1. (a) The position matrix of the object is $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{pmatrix} 3 & 2 & 5 & 5 \\ 1 & 6 & 6 & 1 \end{pmatrix}$

(b) Translate such that P_1 is at the origin

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix} - \begin{bmatrix} d_x \\ d_y \end{bmatrix} = \begin{pmatrix} 3 & 2 & 5 & 5 \\ 1 & 6 & 6 & 1 \end{pmatrix} - \begin{pmatrix} 3 & 3 & 3 & 3 \\ 1 & 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & -1 & 2 & 2 \\ 0 & 5 & 5 & 0 \end{pmatrix}$$

Rotate 30° clockwise

$$\begin{bmatrix} x_\theta \\ y_\theta \end{bmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$= \begin{pmatrix} \cos(-30^o) & -\sin(-30^o) \\ \sin(-30^o) & \cos(-30^o) \end{pmatrix} \begin{pmatrix} 0 & -1 & 2 & 2 \\ 0 & 5 & 5 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix} \begin{pmatrix} 0 & -1 & 2 & 2 \\ 0 & 5 & 5 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & \frac{5}{2} - \frac{\sqrt{3}}{2} & \frac{5}{2} + \sqrt{3} & \sqrt{3} \\ 0 & \frac{1}{2} + \frac{5\sqrt{3}}{2} & -1 + \frac{5\sqrt{3}}{2} & -1 \end{pmatrix}$$

Translate such that the point at the origin returns to P_1

$$\begin{bmatrix} x_\theta' \\ y_\theta' \end{bmatrix} = \begin{bmatrix} x_\theta \\ y_\theta \end{bmatrix} + \begin{bmatrix} d_x \\ d_y \end{bmatrix}$$

$$= \begin{pmatrix} 0 & \frac{5}{2} - \frac{\sqrt{3}}{2} & \frac{5}{2} + \sqrt{3} & \sqrt{3} \\ 0 & \frac{1}{2} + \frac{5\sqrt{3}}{2} & -1 + \frac{5\sqrt{3}}{2} & -1 \end{pmatrix} + \begin{pmatrix} 3 & 3 & 3 & 3 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & \frac{11}{2} - \frac{\sqrt{3}}{2} & \frac{11}{2} + \sqrt{3} & 3 + \sqrt{3} \\ 1 & \frac{3}{2} + \frac{5\sqrt{3}}{2} & \frac{5\sqrt{3}}{2} & 0 \end{pmatrix}$$

2. (a) (i) The number of multiplications $= 20\times50\times10+20\times10\times40=18000$

The number of additions $= 20\times49\times10+20\times9\times40=17000$

(ii) The number of multiplications $= 50\times10\times40+20\times50\times40=60000$

The number of additions $= 50\times9\times40+20\times49\times40=57200$

(b) (i) **R** = $\{$(2, 2), (3, 3), (4, 4), (5, 5), (6, 6), (7, 7), (8, 8),
(5, 2), (2, 5), (6, 3), (3, 6), (7, 4), (4, 7), (8, 5), (5, 8), (8, 2), (2, 8)$\}$

(ii) **Reflexive**
$\because x-x=0$ which is divisible by 3
$\therefore xRx$

Symmetric but not anti-symmetric
$xRy \Rightarrow 3$ divides $x-y$
$\Rightarrow 3$ divides $y-x$
$\Rightarrow yRx$

Transitive
aRb, bRc
$\Rightarrow (a-b)=3n$ and $(b-c)=3m$ where $n,m\in \mathrm{Z}$
$\therefore a-c=a-b+b-c=3n+3m=3(n+m)$
$\Rightarrow 3$ divides $a-c$
$\Rightarrow aRc$

4. (i) $A^2=\begin{pmatrix}6&3&4\\3&5&3\\4&3&3\end{pmatrix}$, $A^3=\begin{pmatrix}16&16&13\\16&9&11\\13&11&10\end{pmatrix}$

(ii) The number of walks of length 2 from v_1 to v_3 is 4.

The number of walks of length 3 from v_1 to v_3 is 13.

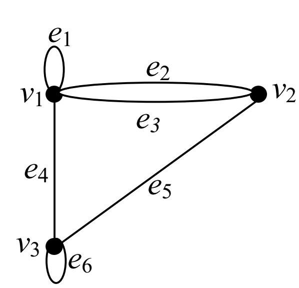
(iii) The walks of length 2 from v_1 to v_3 are:

$v_1\, e_1\, v_1\, e_4\, v_3$

$v_1\, e_2\, v_2\, e_5\, v_3$

$v_1\, e_3\, v_2\, e_5\, v_3$

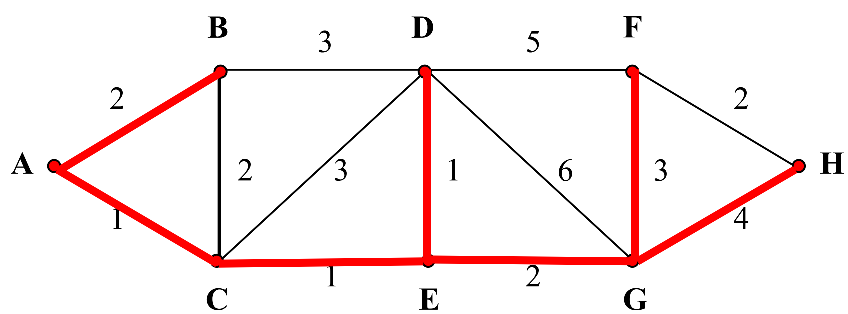
$v_1\, e_4\, v_3\, e_6\, v_3$



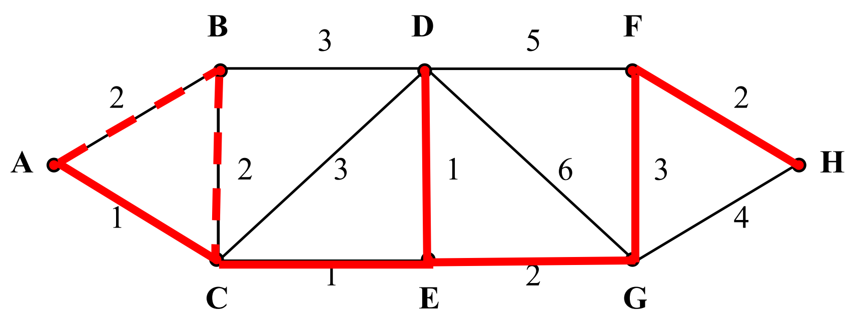
3. (i) The shortest path from A to H

Iteration ***n***	**Solved Nodes Directly Connected to Unsolved Nodes**	**Closest Connected Unsolved Node**	**Total Distance Involved**	***n*th Nearest Node**	**Minimum Distance**	**Last Connection**
1	A	C	1	C	1	AC
2	A C	B E	2 2	B E	2 2	AB CE
3	B C E	D D D	5 4 3	D	3	ED
4	D E	F G	8 4	G	4	EG
5	D G	F F	8 7	F	7	GF
6	F G	H H	9 8	H	8	GH

Graphical representation of the shortest path from A to H and the other points:



(ii) The minimum spanning tree joining the nodes:



Therefore the total minimum distance is 12.