section B: (15% each)**Answer any FOUR Questions.**

7. (a) Among 100 students, 24 study mathematics, 35 study physics, 40 study biology, 7 study mathematics and biology, 12 study mathematics and physics, 5 study all three subjects, and 23 do not study any of the three subjects.
- (i) Represent the above information on a Venn diagram. [4 marks]
- (ii) Find the number of students studying physics and biology only. [2 marks]
- (iii) Find the number of students studying exactly one of the three subjects. [2 marks]
- (b) (i) Convert the following argument to its symbolic form.
If the power is on and the LED is blinking then the printer is defective.
If the LED is blinking then the power is on.
The LED is blinking.
Therefore, the printer is defective. [3 marks]
- (ii) Use a truth table to test for validity of the argument. [4 marks]

8. (a) Let A be a set of books. Let R be a binary relation on A such that (a, b) is in R if book a costs more and contains fewer pages than book b . Determine whether R is reflexive, symmetric, antisymmetric and transitive. Justify your answer. [8 marks]

- (b) Let

$$A = \{a, b, c, d\}$$

be a set of students and

$$B = \{\text{AT\&T}, \text{3Com}, \text{HKT}, \text{IBM}, \text{Orange}\}$$

be a set of companies that came to the Morrison Hill to interview students for jobs.

We have a binary relation R_1 from A to B describing the interviews the companies had with the students, and a binary relation R_2 from A to B describing the jobs of the companies offered to the students as shown in below.

	AT&T	3Com	HKT	IBM	Orange
a	✓	✓		✓	
b	✓	✓	✓	✓	✓
c				✓	✓
d	✓		✓	✓	

R_1

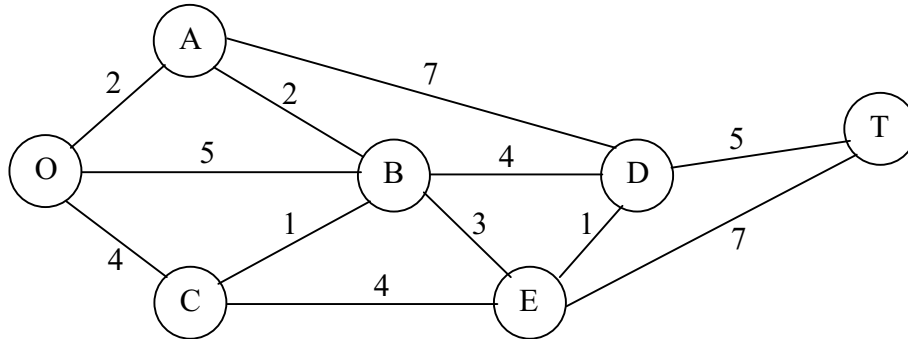
	AT&T	3Com	HKT	IBM	Orange
a					
b			✓	✓	
c		✓		✓	✓
d			✓	✓	✓

R_2

List the members and give the meanings of the binary relations [7 marks]

- (i) $R_1 \cap R_2$
(ii) $R_1 \setminus R_2$
(iii) $R_2 \setminus R_1$

9. Ocean Park has recently been redeveloped. There is a narrow, winding road system for trams driven by the park ranger. This road system is shown below, where location O is the entrance into the park and other letters designate the location of ranger stations. The numbers give the distances of these winding roads in km.



The park contains a scenic wonder at station T. A small number of trams are used to transport sightseers from the park entrance to station T.

- (a) Determine which route from the park entrance to station T has the smallest total distance for the operation of the trams. [8 marks]

Telephone lines must be installed under the roads to establish telephone communication among all the stations. Because the installation is both expensive and disruptive to the natural environment, lines will be installed under just enough roads to provide connections between every station.

- (b) How the lines should be laid to accomplish this with a minimum total number of km of line installed. [7 marks]

10. Only two factories manufacture smart cards. 20 percent of the smart cards from factory I and 5 percent from factory II are defective. The volume of smart cards produced at factory I doubles that at factory II each week.

- (a) What is the probability that a smart card, randomly chosen from a week's production, is satisfactory? [8 marks]
- (b) If the chosen smart card is defective, what is the probability that it is from factory I? [7 marks]

11. (a) Customers arrive at a facility according to a Poisson distribution at the rate of two an hour. Find the following:
- (i) The average number of customers arriving in an 8-hour period. [3 marks]
 - (ii) The probability that there will be at least one customer in an 1-hour period. [5 marks]
- (b) A coin is tossed 3 times, and a head turns up each time with probability 0.4. Find the expected number of heads obtained. [7 marks]
12. In a certain network, the binary digit 0 is sent as a voltage signal of zero volt and the received signal has a level represented by the random variable $X \sim N(0, 0.16^2)$; the binary digit 1 is sent as a voltage signal of 1 volt and the received signal has a level represented by the random variable $Y \sim N(1, 0.09^2)$. The receiver interprets a level of 0.4 volt or lower as a binary digit 0. The receiver interprets a level of 0.8 volt or higher as a binary digit 1. Otherwise the received signal is interpreted as an error.
- (a) Find the probability that a zero is received correctly. [4 marks]
 - (b) Find the probability that a one is received correctly. [4 marks]
- Assuming that 0 and 1 are equally likely to be transmitted, find
- (c) the probability that a digit is received correctly and [3 marks]
 - (d) the probability that the received signal is interpreted as an error. [4 marks]

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