Department of Computing
Discrete Mathematics and Statistics
Assignment 2

Hand-out Date: 4 March, 2002.
Hand-in Date: 18 March, 2002.

You are warned to refrain from plagiarism. Both the plagiarist and the students whose work has been plagiarised will be penalized appropriately.

Answer **ALL** questions. All working must be clearly shown. Total mark is 40.

1. A point in two dimensions is represented by its coordinates, x and y. These values is written as a 2×1 matrix, for example, $\begin{bmatrix} 3 \\ -2 \end{bmatrix}$. Such matrix is referred to as position matrix or position vector. A line can be determined by its two end-points. Hence the matrix $\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$ represents the line joining the points $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 3 \\ 7 \end{bmatrix}$. Similarly the triangle can be represented as a 2×3 matrix $\begin{bmatrix} 2 & 5 & 3 \\ 3 & 6 & -1 \end{bmatrix}$ as shown in the figure below (Figure 1).

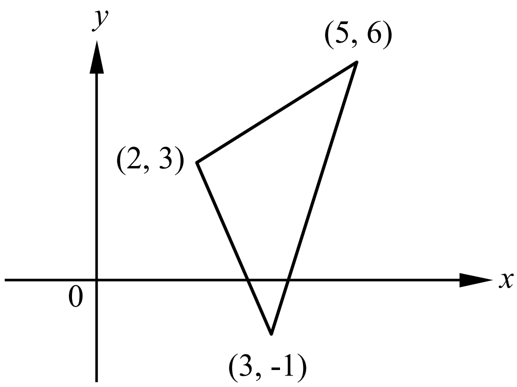


Figure 1

We can translate points in the (x, y) plane to new positions by adding translation amounts to the coordinates of the points. For each point $P(x, y)$ to be moved by d_x units parallel to the x axis and by d_y units parallel to the y axis to the new point $P'(x', y')$, we can write

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} d_x \\ d_y \end{bmatrix}$$

Points can be rotated through an angle θ about the origin. A rotation is defined mathematically by

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Consider the rotation of an object (Figure 2) about point P_1. To rotate about P_1, we need a sequence of three fundamental transformations:

(a) Translate such that P_1 is at the origin

(b) Rotate

(c) Translate such that the point at the origin returns to P_1.

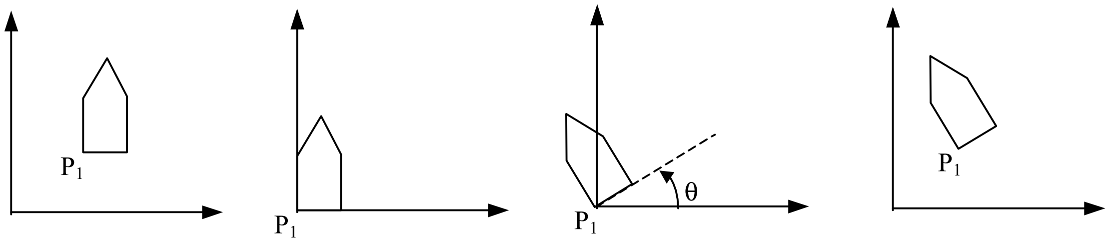
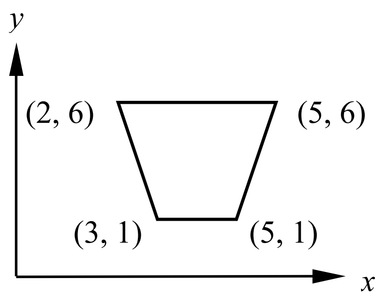


Figure 2 (a) (b) (c)

Given a diagram below (Figure 3): -



(a) Find the position matrix of the above object. [2 marks]

(b) Now, rotate the object about point (3, 1) for 30° clockwise. Use the sequence of three transformations as described above, what is the new position matrix of the rotated object. (**All the steps should be clearly shown**.) [6 marks]

2. (a) Given the matrices A_1, A_2 and A_3 with sizes 20x50, 50x10 and 10x40 respectively. Calculate the computational complexity of

(i) $(A_1 A_2) A_3$ and

(ii) $A_1 (A_2 A_3)$

in terms of the number of multiplications and additions are used. [4 marks]

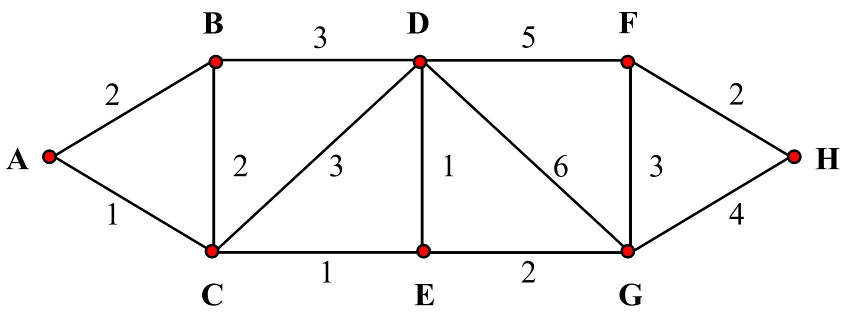
(b) Let $A = \{2,3,4,5,6,7,8\}$ and define a binary relation **R** on A as follows:

"for all $x, y \in A$, $x\mathbf{R}y$ if and only if 3 divides $(x - y)$"

(i) List the ordered pairs in the relation **R**. [2 marks]

(ii) Determine whether the relation **R** is reflexive, symmetric, antisymmetric and transitive. Explain briefly. [6 marks]

3. Given a network as shown below:



(i) Find the shortest path from A to H. [6 marks]

(ii) Find the minimum spanning tree joining the nodes. What is the total minimum distance? [4 marks]

4 Let $A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ be an adjacency matrix of the graph G with vertices v_1, v_2 and v_3.

(i) Calculate A^2 and A^3. [3 marks]

(ii) Find the number of walks of length 2 from v_1 to v_3 and the number of walks of length 3 from v_1 to v_3. [2 marks]

(iii) Draw G and find all the walks of length 2 from v_1 to v_3. [5 marks]

~ End ~