



Interfaces with Other Disciplines

A game-theoretic formulation of joint implementation of environmental projects

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Abstract

The aim of this paper is to provide a game-theoretic interpretation of joint implementation in environmental projects and to assess the merit of such a strategy. More specifically, we consider a two-player game and solve it under three different cases. In the first case, countries play a non-cooperative game and optimize their welfare under an environmental constraint without having access to joint implementation. In the second case, we assume countries do have access to *JI*, which allows us to assess its merits by comparing the players' welfare levels achieved with and without *JI*. In the last case, the players jointly optimize their welfare under a collective environmental constraint. Comparing welfare levels in this case to those in the second case allows us to assess the merits of cooperation.

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1. Introduction

In December 1997, representatives from both developed and transition countries (jointly labelled Annex 1 Parties) met in Kyoto where they agreed on overall Greenhouse Gases (GHG) emission reductions. Under the terms of the agreement, each country committed to at least a 5 per cent reduction in GHG emissions relative to its 1990 levels for the 2008–2012 period. In order to reach their respective targets, countries are allowed to use three different mechanisms, namely *Joint Implementation (JI)*, *Clean Development Mechanism (CDM)* and *Emissions Trading (ET)*. The idea behind the *JI* mechanism, which is the focus of this

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paper, is that it allows countries with high abatement costs to reach their targets by investing in countries where the abatement costs are low. This mechanism is cost effective in the sense that it allows pollution reductions to be achieved at a lower cost. More specifically, Article 6.1 of the Protocol (see UNFCCC, 1997) states that “for the purpose of meeting its commitments under Article 3, any Party included in Annex 1 may transfer to, or acquire from, any other such Party emission reduction units (ERU) resulting from projects aimed at reducing anthropogenic emissions by sources or enhancing anthropogenic removals by sinks of greenhouse gases in any sector of the economy...”. This article essentially allows countries to invest in projects based abroad for the purposes of collecting rewards in the form of ERU.¹

The JI mechanism was formulated in 1991 and introduced at the Earth Summit 1992 in Rio. Later, at the First Conference of the Parties in Berlin (1995), this mechanism was operationalized (as a test pilot for 1995–1999) as an *Activity Jointly Implemented* (AJI) but without the use of credits (Parson and Fisher-Vanden, 1999). The AJI accounted for both the actual JI and CDM. All participating Parties were indeed in a stand-by state because no emissions reductions were credited.

It is worth mentioning some of the by-products of the JI mechanism under the Kyoto Protocol. First, the joint implementation of environmental projects provides additional incentives for research into, and the development of, environmentally friendly technologies. Furthermore, it promotes sustainable economic growth in transition countries via both physical and financial-capital flows.

It is also worth noting that JI is a form of emissions trading (ET). The latter is used to *trade* emissions on a governmental basis while the former is used by private and public entities to *produce* ERU. Moreover, JI is a type of baseline-and-credit system while ET is a cap-and-trade system (Janssen, 2000). Comparing these two mechanisms, Woerdman (2000) argues that JI is more efficient, effective and politically accepted than is ET.

Although contributions by game theorists to the environmental field have been numerous during the past few years, there has been little work done to assess the effectiveness and efficiency of Kyoto's tools. Joint Implementation has been studied but from a single-player perspective whereby the player attempts to maximize his welfare function subject to specific constraints. Furthermore, the literature dealing with joint implementation of environmental projects is focused mainly on the stability of the agreement and not on the modelling of the mechanism itself (see e.g., Janssen, 1999). Lee et al. (1997) provide a game-theoretic interpretation of JI in terms of strategic behaviour. Others have described the mechanism in a non-technical way (see, e.g., Michaelowa, 1995, 1996, 1998; Dudek and Wiener, 1996).

The aim of this paper is to provide a game-theoretic interpretation of joint implementation in environmental projects and to assess the merit of such a strategy. More specifically, we consider a two-player game and solve it under three different modes of play. In the first case, countries play a non-cooperative game and optimize their welfare under an environmental constraint without having access to joint implementation. In the second case, we assume countries do have access to JI, which allows us to assess the merits of JI by comparing the players' welfare levels achieved with and without JI. In the final case, the players jointly optimize their welfare under a collective environmental constraint. By comparing the welfare levels achieved in this last case with those of the previous one we can assess the merits of cooperation, which is an extreme case of a bubble system in the sense that the players also jointly maximize their performance functions.

The paper is organized as follows. In Section 2 we introduce a simple model of pollution control and define the three cases mentioned above. In Section 3 we solve the autarky² non-cooperative game, in Sec-

¹ JI had many precursor mechanisms. For example, in the offset mechanism (also known as external emissions trade), new emissions sources can begin operating only if they offset the new emissions with other existing sources (see, e.g., Grubb, 1992; Heintz and Tol, 1995; Koutstaal, 1997; Janssen, 2000). Another example is the so-called bubble mechanism whereby a set of countries collectively reduce their emissions by creating an *umbrella group*, known as a bubble, instead of doing so individually. In technical terms, this means that instead of having an emission constraint for each country, there is a global constraint for a set of countries. This obviously leaves greater flexibility for reaching the global emissions reduction.

² A dictionary definition of autarky is self-sufficiency, independence.

tion 4 the joint implementation game and in Section 5 we characterize the cooperative game solution. Section 6 provides some comparisons and Section 7 concludes.

2. Model and scenarios

We present a model of pollution control in a two-country setting. To maintain our emphasis on our pursued objective, that is, the game formulation and interpretation of joint implementation, we have purposely kept simple the economics of the pollution model. Furthermore, we wish to parsimoniously capture two of the most important elements that characterize global environmental problems, namely, that emissions by a given country have negative effects on others and that the players are asymmetric with respect to damage cost and investments in emissions reduction.

Let $N = \{1, 2\}$ be the set of players (countries) involved in the effort to control pollution. Let e_i denote the (gross) emissions resulting from the industrial production of Player i . We assume that the by-product gross emissions are proportional to the industrial output. This assumption allows us to express revenue from the industrial output as a function of emissions. Country i 's revenue function is assumed to be concave and increasing, with the following simple functional form:

$$e_i \left(b_i - \frac{1}{2} e_i \right), \quad 0 \leq e_i \leq b_i,$$

where b_i is a given parameter. It is important to note that this functional form implicitly assumes that the concavity parameter in the revenue function is identical for both countries.³

Emissions can be reduced by investing in environmental projects (e.g., installing air filters, cleaning a river basin) at home or abroad. Let I_{ij} denote the investment in an environmental project conducted by country i (foreign investor) in country j (host country). Henceforth, if index i represents a given player, index j denotes the other player, that is $j \equiv 3 - i$, so that I_{ii} represents Player i 's investment at home whereby I_{ij} represents Player i 's abroad. Let the investment cost be convex and increasing. We further assume that the host country has a first-choice option in choosing the available project. Fig. 1 represents how the investment costs are divided between both investors in host country j . Assuming a quadratic cost function yields for Player i

$$\frac{1}{2} (I_{ii}^2 + (I_{jj} + I_{ij})^2 - I_{jj}^2).$$

For the investor, the benefit of this investment lies in so-called *emissions reduction units*, which are assumed to be linear in investment, that is, $\gamma_j I_{ij}$, where $\gamma_j > 0$.⁴ From the outset, it is important to distinguish between a country's actual emissions (physical or local "net" pollution) and this country's emissions "account". The actual or net emissions in country i , denoted R_i , are given by

$$R_i = e_i - \gamma_i I_{ii} - \gamma_i I_{ji},$$

that is, gross emissions minus the reduction resulting from local and foreign investments in the same country. On the other hand, accounted-for-emissions, denoted A_i , are given by

$$A_i = e_i - \gamma_i I_{ii} - \gamma_j I_{ij},$$

³ This assumption does not change the results qualitatively but does simplify the expression of the solutions. Specifically, Propositions 2, 3, 6 and 7 still hold true when different concavities are assumed.

⁴ We have chosen to let the reduction depend on the conditions prevailing in the host country. Another possibility would have been to let this reduction depend on the investor's technology (and hence write that the reduction is given by $\gamma_i I_{ij}$) or on both players' conditions, by defining the reduction as $\gamma_{ij} I_{ij}$.

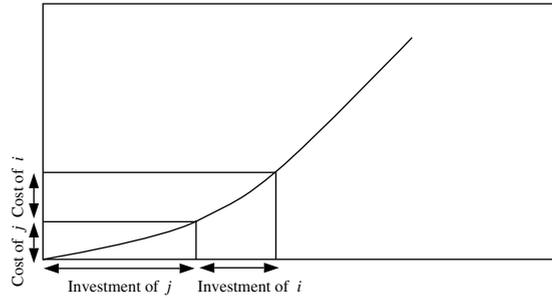


Fig. 1. Representation of a quadratic investment cost function.

that is, emissions minus the reduction resulting from investments at home and the emissions credits obtained from investments (or joint implementation) abroad.

Player i is faced with an *environmental constraint* which states that the country must keep its accounted-for-emissions below or equal to a certain prescribed level $E_i > 0$, that is

$$A_i \leq E_i. \tag{1}$$

To capture the externality associated with pollution, the damage cost incurred by each player is assumed to depend on the total emissions that is, by the emissions of both countries. We assume this cost to be linear, given by $d_i(\sum_{k=1}^2 R_k)$, $i = 1, 2$.⁵

We suppose that Player i maximizes her welfare function, that is, revenue minus investment and damage costs, that is

$$W_i = \left[e_i \left(b_i - \frac{1}{2} e_i \right) - \frac{1}{2} I_{ii}^2 - \frac{1}{2} I_{ij}^2 - I_{ij} I_{jj} - d_i \sum_{k=1}^2 R_k \right]. \tag{2}$$

Next we study three cases sequentially:

- *Autarky*: In this case, each player invests exclusively in local environmental projects. Thus, given the other player's e_j and I_{jj} , Player i optimizes her welfare function subject to the environmental and non-negativity constraints, that is

$$\begin{aligned} \max_{\substack{0 \leq e_i \leq b_i \\ I_{ii} \geq 0}} W_i^a &= \left[e_i \left(b_i - \frac{1}{2} e_i \right) - \frac{1}{2} I_{ii}^2 - d_i \left(\sum_{k=1}^2 e_k - \gamma_k I_{kk} \right) \right] \\ \text{s.t.} \quad & 0 \leq e_i - \gamma_i I_{ii} \leq E_i. \end{aligned} \tag{3}$$

Notice that the interaction between the players appears only in the damage cost. A Nash equilibrium is solved for, where resulting values are superscripted with an a for *autarky*.

- *Joint implementation*: In this case players are free to invest at home and abroad. Player i 's optimisation problem, given actions e_j , I_{jj} and I_{ji} by the other player, is given by

⁵ It has often been argued that the damage cost is non-linear in pollution. As a result, our linearity assumption must be seen as a rough approximation of some more realistic non-linear damage function.

$$\begin{aligned}
 \max_{\substack{0 \leq e_i \leq b_i \\ I_{ii}, I_{ij} \geq 0}} \quad & W_i^n = \left[e_i \left(b_i - \frac{1}{2} e_i \right) - \left(\frac{1}{2} I_{ii}^2 + \frac{1}{2} I_{ij}^2 + I_{ij} I_{jj} \right) - d_i \sum_{k=1}^2 R_k \right] \\
 \text{s.t.} \quad & e_i - \gamma_i I_{ii} - \gamma_j I_{ij} \leq E_i, \\
 & R_i = e_i - \gamma_i (I_{ii} + I_{ji}) \geq 0, \\
 & R_j = e_j - \gamma_j (I_{jj} + I_{ij}) \geq 0.
 \end{aligned} \tag{4}$$

Under **II**, players do not coordinate their emissions and investment strategies. We again solve for the Nash equilibrium where resulting values are superscripted with an *n* for *non-cooperative joint implementation*. Note that each optimisation problem is constrained by all players' net emissions non-negativity constraints ($R_i \geq 0, i = 1, 2$). This requirement is due to the fact that Player *j*'s constraints also involve Player *i*'s decision variables and hence affects the feasibility of Player *i*'s actions.

- *Cooperative game*: In this last case the two countries play a cooperative game and jointly optimize their welfare. The assumption here is that they pool their environmental constraints as if they were one single player. The optimisation problem thus reads as

$$\begin{aligned}
 \max_{\substack{0 \leq e_i \leq b_i \\ I \geq 0}} \quad & W^c = \left(e_1 \left(b_1 - \frac{1}{2} e_1 \right) + e_2 \left(b_2 - \frac{1}{2} e_2 \right) - \frac{1}{2} (I_{11} + I_{21})^2 \right. \\
 & \left. - \frac{1}{2} (I_{22} + I_{12})^2 - (d_1 + d_2)(R_1 + R_2) \right) \\
 \text{s.t.} \quad & R_1 = e_1 - \gamma_1 (I_{11} + I_{21}) \geq 0, \\
 & R_2 = e_2 - \gamma_2 (I_{22} + I_{12}) \geq 0, \\
 & R_1 + R_2 \leq E_1 + E_2,
 \end{aligned} \tag{5}$$

where $e \equiv (e_1, e_2)$ and $I \equiv (I_{11}, I_{12}, I_{21}, I_{22})$. The resulting optimal values are superscripted with a *c* for *cooperative game*.

The differences, in terms of strategies and welfares, between the first two cases will allow us to assess the value of joint implementation under the same (non-cooperative) type of play. The difference between the last two provides a measure of the value of the full coordination of strategies in addition to joint implementation (or foreign investment).

Before characterizing the solution in each case, we make the following remarks with respect to the cases discussed above.

Remark 1. It is obvious that in the autarky game we have $A_i = R_i$.

Remark 2. In the cooperative game, the investment decisions are the sum $I_{ii} + I_{ji}$, so that although the players are jointly implementing environmental projects, joint implementation as it is understood in a Kyoto-like setting, is not really an issue.

Remark 3. In all games, we assume that the net emissions R_i are non-negative. We do not, however, impose such restriction on accounted-for-emissions A_i .

Remark 4. In order to interpret E_i in a Kyoto-Protocol type context, we could set $E_i = (1 - \tau_i) e_i^{1990}$, where e_i^{1990} corresponds to emissions in reference year (1990) and τ_i is the reduction rate for country *i*. These emissions could be obtained, for instance, by assuming that in 1990, country *i* solves the optimisation problem in (2) without investing to reduce emissions.

3. The autarky equilibrium

Recall that in the autarky case the players act independently from one another and invest in reducing pollution exclusively at the local level. The following proposition characterizes the autarky Nash equilibrium.

Proposition 1. *The unique autarky Nash equilibrium is given by*

$$(e_i^a, I_{ii}^a) = \begin{cases} \left(\frac{\gamma_i^2 b_i}{1+\gamma_i^2}, \frac{\gamma_i b_i}{1+\gamma_i^2} \right) & \text{if } b_i - d_i(1 + \gamma_i^2) < 0, \\ (b_i - d_i, \gamma_i d_i) & \text{if } 0 \leq b_i - d_i(1 + \gamma_i^2) \leq E_i, \quad i = 1, 2. \\ \left(\frac{E_i + \gamma_i^2 b_i}{1+\gamma_i^2}, \frac{\gamma_i(b_i - E_i)}{1+\gamma_i^2} \right) & \text{if } b_i - d_i(1 + \gamma_i^2) > E_i, \end{cases}$$

Proof. To determine a Nash equilibrium, we write the Lagrangian of Player i 's optimisation problem as given in (3)

$$\mathcal{L}_i^a(e_i, I_{ii}, \beta_i, \eta_i) = e_i \left(b_i - \frac{1}{2} e_i \right) - \frac{1}{2} I_{ii}^2 - d_i \sum_{k=1}^2 (e_k - \gamma_k I_{kk}) + \beta_i (E_i - (e_i - \gamma_i I_{ii})) + \eta_i (e_i - \gamma_i I_{ii}),$$

where β_i and η_i are Lagrange multipliers associated respectively with the environmental constraint and the non-negativity constraint.

Assuming an interior solution, necessary Nash equilibrium conditions are

$$\frac{\partial \mathcal{L}_i^a}{\partial e_i} = 0 \Rightarrow e_i = b_i - d_i - \beta_i + \eta_i, \quad i = 1, 2, \tag{6}$$

$$\frac{\partial \mathcal{L}_i^a}{\partial I_{ii}} = 0 \Rightarrow I_{ii} = \gamma_i (d_i + \beta_i - \eta_i), \quad i = 1, 2, \tag{7}$$

$$\eta_i \geq 0, \quad (e_i - \gamma_i I_{ii}) \geq 0, \quad \eta_i (e_i - \gamma_i I_{ii}) = 0, \quad i = 1, 2, \tag{8}$$

$$\beta_i \geq 0, \quad (E_i - (e_i - \gamma_i I_{ii})) \geq 0, \quad \beta_i (E_i - (e_i - \gamma_i I_{ii})) = 0, \quad i = 1, 2, \tag{9}$$

Notice that equilibrium conditions are separable by player (decisions of Player i are independent of decisions of Player j). To prove the proposition, we study the four possible combinations of active and non-active constraints for Player i . Recall that the solution is interior if $0 \leq e_i \leq b_i$ and $I_{ii} \geq 0$ and notice that $(e_i - \gamma_i I_{ii}) \geq 0$ implies $e_i \geq 0$.

- $\eta_i = 0$ and $\beta_i = 0$ implying $(e_i - \gamma_i I_{ii}) \geq 0, (E_i - (e_i - \gamma_i I_{ii})) \geq 0$ and using (6) and (7) leads to

$$e_i = b_i - d_i,$$

$$I_{ii} = \gamma_i d_i,$$

$$R_i = b_i - d_i(1 + \gamma_i^2),$$

which is interior and satisfies the constraints if

$$0 \leq b_i - d_i(1 + \gamma_i^2) \leq E_i.$$

- $\eta_i=0$ and $\beta_i>0$ implying $(E_i - (e_i - \gamma_i I_{ii}))=0$ and $(e_i - \gamma_i I_{ii}) \geq 0$ leads to

$$e_i = \frac{E_i + \gamma_i^2 b_i}{1 + \gamma_i^2},$$

$$I_{ii} = \frac{\gamma_i(b_i - E_i)}{1 + \gamma_i^2},$$

$$\beta_i = \frac{b_i - E_i}{(1 + \gamma_i^2)} - d_i,$$

$$R_i = E_i,$$

which is interior if $E_i \leq b_i$ and satisfies $\beta_i > 0$ if

$$E_i < b_i - d_i(1 + \gamma_i^2).$$

- $\eta_i > 0$ and $\beta_i = 0$ implying $(e_i - \gamma_i I_{ii})=0$ and $(E_i - (e_i - \gamma_i I_{ii})) \geq 0$ leads to

$$e_i = \frac{\gamma_i^2 b_i}{1 + \gamma_i^2},$$

$$I_{ii} = \frac{\gamma_i b_i}{1 + \gamma_i^2},$$

$$\eta_i = \frac{d_i(1 + \gamma_i^2) - b_i}{1 + \gamma_i^2},$$

$$R_i = 0,$$

which is interior and satisfies $\eta_i > 0$ if

$$b_i < d_i(1 + \gamma_i^2).$$

- $\eta_i > 0$ and $\beta_i > 0$ implying $(e_i - \gamma_i I_{ii})=0$ and $(E_i - (e_i - \gamma_i I_{ii}))=0$ is a trivial case only possible with $E_i=0$, a possibility we rule out in the model. \square

The proposition shows that a zero-investment strategy is not part of a Nash equilibrium. The unique Nash equilibrium depends on the values of the model's parameters. If $b_i - d_i(1 + \gamma_i^2) < 0$, then the equilibrium strategy corresponds to net zero-emissions and, obviously, Player i is over complying⁶ with the emissions constraint. If $0 \leq b_i - d_i(1 + \gamma_i^2) \leq E_i$, then net emissions are positive and Player i is again over complying with the emissions constraint. Finally, if $b_i - d_i(1 + \gamma_i^2) > E_i$, then the equilibrium strategy is to comply exactly with the emissions constraint.

4. Joint implementation

We now relax the assumption that each player can only invest in local environmental projects and allow for joint implementation in such projects. The assumption in this case is that the players do not coordinate their emissions and investment strategies but do have access to each other's country to invest in emissions reduction projects. Player i 's optimisation problem is then given by (4). Player i 's Lagrangian is given by

⁶ We use the term "overcomplying" when the environmental constraint is not binding. Another interpretation would be that the target imposed on a player by, e.g., the Kyoto Protocol is actually above what the country would nonetheless optimally emit.

$$\mathcal{L}_i^n(e_i, I_{ii}, I_{ij}, \eta_i, \beta_i) = e_i \left(b_i - \frac{1}{2} e_i \right) - \frac{1}{2} I_{ij}^2 - I_{ij} I_{jj} - \frac{1}{2} I_{ii}^2 - d_i \sum_{k=1}^2 R_k + \sum_{k=1}^2 \eta_{ik} R_k + \beta_i (E_i - A_i),$$

where β_i and $\eta_i = (\eta_{i1}, \eta_{i2})$ are Lagrange multipliers corresponding to the environmental and the non-negativity constraints, respectively. Fully characterizing the equilibrium solution amounts to studying all possible combinations of active and non-active constraints. We restrict the analysis to the case where non-negativity constraints on net emissions are not active for either player, that is, $\eta_{ii} = \eta_{ij} = \eta_{ji} = \eta_{jj} = 0$. We discuss the reason behind this restriction further on. Assuming an interior solution, necessary conditions for a Nash equilibrium are given by

$$\begin{aligned} \frac{\partial \mathcal{L}_i^n}{\partial e_i} = 0 &\Rightarrow e_i = b_i - d_i - \beta_i, \quad i = 1, 2, \\ \frac{\partial \mathcal{L}_i^n}{\partial I_{ii}} = 0 &\Rightarrow I_{ii} = \gamma_i (d_i + \beta_i), \quad i = 1, 2, \\ \frac{\partial \mathcal{L}_i^n}{\partial I_{ij}} = 0 &\Rightarrow I_{ij} = \gamma_j (d_i + \beta_i) - I_{jj}, \quad i = 1, 2, \\ (E_i - e_i + \gamma_i I_{ii} + \gamma_j I_{ij}) &\geq 0, \quad \beta_i \geq 0, \quad \beta_i (E_i - e_i + \gamma_i I_{ii} + \gamma_j I_{ij}) = 0, \quad i = 1, 2. \end{aligned} \tag{10}$$

Notice that $e_i \geq 0$ is implied by $R_i \geq 0$ and that necessary conditions imply $e_i < b_i$ and $I_{ii} > 0$. If the sign of I_{ij} is positive (which we verify below), then joint implementation will be used.

Proposition 2. *If a Nash equilibrium exists to the non-cooperative joint-implementation game and if $R_i^n > 0$, $i = 1, 2$, then $I_{ij}^n I_{ji}^n = 0$.*

Proof. Using equilibrium conditions to substitute for I_{ii}^n and I_{jj}^n , we obtain

$$\begin{aligned} \frac{\partial \mathcal{L}_i^n}{\partial I_{ij}} &= -I_{ij} - \gamma_j (-d_i - \beta_i + d_j + \beta_j), \\ \frac{\partial \mathcal{L}_j^n}{\partial I_{ji}} &= -I_{ji} + \gamma_i (-d_i - \beta_i + d_j + \beta_j). \end{aligned}$$

Suppose $-d_i - \beta_i + d_j + \beta_j > 0$, then $\frac{\partial \mathcal{L}_i^n}{\partial I_{ij}}$ is negative for $I_{ij} \geq 0$ and $I_{ij}^n = 0$, with $I_{ji}^n = \gamma_i (-d_i - \beta_i + d_j + \beta_j) > 0$.

Now suppose $-d_i - \beta_i + d_j + \beta_j < 0$, then $\frac{\partial \mathcal{L}_j^n}{\partial I_{ji}}$ is negative for $I_{ji} \geq 0$ and $I_{ji}^n = 0$, with $I_{ij}^n = -\gamma_j (-d_i - \beta_i + d_j + \beta_j) > 0$.

Finally notice that $-d_i - \beta_i + d_j + \beta_j = 0$ implies $I_{ij}^n = I_{ji}^n = 0$. \square

The result of the above proposition shows that one player at most invests in a joint-implementation endeavour at equilibrium. This result is somewhat expected given that the main purpose of JI is to channel investments to countries offering more efficient options for emissions reductions. The merit of the proposition is to show it rigorously in a non-cooperative setting where players do not coordinate their investment strategies. Furthermore, the proposition underlines the conditions determining which player is investing abroad. As will be shown below for the different possible cases, these conditions involve damage costs, investment efficiencies, revenue parameters, environmental targets as well as their shadow prices. One can conclude, therefore, that the selection of a JI project is a far more complex process than simply comparing the investment costs of the two countries involved.

Without loss of generality, suppose that Player 2 has the highest damage cost, $d_2 - d_1 = D > 0$. From equilibrium conditions, we can find all possible equilibriums for two players. We next list different cases for different parameter values, which give rise to all possible solutions when the *non-negativity constraints are not active*. These cases are presented in Fig. 2. The detailed results are provided in Appendix A.

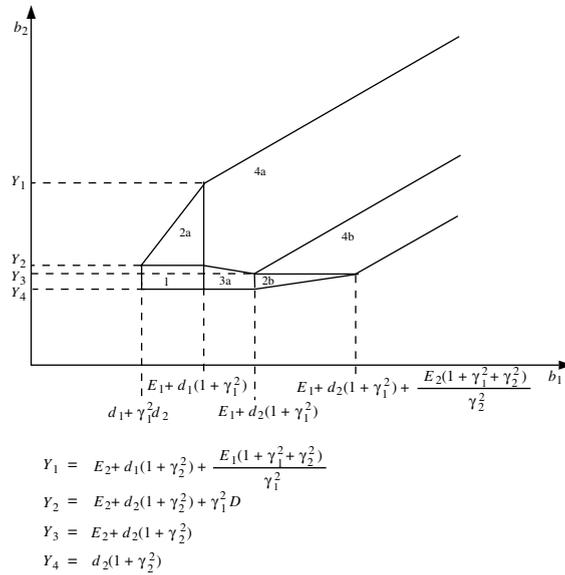


Fig. 2. Characterization of JI equilibrium.

1. Both players are over complying, that is, the environmental constraint is not binding (which implies $\beta_1 = \beta_2 = 0$).
- 2a. Player 1 is over complying and not investing abroad while Player 2 is complying ($\beta_1 = 0, \beta_2 \geq 0, I_{12} = 0$).
- 2b. The symmetric case in which Player 2 is over complying and not investing abroad while Player 1 is complying ($\beta_2 = 0, \beta_1 \geq 0, I_{21} = 0$).
- 3a. Player 1 is complying and not investing abroad while Player 2 is over complying ($\beta_2 = 0, \beta_1 \geq 0, I_{12} = 0$).
- 3b. Player 1 is over complying while Player 2 is complying and not investing abroad ($\beta_1 = 0, \beta_2 \geq 0, I_{21} = 0$).
- 4a. Both players are complying and country 1 is not investing abroad ($\beta_1 \geq 0, \beta_2 \geq 0, I_{12} = 0$).
- 4b. Both players are complying and country 2 is not investing abroad ($\beta_1 \geq 0, \beta_2 \geq 0, I_{21} = 0$).

Foreign investment strategies for the non-cooperative joint-implementation case can actually be synthesized as follows.

- If environmental constraints are not active (that is, $\beta_1 = \beta_2 = 0$) the player investing abroad is the one with the highest damage cost.
- If both environmental constraints are active, the player investing abroad is the player with the highest ratio $\frac{(b_i - E_i)}{(1 + \gamma_i^2)}$.
- If the environmental constraint is active for the player with the highest damage cost, then the player investing abroad is the one with the highest damage cost.
- If the environmental constraint is active for the player with the lowest damage cost (say Player 1), then the player with the highest damage cost (Player 2) invests abroad if her damage cost is higher than the ratio $\frac{b_1 - E_1}{(1 + \gamma_1^2)}$, otherwise Player 1 invests abroad.

As mentioned above, this characterization of equilibrium strategies is for the case where non-negativity constraints are not active. It is worth noticing that, outside the regions where this is true (showed in Fig. 2), the net emissions are zero for at least one player. Consequently, the same non-negativity constraints are active in both players' optimisation problems, which leads to an infinite number of equilibrium solutions. Recall that we are in a non-cooperative setting. As a result, it is not clear at all how the players, who can neither communicate nor coordinate their emissions and investment strategies, would select a particular equilibrium to implement. From a game-theoretic point of view, this remains an intriguing and yet unresolved methodological problem. However, note that this case is not the most empirically interesting case since it is unlikely that a country would drive its emissions level to zero.

5. Cooperative game

In this case, the two players agree to play a cooperative game, that is, they accept to coordinate their emissions and investment strategies and agree on a rule for sharing the total cooperative reward. In such a setting, one usually derives the optimal cooperative strategies through the joint optimisation of a weighted sum of the players' objectives, where a player's weight reflects her strategic force or power. We assume here that the players maximize the sum of their welfares subject to a joint environmental constraint. The joint optimisation problem is thus given by

$$\begin{aligned} \max_{\substack{0 \leq e \leq b \\ I \geq 0}} \quad & W^c = \sum_{i=1}^2 W_i^c = \left[\sum_{i=1}^2 e_i \left(b_i - \frac{1}{2} e_i \right) - \frac{1}{2} (I_{ii} + I_{ji})^2 - d_i (R_1 + R_2) \right] \\ \text{s.t.} \quad & R_1 = e_1 - \gamma_1 (I_{11} + I_{21}) \geq 0, \\ & R_2 = e_2 - \gamma_2 (I_{22} + I_{12}) \geq 0, \\ & A_1 + A_2 = R_1 + R_2 \leq E_1 + E_2. \end{aligned}$$

It is easy to see that this problem has redundant decision variables and can be solved by using the total investment in country i , $I_i \equiv I_{ii} + I_{ji}$, with the associated Lagrangian

$$\mathcal{L}^c(e, I, \eta, \beta) = \sum_{i=1}^2 e_i \left(b_i - \frac{1}{2} e_i \right) - \frac{1}{2} (I_i)^2 - d_i (R_1 + R_2) + \eta_i R_i + \beta (E_i - R_i),$$

where β and $\eta = (\eta_1, \eta_2)$ are Lagrange multipliers associated with the environmental and non-negativity constraints, respectively. The necessary and sufficient optimality conditions are given by

$$\begin{aligned} \frac{\partial \mathcal{L}^c}{\partial e_i} = 0 &\Rightarrow e_i = b_i - \beta + \eta_i - d_1 - d_2, \quad i = 1, 2, \\ \frac{\partial \mathcal{L}^c}{\partial I_i} = 0 &\Rightarrow I_i = \gamma_i (\beta - \eta_i + d_1 + d_2), \quad i = 1, 2, \\ R_i \geq 0, \quad \eta_i \geq 0, \quad \eta_i R_i = 0, \quad i = 1, 2, \\ \sum_{i=1}^2 (E_i - R_i) \geq 0, \quad \beta \geq 0, \quad \beta \sum_{i=1}^2 (E_i - R_i) = 0. \end{aligned} \tag{11}$$

Proposition 3. *The cooperative solution is such that $I_i^c > 0$, $i = 1, 2$.*

Proof. Rearranging (11) yields the following:

$$I_i = \gamma_i(d_1 + d_2 + \beta - \eta_i), \quad i = 1, 2,$$

$$R_i = b_i - (1 + \gamma_i^2)(d_1 + d_2 + \beta - \eta_i), \quad i = 1, 2,$$

where it is apparent that $\eta_i=0$ implies $I_i>0$. On the other hand, $R_i=0$ yields $\eta_i = \frac{-b_i}{1+\gamma_i^2} + d_1 + d_2 + \beta$ and $I_i = \gamma_i \frac{b_i}{1+\gamma_i^2} > 0$. \square

Proposition 4. *If the cooperative solution is such that $R_i^c > 0$, $i = 1, 2$, it then satisfies*

$$e_i^c = b_i - \beta^c - (d_1 + d_2), \quad i = 1, 2,$$

$$I_i^c = \gamma_i(\beta^c + d_1 + d_2), \quad i = 1, 2,$$

$$\beta^c = \left[\frac{-E + b_1 + b_2 - (2 + \gamma_1^2 + \gamma_2^2)(d_1 + d_2)}{2 + \gamma_1^2 + \gamma_2^2} \right]^+,$$

$$R_i^c = b_i - (1 + \gamma_i^2)(d_1 + d_2 + \beta^c) > 0.$$

Proof. The proof, along with the characterization of all other possible cases, is given in [Appendix B](#). \square

Fig. 3 presents the regions for the parameter values giving rise to all possible solutions.

The results of the above proposition may be interpreted as follows. Whenever the environmental constraint is not active ($\beta=0$), meaning that the players are collectively over complying with the environmental target, optimal emissions are such that the marginal revenue ($b_i - e_i$) is equal to the sum of marginal damage costs ($d_1 + d_2$). If the constraint is active, the relationship takes into account the shadow price of the environmental constraint (β).

Optimal investment is also determined by equating marginal investment cost (I_{ij}) to marginal benefit. The latter is measured by the reduction in the sum of damage costs when the environmental constraint is not active. As for emissions, the investment decisions incorporate the shadow price of the environmental constraint when it is active.

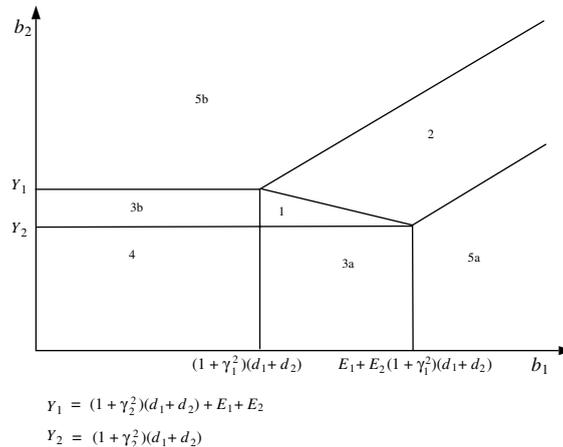


Fig. 3. Characterization of cooperative solution.

6. Comparison

In this section we compare the results obtained under the three cases in order to assess the “value” of joint implementation and cooperation relative to the autarky case. Although it is technically possible to compare all combinations of results, we focus on the most interesting case where

- net emissions are not completely eliminated ($R_i > 0, i = 1, 2$),
- the environmental constraint is individually active in the autarky case ($A_i^a = E_i$),
- the environmental constraint is active in the non-cooperative joint implementation case ($A_i^n = E_i$),
- the environmental constraint is collectively active in the cooperative game ($\sum R_i^c = E_i + E_j$).

We focus on this case because it appears to us to be the most realistic. The intersection of the conditions satisfied by the parameters in that case yields the following:

$$b_1 + b_2 > E_1 + E_2 + (2 + \gamma_1^2 + \gamma_2^2)(d_1 + d_2),$$

$$-(E_1 + E_2)(1 + \gamma_1^2) < b_1(1 + \gamma_2^2) - b_2(1 + \gamma_1^2) < (E_1 + E_2)(1 + \gamma_2^2)$$

and either ($I_{12}^n = 0$)

$$b_2(1 + \gamma_2^2) - b_1(1 + \gamma_1^2) \leq E_1(1 + \gamma_2^2) - E_2(1 + \gamma_1^2)$$

or ($I_{21}^n = 0$)

$$b_2(1 + \gamma_1^2) - b_1(1 + \gamma_2^2) \leq E_2(1 + \gamma_1^2) - E_1(1 + \gamma_2^2).$$

Recall that when both environmental constraints are active, the player investing abroad has the highest ratio $(b_i - E_i)/(1 + \gamma_i^2)$. Since cooperative and autarky solutions are symmetric, we assume, without loss of generality, that Player 2 is the one investing abroad in the non-cooperative case. We, however, no longer suppose that she has the highest damage cost. Denote $(b_i - E_i) = B_i$ and $F = (1 + \gamma_1^2)B_2 - (1 + \gamma_2^2)B_1 > 0$, we have the following results:

Proposition 5. *Gross emissions and net emissions compare as follows:*

$$e_1^a = e_1^n > e_1^c \quad e_2^a < e_2^n < e_2^c,$$

$$R_1^n = E_1 - \frac{\gamma_1^2 F}{(1 + \gamma_1^2)(1 + \gamma_1^2 + \gamma_2^2)} < R_1^a = R_1^c = E_1,$$

$$R_2^a = R_2^c = E_2 < R_2^n = E_2 + \frac{\gamma_1^2 F}{(1 + \gamma_1^2)(1 + \gamma_1^2 + \gamma_2^2)}.$$

Proof. Straightforward from the optimality conditions of the different cases. □

The above proposition shows that the levels of net emissions in the two countries are the same in autarky and in full cooperation. This is to be expected since we are concentrating on the case where the environmental constraints are active in both countries. The proposition also shows that Player 1 (2)’s net emissions are lower (higher) in the non-cooperative joint implementation equilibrium. This result is a by-product of the assumption that Player 2 is the one investing abroad. It is easy to verify that the non-cooperative and cooperative joint implementation investment strategies are related as follows:

$$I_{11}^n + (I_{12}^n = 0) < I_{11}^c, \quad I_{22}^n + I_{21}^n > I_{22}^c,$$

showing that Player 1 invests less in the non-cooperative game than in the cooperative one. The result is reversed for Player 2.

We next move to comparing welfares across the different cases. The following proposition shows that one player is indifferent between autarky and non-cooperative joint implementation, while the second prefers the latter case. Clearly, the player who invests abroad is getting the total dividend of joint implementation.

Proposition 6. *Non-cooperative joint implementation is weakly Pareto-improving with respect to autarky, that is, $W_1^n = W_1^a$ and $W_2^n > W_2^a$.*

Proof. For Player 1 the result is obvious since $e_1^a = e_1^n$, $I_{11}^a = I_{11}^n$, $I_{12}^a = I_{12}^n = 0$ and $R_1^a + R_2^a = R_1^n + R_2^n$. For Player 2, it suffices to compute

$$W_2^n - W_2^a = \frac{\gamma_1^2 F^2}{2(1 + \gamma_2^2)(1 + \gamma_1^2 + \gamma_2^2)(1 + \gamma_1^2)^2} > 0. \quad \square$$

Comparing now full cooperation to non-cooperative joint implementation, we get the following result.

Proposition 7

- (i) *Total cooperative welfare is higher than total non-cooperative joint implementation welfare.*
- (ii) *Player 2 increases her welfare under cooperation with respect to non-cooperative joint implementation. The reverse holds true for Player 1.*

Proof

- (i) Compute the difference between total welfares to get

$$W^c - W^n = \frac{1}{2} \frac{F^2}{(2 + \gamma_1^2 + \gamma_2^2)(1 + \gamma_1^2 + \gamma_2^2)(1 + \gamma_1^2)^2} > 0.$$

- (ii) It is easy to check that Player 1 (2)'s welfare is lower (higher) in cooperation than under non-cooperative joint implementation. Indeed,

$$W_1^c - W_1^n = -\frac{1}{2} F \frac{2B_1\gamma_1^2 + 2B_1\gamma_2^2 + 4B_1 + F}{(2 + \gamma_1^2 + \gamma_2^2)^2(1 + \gamma_1^2)} < 0,$$

$$W_2^c - W_2^n = \frac{1}{2} F \left(\frac{2B_1}{(2 + \gamma_1^2 + \gamma_2^2)(1 + \gamma_1^2)} + \frac{(\gamma_2^2\gamma_1^2 + 2\gamma_2^2 + 3\gamma_1^2 + 3 + \gamma_1^4)}{(2 + \gamma_1^2 + \gamma_2^2)^2(1 + \gamma_1^2 + \gamma_2^2)(1 + \gamma_1^2)^2} \right) > 0. \quad \square$$

The result in item (i) is not surprising since joint optimisation provides a welfare level which is at least as high as the one obtained in a decentralized optimisation. This result is consistent with much of the existing environmental literature (see, e.g., van der Ploeg and de Zeeuw, 1992; Dockner and Van Long, 1993; Kaitala and Pohjola, 1995; Jørgensen and Zaccour, 2001; and Petrosjan and Zaccour, 2003). The consequence of item (ii) is that unless the players agree on a side payment scheme (from Player 2 to Player 1), the cooperative emissions and investment strategies will not be accepted by Player 1 and will not, therefore, be implemented.

The total cooperative welfare could be shared by invoking, for instance, an egalitarian principle that allocates to each player his non-cooperative welfare and half of the cooperative dividend, that is

$$W_i^n + \frac{1}{2}[(W_i^c + W_j^c) - (W_i^n + W_j^n)], \quad i, j = 1, 2; \quad i \neq j$$

which is obviously Pareto-improving.

This comparison was done for the particular (but probably most interesting) setting where the players comply with their environmental targets yet cannot totally eliminate pollution. One can follow the same steps and compute the differences in strategies and welfares for the other possible solutions identified in this paper. Note however that the regions where the different cases intersect give rise to many possibilities.

7. Conclusion

The aim of this paper was to formalize in a game-theoretic setting the so-called joint implementation mechanism; a mechanism that aims to make it easier for high damage or abatement cost countries to reach their environmental targets. Our results are consistent with some intuitive and expected outcomes of joint implementation. Our setting, which was deliberately kept simple in order to keep the focus on the game-theoretic methodological issues, can be extended in several ways.

- The consideration of more than two countries is clearly of interest. It would probably give rise to a solution where a given country could be at once host to a foreign investment, by a country with a higher damage cost, and itself an investor in a foreign country offering interesting possibilities to help it reach its target.
- Joint implementation projects are intimately related to the technological capabilities of the players. Extending the model to accommodate for multiple technologies is definitely worthy if investigation.
- A case in which players pool their environmental constraints without jointly optimizing their objective is methodologically challenging because it requires the elaboration of a solution concept that accommodates a situation that is half-cooperative and half-not.

Appendix A. Characterization of joint implementation equilibrium

From equilibrium conditions, we study all possible combinations of active and non-active constraints. In each case, a unique equilibrium solution is found and the region for the parameter values where it applies is characterized. These regions are mutually exclusive and collectively exhaustive.

1. Both players are over complying, that is, $\beta_1 = \beta_2 = 0$. In that case, the player investing abroad is the one with the highest damage cost (Player 2). Using the equilibrium necessary conditions, we get

$$e_1 = b_1 - d_1 \quad e_2 = b_2 - d_2,$$

$$I_{11} = \gamma_1 d_1 \quad I_{12} = 0,$$

$$I_{21} = \gamma_1(d_2 - d_1) \quad I_{22} = \gamma_2 d_2,$$

$$A_1 = b_1 - d_1(1 + \gamma_1^2) \quad A_2 = b_2 - d_2(1 + \gamma_1^2 + \gamma_2^2) + \gamma_1^2 d_1,$$

$$R_1 = b_1 - d_1 - \gamma_1^2 d_2 \quad R_2 = b_2 - d_2(1 + \gamma_2^2).$$

This solution satisfies the constraints if

$$\begin{aligned}
 I_{21} &\geq 0 : \gamma_1 D \geq 0 \text{ (redundant)}, \\
 A_1 &\leq E_1 : b_1 \leq E_1 + d_1(1 + \gamma_1^2), \\
 A_2 &\leq E_2 : b_2 \leq E_2 + \gamma_1^2 D + d_2(1 + \gamma_2^2), \\
 R_1 &\geq 0 : b_1 \geq d_1 + \gamma_1^2 d_2, \\
 R_2 &\geq 0 : b_2 \geq d_2(1 + \gamma_2^2).
 \end{aligned}$$

Notice that for this solution to be possible, we must have $E_1 \geq \gamma_1^2 D$.

2a. Player 1 is over complying and not investing abroad and Player 2 is complying, that is, $\beta_1 = 0, \beta_2 \geq 0, I_{12} = 0$. Using the equilibrium necessary conditions, we get

$$\begin{aligned}
 e_1 &= b_1 - d_1 \quad e_2 = \frac{b_2(\gamma_2^2 + \gamma_1^2) - d_1\gamma_1^2 + E_2}{1 + \gamma_1^2 + \gamma_2^2}, \\
 I_{11} &= \gamma_1 d_1 \quad I_{12} = 0, \\
 I_{22} &= \gamma_2 \frac{b_2 - E_2 + \gamma_1^2 d_1}{1 + \gamma_1^2 + \gamma_2^2} \quad I_{21} = \gamma_1 \frac{b_2 - E_2 - d_1(1 + \gamma_2^2)}{1 + \gamma_1^2 + \gamma_2^2}, \\
 A_1 &= b_1 - d_1(1 + \gamma_1^2) \quad \beta_2 = \frac{b_2 - E_2 - d_2(1 + \gamma_1^2 + \gamma_2^2) + \gamma_1^2 d_1}{1 + \gamma_1^2 + \gamma_2^2}, \\
 R_1 &= b_1 - \frac{d_1(1 + \gamma_1^2 + \gamma_2^2 + \gamma_1^4) + \gamma_1^2(b_2 - E_2)}{1 + \gamma_1^2 + \gamma_2^2} \quad R_2 = \frac{E_2(1 + \gamma_2^2) - \gamma_1^2(d_1(1 + \gamma_2^2) - b_2)}{1 + \gamma_1^2 + \gamma_2^2}.
 \end{aligned}$$

This solution satisfies the constraints if

$$\begin{aligned}
 I_{21} &\geq 0 : b_2 \geq d_1(1 + \gamma_2^2) + E_2 \text{ (redundant)}, \\
 \beta_2 &\geq 0 : b_2 \geq d_2(1 + \gamma_2^2) + \gamma_1^2 D + E_2, \\
 A_1 &\leq E_1 : b_1 \leq d_1(\gamma_1^2 + 1) + E_1, \\
 R_1 &\geq 0 : d_1(1 + \gamma_1^2 + \gamma_2^2 + \gamma_1^4) + \gamma_1^2(b_2 - E_2) \leq b_1(1 + \gamma_1^2 + \gamma_2^2), \\
 R_2 &\geq 0 : b_2 \geq \frac{(1 + \gamma_2^2)(\gamma_1^2 d_1 - E_2)}{\gamma_1^2} \text{ (redundant)}.
 \end{aligned}$$

2b. The symmetric case, that is, Player 2 is over complying and not investing abroad and Player 1 is complying ($\beta_2 = 0, \beta_1 > 0, I_{21} = 0$) requires the following conditions on the parameter values:

$$\begin{aligned}
 I_{12} &\geq 0 : b_1 \geq d_2(1 + \gamma_1^2) + E_1, \\
 \beta_1 &> 0 : b_1 > d_1(1 + \gamma_1^2) - \gamma_2^2 D + E_1 \text{ (redundant)}, \\
 A_2 &\leq E_2 : b_2 \leq d_2(1 + \gamma_2^2) + E_2, \\
 R_1 &\geq 0 : b_1 \geq \frac{(1 + \gamma_1^2)(\gamma_2^2 d_2 - E_1)}{\gamma_2^2} \text{ (redundant)}, \\
 R_2 &\geq 0 : d_2(1 + \gamma_1^2 + \gamma_2^2 + \gamma_2^4) + \gamma_2^2(b_1 - E_1) \leq b_2(1 + \gamma_1^2 + \gamma_2^2).
 \end{aligned}$$

- 3a. Player 1 is complying and not investing abroad and Player 2 is over complying, that is, $\beta_2=0$, $\beta_1 \geq 0$, $I_{12}=0$ yields

$$e_1 = \frac{E_1 + \gamma_1^2 b_1}{1 + \gamma_1^2} \quad e_2 = b_2 - d_2,$$

$$I_{11} = \gamma_1 \frac{b_1 - E_1}{1 + \gamma_1^2} \quad I_{12} = 0,$$

$$I_{22} = \gamma_2 d_2 \quad I_{21} = \gamma_1 \frac{E_1 - b_1 + d_2(1 + \gamma_1^2)}{1 + \gamma_1^2},$$

$$\beta_1 = \frac{b_1 - E_1 - d_1(1 + \gamma_1^2)}{1 + \gamma_1^2} \quad A_2 = b_2 - d_2(1 + \gamma_1^2 + \gamma_2^2) + \frac{\gamma_1^2(b_1 - E_1)}{1 + \gamma_1^2},$$

$$R_1 = \frac{E_1 - \gamma_1^2(d_2(1 + \gamma_1^2) - b_1)}{1 + \gamma_1^2} \quad R_2 = b_2 - d_2(1 + \gamma_2^2) \geq 0.$$

This solution satisfies the constraints if

$$I_{21} \geq 0 : b_1 \leq E_1 + d_2(1 + \gamma_1^2),$$

$$\beta_1 \geq 0 : b_1 \geq E_1 + d_1(1 + \gamma_1^2),$$

$$A_2 \leq E_2 : b_2 \leq E_2 + d_2(1 + \gamma_1^2 + \gamma_2^2) - \frac{\gamma_1^2(b_1 - E_1)}{1 + \gamma_1^2},$$

$$R_1 \geq 0 : b_1 \geq d_2(1 + \gamma_1^2) - \frac{E_1}{\gamma_1^2} \text{ (redundant),}$$

$$R_2 \geq 0 : b_2 \geq d_2(1 + \gamma_2^2).$$

- 3b. Player 1 is over complying and Player 2 is complying and not investing abroad, that is $\beta_1=0$, $\beta_2 \geq 0$, $I_{21}=0$. As shown in Proposition 2, this leads to a contradiction ($d_2 - d_1 + \beta_2 - \beta_1 > 0$ implies $I_{21} > 0$).
- 4a. Both players are over complying and Player 1 is not investing abroad, that is, $\beta_1 \geq 0$, $\beta_2 \geq 0$, $I_{12}=0$, yields

$$e_1 = \frac{E_1 + \gamma_1^2 b_1}{1 + \gamma_1^2},$$

$$I_{11} = \gamma_1 \frac{-E_1 + b_1}{1 + \gamma_1^2},$$

$$I_{22} = \gamma_2 \frac{\gamma_1^2(b_1 - E_1) + (1 + \gamma_1^2)(b_2 - E_2)}{(1 + \gamma_1^2)(1 + \gamma_1^2 + \gamma_2^2)},$$

$$\beta_1 = \frac{b_1 - E_1 - d_1(1 + \gamma_1^2)}{1 + \gamma_1^2},$$

$$R_1 = \frac{E_1(1 + \gamma_1^2 + \gamma_1^4 + \gamma_2^2) + \gamma_1^2 b_1(1 + \gamma_2^2) - \gamma_1^2(1 + \gamma_1^2)(b_2 - E_2)}{(1 + \gamma_1^2)(1 + \gamma_1^2 + \gamma_2^2)},$$

$$e_2 = \frac{b_2(1 + \gamma_1^2)(\gamma_1^2 + \gamma_2^2) - \gamma_1^2(b_1 - E_1) + E_2(1 + \gamma_1^2)}{(1 + \gamma_1^2)(1 + \gamma_1^2 + \gamma_2^2)},$$

$$I_{12} = 0,$$

$$I_{21} = \gamma_1 \frac{(1 + \gamma_1^2)(b_2 - E_2) - (1 + \gamma_2^2)(b_1 - E_1)}{(1 + \gamma_1^2)(1 + \gamma_1^2 + \gamma_2^2)},$$

$$\beta_2 = \frac{\gamma_1^2(b_1 - E_1) + (1 + \gamma_1^2)(b_2 - E_2 - d_2(1 + \gamma_1^2 + \gamma_2^2))}{(1 + \gamma_1^2)(1 + \gamma_1^2 + \gamma_2^2)},$$

$$R_2 = \frac{(1 + \gamma_1^2)(b_2\gamma_1^2 + E_2(1 + \gamma_2^2)) + \gamma_1^2(1 + \gamma_2^2)(b_1 - E_1)}{(1 + \gamma_1^2)(1 + \gamma_1^2 + \gamma_2^2)}$$

which satisfies the constraints if

$$I_{21} \geq 0 : b_1(1 + \gamma_2^2) - b_2(1 + \gamma_1^2) \leq E_1(1 + \gamma_2^2) - E_2(1 + \gamma_1^2),$$

$$\beta_1 \geq 0 : b_1 \geq d_1(1 + \gamma_1^2) + E_1,$$

$$\beta_2 \geq 0 : b_2 \geq E_2 + d_2(\gamma_2^2 + 1 + \gamma_1^2) - \frac{\gamma_1^2(b_1 - E_1)}{1 + \gamma_1^2},$$

$$R_1 \geq 0 : b_2(1 + \gamma_1^2) - b_1(1 + \gamma_2^2) \leq \frac{E_1(1 + \gamma_2^2)}{\gamma_1^2} + E_1(1 + \gamma_1^2) + E_2(1 + \gamma_1^2),$$

$$R_2 \geq 0 : b_1(1 + \gamma_2^2) - b_2(1 + \gamma_1^2) < E_1(1 + \gamma_2^2) + (1 + \gamma_2^2)E_2 \frac{1 + \gamma_1^2}{\gamma_1^2}.$$

4b. The symmetric case $\beta_1 \geq 0, \beta_2 \geq 0, I_{21} = 0$ requires the following conditions on the parameters:

$$I_{12} \geq 0 : b_2(1 + \gamma_1^2) - b_1(1 + \gamma_2^2) \leq E_2(1 + \gamma_1^2) - E_1(1 + \gamma_2^2)$$

$$\beta_1 \geq 0 : \gamma_2^2 b_2 + b_1(1 + \gamma_2^2) \geq \gamma_2^2 E_2 + (1 + \gamma_2^2)(E_1 + d_1(1 + \gamma_1^2 + \gamma_2^2)) \text{ (redundant)}$$

$$\beta_2 \geq 0 : b_2 \geq d_2(1 + \gamma_2^2) + E_2$$

$$R_1 \geq 0 : b_2(1 + \gamma_1^2) - b_1(1 + \gamma_2^2) < E_2(1 + \gamma_1^2) + (1 + \gamma_1^2)E_1 \frac{1 + \gamma_2^2}{\gamma_2^2} \text{ (redundant)}$$

$$R_2 \geq 0 : b_2(1 + \gamma_1^2) - b_1(1 + \gamma_2^2) > -\frac{E_2(1 + \gamma_1^2)}{\gamma_2^2} - E_2(1 + \gamma_2^2) - E_1(1 + \gamma_2^2).$$

Appendix B. Computation of cooperative solution

We study all possible combinations of active and non-active constraints, which are the following:

1. Environmental and non-negativity constraint not active, $\beta = \eta_1 = \eta_2 = 0$ yields the following solution for $i = 1, 2$:

$$e_i = b_i - (d_1 + d_2),$$

$$I_i = \gamma_i(d_1 + d_2),$$

$$R_i = b_i - (1 + \gamma_i^2)(d_1 + d_2)$$

which is interior and satisfies the constraints if

$$R_i \geq 0 : b_i \geq (1 + \gamma_i^2)(d_1 + d_2), \quad i = 1, 2$$

$$R_1 + R_2 \leq E_1 + E_2 : b_1 + b_2 - (2 + \gamma_1^2 + \gamma_2^2)(d_1 + d_2) \leq E_1 + E_2.$$

2. Environmental constraint active, net emissions non-negative, $\beta \geq 0$, $\eta_i = \eta_j = 0$ yields the following solution for $i=1,2, j=3-i$:

$$e_i = \frac{b_i(1 + \gamma_1^2 + \gamma_2^2) + E_1 + E_2 - b_j}{2 + \gamma_1^2 + \gamma_2^2},$$

$$I_i = \gamma_i \frac{b_1 + b_2 - E_1 - E_2}{2 + \gamma_1^2 + \gamma_2^2},$$

$$R_i = \frac{b_i(1 + \gamma_i^2) - (1 + \gamma_i^2)(b_j - E_2 - E_2)}{2 + \gamma_1^2 + \gamma_2^2},$$

$$\beta = \frac{b_1 + b_2 - E_1 - E_2 - (2 + \gamma_1^2 + \gamma_2^2)(d_1 + d_2)}{2 + \gamma_1^2 + \gamma_2^2}$$

which satisfies the constraints if

$$\beta \geq 0 : b_1 + b_2 - (2 + \gamma_1^2 + \gamma_2^2)(d_1 + d_2) \geq E_1 + E_2, \text{ implying } I_i > 0 \text{ and } e_i < b_i,$$

$$R_i \geq 0 : b_i - \frac{b_j(1 + \gamma_i^2)}{1 + \gamma_j^2} \leq E_1 + E_2, \quad i = 1, 2.$$

3. Environmental constraint not active and Player i 's net emissions equal 0, $\beta=0$, $\eta_i=0$, $\eta_j \geq 0$, yields the following solution for $i=1,2, j=3-i$:

$$e_i = b_i - (d_1 + d_2) \quad e_j = \gamma_j^2 \frac{b_j}{1 + \gamma_j^2},$$

$$I_i = \gamma_i(d_1 + d_2) \quad I_j = \gamma_j \frac{b_j}{1 + \gamma_j^2},$$

$$R_i = b_i - (1 + \gamma_i^2)(d_1 + d_2) \quad \eta_j = \frac{-b_j + (1 + \gamma_j^2)(d_1 + d_2)}{1 + \gamma_j^2}$$

which is interior and satisfies the constraints if

$$R_i \geq 0 : (1 + \gamma_i^2)(d_1 + d_2) < b_i,$$

$$R_i \leq E_1 + E_2 : b_i < E_1 + E_2 + (1 + \gamma_i^2)(d_1 + d_2),$$

$$\eta_j \geq 0 : (1 + \gamma_j^2)(d_1 + d_2) > b_j.$$

4. Environmental constraint not active and both players net emissions at 0, $\beta=0$, $\eta_i \geq 0$, $\eta_j \geq 0$, yields the following solution for $i=1,2, j=3-i$:

$$e_i = \gamma_i^2 \frac{b_i}{1 + \gamma_i^2},$$

$$I_i = \gamma_i \frac{b_i}{1 + \gamma_i^2},$$

$$\eta_i = -\frac{b_i - (1 + \gamma_i^2)(d_1 + d_2)}{1 + \gamma_i^2}$$

which is interior and satisfies the constraints if

$$\eta_i \geq 0 : b_i \geq (1 + \gamma_i^2)(d_1 + d_2), \quad i = 1, 2.$$

5. Environmental constraint active and Player j 's net emissions at 0, $\beta \geq 0$, $\eta_i = 0$, $\eta_j \geq 0$, yields the following solution for $i=1, 2$, $j=3-i$:

$$e_i = \frac{E_1 + E_2 + \gamma_i^2 b_i}{1 + \gamma_i^2} \quad e_j = \gamma_j^2 \frac{b_j}{1 + \gamma_j^2},$$

$$I_i = \gamma_i \frac{b_i - (E_1 + E_2)}{1 + \gamma_i^2} \quad I_j = \gamma_j \frac{b_j}{1 + \gamma_j^2},$$

$$\beta = \frac{b_i - (E_1 + E_2)}{1 + \gamma_i^2} - (d_1 + d_2) \quad \eta_j = \frac{b_j - (E_1 + E_2)}{1 + \gamma_j^2} - \frac{b_j}{1 + \gamma_j^2}$$

which is interior for Player j and satisfies the constraints if

$$\eta_j \geq 0 : b_j(1 + \gamma_j^2) - b_i(1 + \gamma_i^2) \geq (1 + \gamma_j^2)(E_1 + E_2),$$

$$\beta \geq 0 : b_i \geq E_1 + E_2 + (1 + \gamma_i^2)(d_1 + d_2), \text{ implying } e_i < b_i \text{ and } I_i > 0.$$

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