Coordination of cooperative advertising in a two-level supply chain when manufacturer offers discount

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Abstract

We studied the coordination of cooperative advertisement in a manufacturer–retailer supply chain when the manufacturer offers price deductions to customers. With a price sensitive market, the expected demand with cooperative advertising and price deduction is demonstrated. When the manufacturer is a leader, we obtained the optimal national brand name investment, local advertisement and associated manufacturer’s allowance with any given price deduction. When the manufacturer offers more price deduction to customers, the retailer will increase local advertisement if the manufacturer provides the same portion of the local advertising allowance. We obtained the necessary and sufficient condition for the price deduction to ensure an increase of manufacturer’s profit, and a search procedure for determining such an optimal price deduction is provided as well. When the manufacturer and retailer are partners, we obtained the optimal national brand name investment and local advertisement. For any given price deduction, the total profit for the supply chain with cooperative scheme is always higher than that with the non-cooperative scheme. When price elasticity of demand is larger than one, the resulting closed form optimal price deduction with partnership is also obtained. To increase profits for both parties in a supply chain, we recommend that coordination in local and national cooperative advertising with a partnership relationship between manufacturer and retailer is the best solution. The bargaining results show how to share the profit gain between the manufacturer and the retailer, and determine the associated pricing and advertising policies for both parties.

Keywords: Supply chain; Co-op advertising; Price discount; Game theory

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1. Introduction

1.1. Non-cooperative vs. cooperative supply chain

Two-level supply chains have been extensively studied in recent operations management literature. In a two-level supply chain, the manufacturer and retailer make interactive actions to benefit themselves or the whole chain. These relationships may be non-cooperative or cooperative. In non-cooperative situations, the party with the manipulative power in the chain controls the other party who becomes the follower in the relationship. The leader of the chain estimates the reactions of the follower and decides the first move and then prescribes the behavior of the follower (Gaski, 1984; Munson and Rosenblatt, 2001).

In manufacturer and retailer two-level supply chains, traditionally manufacturers hold manipulative powers, act as leaders, and are followed by retailers. For example, automobile makers may coerce their small, family run dealership networks by using slow delivery or by removing business from poorer performing dealers (Lamb et al., 1996). McDonalds typically impose strong control over its franchisees (Love, 1986). In recent years, more leading powers have shifted from manufacturers to retailers (Huang et al., 2002). For example, Wal-Mart effectively uses its power to get reduced prices from its suppliers (manufacturers) (Lucas, 1995). When both parties in the supply chain have negotiation power, they may finally agree to cooperate rather than not cooperate with each other.

In a cooperative environment, the manufacturer and retailer work together advantageously in determining price, order quantity, advertising, etc., to achieve maximum savings and/or to enhance profit for the whole chain. For example, Procter & Gamble (P&G), a former leader in the supply chain (Walton and Huey, 1992), now partners with Wal-Mart, through demand monitoring, JIT delivery, and information sharing (Foley and Mahmood, 1996; Coyle et al., 1996). This new partnership approach for P&G and Wal-Mart has changed the relationship from a win–lose situation into a win–win situation for cost saving and revenue increases for both parties (Huang et al., 2002).

1.2. Co-op advertising and price discount in the supply chain

There are many possible interactive actions in a manufacturer and retailer supply chain. In this research, we focus on cooperative (co-op) advertising between the two parties and price discount. In several supply chain management studies (Monahan, 1984; Lee and Rosenblatt, 1986; Drezner and Wesolowsky, 1989; Chiang et al., 1994), price discounts were discussed as part of the lot sizing decisions. In these studies, the total market demand was assumed to be unaffected by price discounts when the manufacturer provided the discount price to the retailer to compensate for the increase of inventory costs for ordering at a large lot size. Abad (1994) and Li et al. (1996), however, discussed pricing and lot sizing when demand was price sensitive. In fact, for price sensitive demand, the manufacturer and/or retailer could give deeper discounts to customers to increase the market share and profit in the chain.

It is becoming commonplace for manufacturers to offer price deductions directly to customers instead of through retailers. These direct price deductions may be in the form of direct price discounts, sales coupons, mail-in rebates, on-site rebates, etc. The manufacturer’s price deduction is a reflection of the competitive pressures; furthermore, price deduction produces more competitiveness and stimulates the market for the brand name, especially when demand for the product is price sensitive (Lamb et al., 1996). In the automobile industry, for example, at the end of 2002, General Motors (GM) provided rebates of $3000 or no interest for five years for 12 car models and five truck models. During the same time, Ford rebates of $3000 were available for only three models of Ford vehicles and three models of Mercury vehicles. Chrysler provided $3000 rebates for five vehicles models (Teahen, 2003). In 2003, GM is not only providing a $500 rebate toward the purchase of a new truck, but also waiving lease payments for some customers as an incentive to purchase. GM launched a credit card loyalty program in Canada in the early 1990s as way
to provide customer rebates. Rebate dollars can be earned by making purchases on the credit card (5% on all credit card purchases made) and these credits can be applied toward the purchase of a GM vehicle. Originally, a maximum of $3500 could be accumulated over 7 years, but new rules reduce the rebate to 3% and the $3500 allowance is only applicable toward the purchase of a Cadillac (Menzes, 2002). Some analysts are concerned that American automakers’ practice of providing incentives for consumers may create an expectation among consumers that these incentives are permanent and this will create profitability problems for automakers over the long term. CNW Marketing Research estimated that the average rebate expected by consumers when they purchase a new vehicle is $3700 (Grant, 2003). Although the retailer selling the brand name product does not receive a price discount from the manufacturer, the retailer will benefit from an enhancement of the brand name and an increase in sales.

Co-op advertising is typically defined as a cost sharing and promotion mechanism used by manufacturers and retailers. It allows them to focus on building image by national brand name investment and to focus on short-term sales incentives by local advertisement. Cooperative advertising has been in use in the United States since the early 1900s. Warner Brothers, a maker of corsets, issued the first co-op agreement in 1903. The use of co-op advertising spread to grocery stores and then to fashion and hard good stores. The automobile industry is the most common user of cooperative advertising. It was not until after World War II that co-op advertising became commonplace in the United States (Wirebach, 1983). In recent studies, Dant and Berger (1996) used game theory to obtain the Stackelberg equilibrium in advertising cost sharing where allowance from the manufacturer promotes the retailer’s advertising expense and increases the profit for the whole chain. Huang et al. (2002) observed that manufacturers not only paid brand name investments, but also paid part of local advertisement costs incurred by retailers. They used a game theory approach to discuss both leader–follower and co-op partnership advertising models, and provided optimal brand name investments, local advertisement with manufacturer’s allowance in both cases.

In this research, we consider cooperative advertising in a two-level manufacturer retailer supply chain when demand is price sensitive. Game theory is used to analyze both leader–follower and partnership co-op cases. When the manufacturer provides a price deduction directly to customers, the optimal brand name investments and local advertisement with the manufacturer’s local advertising allowance are obtained. The optimal price deductions are also determined for both cooperative and non-cooperative cases.

The rest of the paper is organized as follows: Section 2 gives the demand function with brand name investments and local advertising effects when demand is price sensitive. The profit functions for both the manufacturer and retailer when the manufacturer offers a price deduction directly to customers are also obtained. Section 3 provides the Stackelberg equilibrium when the manufacturer is the leader and retailer is the follower. The condition that a manufacturer’s profit will increase when providing a price deduction is obtained. A search procedure is also provided to obtain the best price deduction to customers by the manufacturer. Section 4 discusses the manufacturer and retailer partnership co-op advertising decision. A closed form optimal price deduction is also obtained. Section 5 discusses the bargaining results to determine the shares of profits between the manufacturer and the retailer. Conclusion remarks are given in Section 6.

2. Demand function and profit function determination

In this section, we will determine the demand function with local advertisement, brand name investment, and price deduction efforts. When demand is not price sensitive, Huang et al. (2002) suggest that the one period expected demand (sale volume) $S$, influenced by local advertising effects and brand name investments, is determined by

$$S(a, q) = a - a^{-\gamma}q^{-\delta},$$
where \( a, \gamma \) and \( \delta \) are positive constants, \( a \) and \( q \) represent local advertisement and brand name investments, respectively. Furthermore, \( \gamma \) and \( \delta \) are called the quasi-advertising elasticity and the quasi-investment elasticity, respectively, and \( S(a,q) \) is a non-decreasing function with respect to \( a \) and \( q \). When either or both local advertisement and brand name investments tend to infinity, the demand \( S \) tends to the constant \( a \).

If demand is price sensitive with constant price elasticity (Abad, 1994; Li et al., 1996), we can express the demand function as

\[
S(P) = \omega P^{-\epsilon},
\]

where \( P \) is the price charged to customers, \( \omega \) is a scaling parameter, and \( \epsilon \) is the price elasticity, which is always positive.

Integrating the local advertisement, brand name investments and price sensitivity effects together, we suggest that the one period expected demand (sale volume) is determined by

\[
S(a,q,P) = \left[ a - \beta a^{-\gamma} q^{-\delta} \right] \left( \frac{P}{P_0} \right)^{-\epsilon},
\]

where \( \beta \) is a scaling parameter, \( P_0 \) is the full price for customers and other parameters are defined the same as before. In many situations, \( P_0 \) can be considered as the manufacturer suggested retail price (MSRP). Scaling parameter \( \beta \) is multiplied to the factor \( a^{-\gamma} q^{-\delta} \) so that it has the same dimensionality as \( a \). The value of \( \beta \) will also determine the impact of brand name investment and local advertisement to the expected market demand.

Note that \( S(a,q,P) \) in (2) is an increasing function for local advertisement \( a \) and brand name investments \( q \), but a decreasing function for price \( P \). Furthermore, \( a \) and \( q \) in (2) must be no less than some positive values \( a \geq a_0 \) and \( q \geq q_0 \) since otherwise \( S(a,q,P) \) there will be negative. Therefore, the above function (2) is valid only when brand name investment and local advertisement are no less than some appropriate levels.

A number of remarks can be made regarding the demand function (2). Observe that the local advertisement \( a \) and brand name investment \( q \) determine the potential customer base for the brand name. Price determines the demand elasticity to the brand name with the determined customer base. When either or both \( a \) and \( q \) increase, the potential customer base increases as well. Furthermore, price deduction makes the customers more willing to buy the brand name with the customer base determined by \( a \) and \( q \). Therefore the multiplicative rule is applied to integrate the effects of \( a \) and \( q \) with the effect of \( P \). If price is the full price \( (P = P_0) \), (2) has exactly the same form as Huang et al. (2002) (here a parameter \( \beta \) is multiplied to the \( a^{-\gamma} q^{-\delta} \) in Huang’s form with the reason given before). If price is reduced \( (P < P_0) \), \( S(a,q,P) \) can be rewritten as \( \{ a(P/P_0)^{\epsilon} - \beta (P/P_0)^{-\epsilon} a^{-\gamma} q^{-\delta} \} \), which still has the same form as of Huang et al. (2002) but with different parameter values. In this case, the demand is consistently higher than that with full price \( \{(a(P/P_0)^{\epsilon} - \beta (P/P_0)^{-\epsilon} a^{-\gamma} q^{-\delta}) > (a - \beta a^{-\gamma} q^{-\delta}) \} \), and it has a higher upper limit \( (a(P/P_0)^{\epsilon} > a) \) when either or both \( a \) and \( q \) tend to infinity. This multiplication really makes sense for a demand sensitive market.

Furthermore, for fixed \( a \) and \( q \), define \( \{ a - \beta a^{-\gamma} q^{-\delta} P_0^\epsilon \} \) as \( \omega \) in (1), then (2) has the same form of (1), which shows the price effect for a sensitive demand with constant elasticity. Since \( \{ a - \beta a^{-\gamma} q^{-\delta} P_0^\epsilon \} \) is an increasing function of \( a \) and \( q \) with a given deductive price, the higher the value of either or both \( a \) and \( q \), the more the market demand.

We discuss a two-level supply chain with one manufacturer and one retailer. Similar supply chain structure is seen in other literatures; for example, Monahan (1984), Lee and Rosenblatt (1986), Abad (1994), Chiang et al. (1994), Li et al. (1996), and Huang et al. (2002). If the manufacturer offers price deductions directly to customers, the price deduction can always be shown as a portion of the full price. Therefore, the price received by customer, \( P \), can be expressed as \( P = (1 - \epsilon) P_0 \) where \( \epsilon \) is the price deduction percentage offered by the manufacturer. Therefore, the expected market demand function (2) is then rewritten as
\[ S(a, q, P) = S(a, q, \varepsilon) = (x - \beta a^{-\gamma} q^{-\delta})(1 - \varepsilon)^{-c}. \]  \hspace{1cm} (3)

Let \( \rho_m \) be the manufacturer’s dollar profit margin of each product unit sold and \( \rho_r \) be the retailer’s dollar profit margin of each product unit sold when full price \( P_0 \) is charged (\( \varepsilon = 0 \)). When the manufacturer offers \( \varepsilon \) percentage price deduction directly to customers, the manufacturer’s one period expected gross profit is

\[ \pi_m(\varepsilon) = (\rho_m - \varepsilon P_0)(x - \beta a^{-\gamma} q^{-\delta})(1 - \varepsilon)^{-c} - ta - q, \]  \hspace{1cm} (4)

where \( ta \) is the local advertisement cost shared by the manufacturer with \( t \) percent of the cost and \( q \) is the national brand name investment purely incurred to the manufacturer.

When the retailer does not receive a price deduction from the manufacturer and is willing to keep its profit margin, the retailer’s expected gross profit is

\[ \pi_r(\varepsilon) = \rho_r(x - \beta a^{-\gamma} q^{-\delta})(1 - \varepsilon)^{-c} - (1 - t)a. \]  \hspace{1cm} (5)

The whole chain’s one period expected gross profit is simply the sum of expected gross profits of the manufacturer and retailer, which is

\[ \pi(\varepsilon) = \pi_m(\varepsilon) + \pi_r(\varepsilon) = (\rho_m - \varepsilon P_0 + \rho_r)(x - \beta a^{-\gamma} q^{-\delta})(1 - \varepsilon)^{-c} - a - q. \]  \hspace{1cm} (6)

The manufacturer’s one period expected gross profit \( \pi_m(\varepsilon) \) defined in (4), the retailer’s one period expected gross profit \( \pi_r(\varepsilon) \) in (5) and the whole chain’s one period expected gross profit \( \pi(\varepsilon) \) in (6) are gross profits that do not subtract manufacturer’s fixed costs (such as overhead, rental, capital interest payment, etc.), retailer’s fixed costs, and whole supply chain’s fixed costs, respectively. Therefore, the gross profits introduced here are less than the “gross margins” defined in accounting (Eskew and Jensen, 1989), since the gross profits defined here subtract the advertising cost but the gross margins in accounting do not. However, the gross profits introduced here are higher than the “net profits (incomes)” in accounting since the gross profits do not subtract other selling expenses, administrative expenses, financial expenses, and tax expense as well but the net profits do so. For simplification purposes, we simply use “the profit” to stand for “the expected gross profit” in the rest of the paper.

Let \( VC_w \) represents the variable cost of one product in the whole supply chain, we have

\[ VC_w = P_0 - (\rho_m + \rho_r). \]  \hspace{1cm} (7)

Eq. (7) is the difference between full price \( P_0 \) and the sum of the marginal profits of the manufacturer and the retailer \( (\rho_m + \rho_r) \), which is the variable cost of one product in the whole supply chain.

With the profit functions (4)–(6), we can determine the optimal amount of advertising and price discount via a game theory approach in the following sections.

### 3. Optimal advertising and price deduction decision when manufacturer is a leader

In this section, we consider the manufacturer as leader and followed by the retailer in a two-level supply chain. For a given price deduction, the manufacturer determines its optimal brand name investment and local advertising allowance based on an estimation of the local advertisement from the retailer to maximize profit. The retailer, as a follower on the other hand, based on the information from the manufacturer, finds out the optimal local advertisement cost to maximize its profit as well. The optimal solution is obtained by the Stackelberg equilibrium in game theory.
3.1. Stackelberg equilibrium

Assume that the manufacturer decides to provide \( \epsilon \) percentage of price deduction to the customers. Note that \( \epsilon \) is often times a given value, not a decision variable, since market pressure from competitors forces the manufacturer to make this kind of decision. For a given manufacturer’s local advertising allowance \( t \) and brand name investments \( q \), retailer’s local advertisement cost \( a \) will be determined by

\[
\max_{a} \pi_t(a) = \rho_t(\alpha - \beta a^{-\gamma} q^{-\delta})(1 - \epsilon)^{-\gamma} - (1 - t)a
\]

subject to \( a \geq 0 \). Equating to zero first partial derivative of \( \pi_t(a) \) with respect to \( a \), the optimal local advertisement \( a^*(\epsilon) \) is obtained as

\[
a^*(\epsilon) = \left[ \frac{\beta \gamma \rho_t}{(1 - t)^{\gamma}(1 - \epsilon)^{\gamma+1}} \right]^{1/(\gamma+1)}.
\]  
(8)

Taking first partial derivatives of \( a^*(\epsilon) \) in (8) with respect to \( t \) and \( q \), we observe that \( \partial a^*(\epsilon)/\partial t > 0 \) and \( \partial a^*(\epsilon)/\partial q < 0 \). That is, the local advertisements increase when (i) the manufacturer’s local advertising allowance \( t \) increases, or (ii) the manufacturer’s brand name investment \( q \) decreases. Furthermore, taking the first partial derivative of \( a^*(\epsilon) \) in (8) with respect to \( \epsilon \), we have

\[
\frac{\partial a^*(\epsilon)}{\partial \epsilon} = \frac{e}{\gamma + 1} \left( \beta \gamma \rho_t \right)^{1/(\gamma+1)} \left( 1 - t \right)^{-1/(\gamma+1)} q^{-\delta/(\gamma+1)} \left( 1 - \epsilon \right)^{-1/(\gamma+1)}> 0.
\]  
(9)

Eq. (9) simply implies that the retailer selling the brand name will increase the local advertisement whenever the manufacturer reduces the price and provides the same level of local advertisement allowance.

In a leader–follower game scheme, the retailer’s reaction is well known by the manufacturer. Given this knowledge, the manufacturer will maximize its profit by deciding the optimal brand name investment, and its allowance to local advertisement. Now, substituting \( a^*(\epsilon) \) of (8) for \( a \) in (4), we have the manufacturer’s expected profit objective function

\[
\max_{q, t} \pi_m(q, t) = (\rho_m - \epsilon P_0)(\alpha - \beta a^{-\gamma} q^{-\delta})(1 - \epsilon)^{-\gamma} - ta - q
\]

\[
= (\rho_m - \epsilon P_0)\{\alpha(1 - \epsilon)^{-\gamma} - [\beta(\gamma \rho_t)^{-\gamma}(1 - t)^{-\delta} q^{-\delta}]^{1/(\gamma+1)}\}
\]

\[
- t[\beta(\gamma \rho_t)(1 - t)^{-1} q^{1/(\gamma+1)}] - q
\]

subject to \( 0 \leq t \leq 1, q \geq q_0 \).

By setting the first partial derivatives of \( \pi_m(q, t) \) in (10) with respect to \( q \) and \( t \) to zeros, respectively, we have the optimal manufacturer’s decision in national brand name investment \( q^*(\epsilon) \), local advertising allowance \( t^*(\epsilon) \), and therefore, the retailer’s decision in local advertisement \( a^*(\epsilon) \) as well. The optimal solution that provides an equilibrium of the two-stage game with leader and follower can be referred to as the Stackelberg equilibrium (Stackelberg, 1934).

**Proposition 1.** If the manufacturer provides \( \epsilon \) percentage price discount directly to customers, the Stackelberg equilibrium for the manufacturer as a leader and the retailer as a follower is given as follows:

\[
q^*(\epsilon) = \left[ \beta \delta^{(\gamma+1)} \gamma^{-\gamma} (\rho_m - \epsilon P_0) (1 - \epsilon)^{-\epsilon} \right]^{1/(\delta + \gamma + 1)},
\]

\[
t^*(\epsilon) = \begin{cases} 
\left[ \rho_m - (1 + \gamma) \rho_t - \epsilon P_0 \right] / \left[ \rho_m - \epsilon P_0 - \gamma \rho_t \right] & \text{when } (\rho_m - \epsilon P_0) / \rho_t > (1 + \gamma), \\
0 & \text{otherwise},
\end{cases}
\]

\[
a^*(\epsilon) = \left[ \beta \delta^{-\delta} (\rho_m - \epsilon P_0 - \gamma \rho_t) (1 - \epsilon)^{-\epsilon} \right]^{1/(\delta + \gamma + 1)}.
\]  
(11)
In (11), \( a'(e) \) is obtained by substituting \( q'(e) \) and \( t'(e) \) into (8). Obviously, Proposition 1 is an extension of the results of the two-stage game in Huang et al. (2002) when price deduction from the manufacturer is considered. If the manufacturer does not provide any price deduction \((e = 0)\) and lets \( \beta = 1 \), the above solutions are exactly same as the solution in Table 1 of Huang et al. (2002). If the manufacturer offers price deduction of \( e \) percent, Theorem 1 of Huang et al. (2002) remains true. The gross profit margin of the manufacturer now becomes \((\rho_m - eP_0)\). From Proposition 1, we can see that the ratio of the optimal local advertisements and brand name investment always equals the ratio of quasi-advertising elasticity over quasi-investment elasticity, which is

\[
\frac{a'(e)}{q'(e)} = \frac{\gamma}{\delta},
\]

regardless of whether there is a price deduction from the manufacturer. Furthermore, when a price deduction is available, and if the marginal profit ratio of the manufacturer and retailer is higher than the quasi-advertising elasticity by unity \( ((\rho_m - eP_0)/(\rho_r) > (1 + \gamma)) \), the manufacturer should provide local advertising allowance to make sure that the ratio of the local advertisement and brand name investment is kept at a constant level of \((\gamma/\delta)\). When the manufacturer offers a price deduction to the customer, the proposition tells us that the local advertising allowance from the manufacturer is still determined by the marginal profit ratio of the manufacturer and retailer. In fact, when this ratio is higher than the quasi-advertising elasticity by unity, the manufacturer’s offer to the local advertising allowance is positively correlated to the manufacturer’s profit margin, and it is negatively correlated with the retailer’s profit margin.

From (11), local advertisement \( a'(e) \) and national brand investment \( q'(e) \) at Stackelberg equilibrium with price deduction \( e \) are higher than that without the price deduction when \([((\rho_m - eP_0 - \gamma \rho_r) > (1 - e)^{-\gamma}]) \) is larger than \((\rho_m - \gamma \rho_r)\). Furthermore, the condition that \( a'(e) \) and \( q'(e) \) are increasing functions of \( e \) is

\[
e < \frac{e(\rho_m - \gamma \rho_r) - P_0}{(e - 1)P_0} \quad \text{if} \ e > 1,
\]
\[
e > \frac{P_0 - e(\rho_m - \gamma \rho_r)}{(1 - e)P_0} \quad \text{if} \ e < 1.
\]

Eq. (13) is obtained simply by setting the first derivative of \([((\rho_m - eP_0 - \gamma \rho_r) > (1 - e)^{-\gamma}]) \) with respect to \( e \) larger than zero. Furthermore, since in local advertisement \( a'(e) \), \([1 - t'(e)]) \) portion is from the retailer, the retailer’s expense in local advertisement is

\[
[1 - t'(e)]a'(e) = \frac{P_0}{[\rho_m - eP_0 - \gamma \rho_r]}[\beta \delta^{-\delta}(\delta + 1)(\rho_m - eP_0 - \gamma \rho_r)(1 - e)^{-\delta}]^{1/(\delta + \gamma + 1)}
\]
\[
= P_0[\beta \delta^{-\delta}(\delta + 1)(1 - e)^{-\delta}]^{1/(\delta + \gamma + 1)}(\rho_m - eP_0 - \gamma \rho_r)^{-\delta},
\]

which is an increasing function of \( e \). Therefore, it is always true that the retailer is willing to pay more money in advertisement when the manufacturer provides price deduction to customers, even the manufacturer’s local advertisement allowance may reduce. When the manufacturer offers a larger price deduction to the customers, the retailer has a stronger incentive to spend more money in local advertisement. This actually happened in practice. For example, Toyota has started a sequence of marketing promotion by offering rebates and/or low interest finance directly to customers nationwide since November 2002. In the meantime, Beaman Toyota, a local dealer in Nashville, TN, has also increased its local advertisement expense in response to the manufacturer’s campaign. Eq. (14) justifies the fact that the retailer’s advertising behavior can be determined by the manufacturer’s decision.
3.2. Price deduction determination when manufacturer is a leader

The manufacturer offering a price deduction to customers may be based on several considerations, such as to increase brand name market share, to response to competitors’ pressure, or others. In this section, we are interested in determining how much a deduction in price will maximize the manufacturer’s profit when the brand name investment, local advertisement, and manufacturer’s local allowance are determined by (11).

When the manufacturer is a leader and the retailer is a follower, it follows from (4) to (6), the profits of the manufacturer, retailer and whole chain at the Stackelberg equilibrium (see Proposition 1) are given by (11).

See the proof in Appendix B.

The manufacturer offering a price deduction to customers may be based on several considerations, such as to increase brand name market share, to response to competitors’ pressure, or others. In this section, we are interested in determining how much a deduction in price will maximize the manufacturer’s profit when the brand name investment, local advertisement, and manufacturer’s local allowance are determined by (11).

When the manufacturer is a leader and the retailer is a follower, it follows from (4) to (6), the profits of the manufacturer, retailer and whole chain at the Stackelberg equilibrium (see Proposition 1) are given by

\[
\pi^*_m(\epsilon) = \alpha(\rho_m - \epsilon P_0)(1 - e)^{-\epsilon} - \delta + \gamma + 1
\]

\[
\times [\beta \delta^{-\gamma}(\rho_m - \epsilon P_0 - \gamma \rho_t)(1 - e)^{-\epsilon}]^{1/(\delta + \gamma + 1)},
\]

\[
\pi^*_i(\epsilon) = \alpha \rho_t (1 - e)^{-\epsilon} - (1 + \gamma) \rho_t
\]

\[
\times [\beta \delta^{-\gamma}(\rho_m - \epsilon P_0 - \gamma \rho_t)^{-(\delta + \gamma)}(1 - e)^{-\epsilon}]^{1/(\delta + \gamma + 1)},
\]

\[
\pi^*_e(\epsilon) = \pi^*_m(\epsilon) + \pi^*_i(\epsilon),
\]

respectively. In the rest of the paper, we only discuss the situation of non-negative profit, that is, \( \pi^*_m(\epsilon), \pi^*_i(\epsilon), \pi^*_e(\epsilon) \) in (15) are all greater than or equal to zero. We have the following theorem to provide the necessary and sufficient condition that the manufacturer’s profit will increase by offering a price deduction.

**Theorem 1.** Under a certain condition, the manufacturer’s profit will increase by offering a price deduction.

Furthermore, this given condition is

\[
(e \rho_m - P_0)[\alpha - (\beta \delta^{-\gamma}(\rho_m - \gamma \rho_t)^{-(\delta + \gamma)/(\delta + \gamma + 1)}] + e(\gamma \rho_t)(\beta \delta^{-\gamma}(\rho_m - \gamma \rho_t)^{-(\delta + \gamma)/(\delta + \gamma + 1)} - (1 - e)^{-\epsilon}]^{1/(\delta + \gamma + 1)}
\]

\[
\geq 0.
\]

See the proof in Appendix A.

If the condition in Theorem 1 is satisfied, the manufacturer needs to decide the price deduction percentage precisely to maximize its profit. The best price deduction percentage (denoted as \( \epsilon^* \)) can be determined by setting the first partial derivative of \( \pi^*_m(\epsilon) \) with respect to \( \epsilon \) to zero. However, there is no closed form solution. In practice, \( \epsilon^* \) can be obtained by a searching algorithm in a range for \( \epsilon^* \) by the following proposition.

**Proposition 2.** If the condition of Theorem 1 is satisfied, the best price deduction percentage \( \epsilon^* \) is restricted in a suitable range \( \epsilon_1 \leq \epsilon^* \leq \epsilon_u \). The lower bound of the range is

\[
\epsilon_1 = \max \left\{ (e \rho_m - P_0)/[P_0(e - 1)], \ 0 \right\},
\]

and the upper bound of the range is

\[
\epsilon_u = \rho_m/P_0.
\]

See the proof in Appendix B.

Although there are many searching algorithms available (such as gradient algorithm (Hillier and Lieberman, 2001 and others), we suggest a simple algorithm below for obtaining the optimal price deduction percentage.
[Searching Algorithm to Obtain the Best $\epsilon^*$] (A)

**Step 1.** In the condition of Theorem 1, determine if a price deduction will increase the manufacturer's profit. If no, set $\epsilon^* = 0$ and stop. Otherwise, go to next step.

**Step 2.** Define $\theta$ as a small positive number representing a desired accuracy. Let $\epsilon = \epsilon_f$.

**Step 3.** Calculate $\pi_m^*(\epsilon)$ by (15).

**Step 4.** If $(\epsilon + \theta) > \epsilon_u$, set $\epsilon^* = \epsilon$, stop. Otherwise, calculate $\pi_m^*(\epsilon + \theta)$ by (15).

**Step 5.** If $\pi_m^*(\epsilon + \theta) \leq \pi_m^*(\epsilon)$, set $\epsilon^* = \epsilon$, stop. Otherwise, let $\epsilon = \epsilon + \theta$, go to Step 4.

The above search algorithm will ensure that we obtain a price deduction percentage which is no more than $\theta$ away from the true optimal price deduction percentage. When $\epsilon$ in (11) is substituted by $\epsilon^*$, we obtain the manufacturer's optimal brand name investment, local advertisement, and manufacturer's local advertising allowance. Similarly, we obtain the optimal profits of the manufacturer, retailer and whole chain by substituting $\epsilon^*$ for $\epsilon$ in (15). The results of the two-stage game when the manufacturer is a leader are summarized in Table 1.

Please keep in mind that the above optimal solutions are for the manufacturer's best interests when the manufacturer is the leader of the chain. In the following section, we discuss what will happen when the manufacturer and the retailer work together as partners.

4. Manufacturer and retailer cooperate as partners

In many industries the balance of power between manufacturers and retailers is shifting (Achenbaum and Mitchel, 1987; Buzzell et al., 1990; Fulop, 1988; Kumar, 1996, Huang et al., 2002). Many manufacturers who had dominated their retailers are finding that retailers hold the upper hand. Retailers often now control access to enormous numbers of consumers. Using local advertisement and introducing product information such as on-site presentations, on-site tasting samples, and personal recommendations, retailers can influence consumers' purchasing behavior, and thereby, influence consumers' purchasing decisions.

The shift of power between manufacturers and retailers has raised an important question: Should powerful companies use their strength to wring concessions from their counterparts? Numerous cases can be used to illustrate that although exploiting power may be advantageous in a short term, it will be self-defeating in a long term. A typical example is P&G. Before retailers developed sophisticated point-of-sale systems which generate a wealth of information on consumers, P&G would bring its comprehensive research on consumers to retailers such as Wal-Mart and use this information to argue for increased shelf space for its brands in Wal-Mart, to pressure Wal-Mart carry of all sizes of a certain product of P&G, and to demand the participation of Wal-Mart in P&G promotional programs. P&G would use its enormous power to dictate to Wal-Mart how much P&G would sell, at what prices, and under what terms. After the point-of-sale systems were developed by retailers, retailers have become more powerful and P&G had to battle for shelf space intensively with other manufacturers. Wal-Mart now held the upper hand and could demand that P&G offer Wal-Mart rock-bottom prices, extra service, and preferred credit terms.

It was not until the mid-1980s that this adversarial relationship began to change. The relationship between Wal-Mart and P&G evolved into partnership and coordination. The result of this relationship was the electronic-data-interchange (EDI) link. EDI enables P&G to receive continuous data by satellite on sales, inventory, and prices of particular products at individual Wal-Mart stores. This link allows P&G to take responsibility for managing Wal-Mart's inventory, to anticipate P&G product sales at Wal-Mart, to determine the number of shelf racks and quantity required, and to automatically ship the
orders—often directly from the factory to individual stores. By working together as partners, Wal-Mart and P&G have turned what used to be a win–lose situation of each striving to lower its own costs regardless of the effect on the other’s costs into a win–win situation of reduced costs and greater revenues for both partners (Coyle et al., 1996; Kumar, 1996). Instead of pursuing the equilibrium results for a leader–follower game structure as we studied previously, the remainder of this section is focused on a new model development to address the issue of coordination and partnership in the content of co-op advertising.

4.1. Cooperation with fixed price deduction

First, let us still consider that the manufacturer offers a fixed price deduction percentage ε directly to customers. The manufacturer adopts this strategy for reasons of increasing market share, improving competitiveness, or others. In this situation, it follows from (6), that the optimal local advertisement and optimal brand name investment, denoted as \( a^*(\varepsilon) \) and \( q^*(\varepsilon) \), respectively, can be obtained by

\[ \max_{\varepsilon, a} \pi(\varepsilon, a) = (\rho_m - \varepsilon P_0 + \rho_c)(x - b \alpha^{-\gamma} \delta^{-\gamma} q^{-\delta})(1 - \varepsilon)^{-\gamma} - a - q \]

subject to \( a \geq 0 \), and \( q \geq 0 \). We use overline here to indicate the manufacturer and the retailer cooperation scheme.

**Table 1**
Two-stage game optimal advertising and price determination

If fixed price deduction ε is provided by the manufacturer,

\[ q^*(\varepsilon) = \frac{[b \delta^{(\varepsilon+1)} - \gamma \gamma (\rho_m - \varepsilon P_0 - \gamma \rho_c) (1 - \varepsilon)^{-\gamma}]^{1/(\delta + \gamma + 1)}}{1}, \]

\[ t^*(\varepsilon) = \begin{cases} \frac{[\rho_m - (1 + \gamma) \rho_c - \varepsilon P_0]}{[\rho_m - \varepsilon P_0 - \gamma \rho_c]} & \text{when } (\rho_m - \varepsilon P_0) / \rho_c > (1 + \gamma), \\ 0 & \text{otherwise}, \end{cases} \]

\[ a^*(\varepsilon) = \frac{[b \delta^{-\gamma} \gamma (\rho_m - \varepsilon P_0 - \gamma \rho_c) (1 - \varepsilon)^{-\gamma}]^{1/(\delta + \gamma + 1)}}{1}, \]

and the profits are

\[ \pi_m^*(\varepsilon) = x(\rho_m - \varepsilon P_0) (1 - \varepsilon)^{-\gamma} - (\delta + \gamma + 1)[b \delta^{-\gamma} \gamma (\rho_m - \varepsilon P_0 - \gamma \rho_c) (1 - \varepsilon)^{-\gamma}]^{1/(\delta + \gamma + 1)}, \]

\[ \pi_r^*(\varepsilon) = \alpha \rho_c (1 - \varepsilon)^{-\gamma} - (1 + \gamma) \rho_c [b \delta^{-\gamma} \gamma (\rho_m - \varepsilon P_0 - \gamma \rho_c) (1 - \varepsilon)^{-\gamma}]^{1/(\delta + \gamma + 1)}, \]

\[ \pi^*(\varepsilon) = \pi_m^*(\varepsilon) + \pi_r^*(\varepsilon). \]

Optimal price deduction ε^* can be obtained by the Searching Algorithm (A) and is between

\[ \max \left\{ \frac{(\rho_m - \varepsilon P_0) [P_0 (1 - \varepsilon) - 1]}{P_0} \right\} \leq \varepsilon^* \leq \rho_m / P_0. \]
Setting the first partial derivatives of \( \pi(e) \) of (6) with respect to \( a \) and \( q \), respectively, to zero, local advertisement \( \bar{a}^*(e) \) and brand name investment \( \bar{q}^*(e) \) with the manufacturer and the retailer cooperation scheme are

\[
\bar{a}^*(e) = [\beta \delta^{-\delta \gamma} \gamma^{-1}(\rho_m - eP_0 + \rho_r)(1 - e)^{-e}]^{1/(\delta + \gamma + 1)},
\]

and

\[
\bar{q}^*(e) = [\beta \delta^{-\delta \gamma} \gamma^{-1}(\rho_m - eP_0 + \rho_r)(1 - e)^{-e}]^{1/(\delta + \gamma + 1)},
\]

respectively. When \((\rho_m - eP_0 + \rho_r)(1 - e)^{-e}\) is larger than \((\rho_m + \rho_r)\), the local advertisement and brand name investment with price deduction are larger than that without price deduction. The largest profit for the entire supply chain is

\[
\bar{\pi}(e) = \pi(\rho_m - eP_0 + \rho_r)(1 - e)^{-e} - (\delta + \gamma + 1)[\beta \delta^{-\delta \gamma} \gamma^{-1}(\rho_m - eP_0 + \rho_r)(1 - e)^{-e}]^{1/(\delta + \gamma + 1)}.
\]

When \(\epsilon = 0\) and \(\beta = 1\), (19)–(21) have the same results of the partnership game as in Huang et al. (2002). The ratio of optimal brand name investment and local advertisement is the same as that in (12):

\[
\frac{\bar{a}^*(e)}{\bar{q}^*(e)} = \frac{\gamma}{\delta}.
\]

We also have the following proposition.

**Proposition 3.** When the manufacturer offers a fixed price deduction to the customer, comparing with the manufacturer leading case, both local advertisement and brand name investment will increase when the manufacturer and retailer are partners. Furthermore, the profit of the whole chain will increase also.

See the proof in Appendix C.

Proposition 3 indicates that Theorem 2 of Huang et al. (2002) is still true when the manufacturer offers direct price deductions to customers. Theoretically, the manufacturer’s local advertising allowance can be any \(\bar{t}(e)\) between zero and one if the manufacturer and retailer belong to the same organization. However, if they are not in the same organization and both parties require that their profits will not be reduced by using a cooperation scheme, \(\bar{t}(e)\) may belong to another restricted range. In the following, we will attempt to find the manufacturer minimum and maximum share of local advertising cost.

Let \(\bar{\pi}_m^*(e)\) and \(\bar{\pi}_r^*(e)\) stand for the profits of the manufacturer and retailer, respectively under the optimal cooperation scheme. We have

\[
\bar{\pi}_m^*(e) = \pi(\rho_m - eP_0)(1 - e)^{-e} - \left(\bar{t}(e)\gamma + \delta + \frac{\rho_m - eP_0}{\rho_m - eP_0 + \rho_r}\right)[\beta \delta^{-\delta \gamma} \gamma^{-1}(\rho_m - eP_0 + \rho_r)(1 - e)^{-e}]^{1/(\delta + \gamma + 1)}
\]

\[
\geq \pi(\rho_m - eP_0)(1 - e)^{-e} - (\delta + \gamma + 1)[\beta \delta^{-\delta \gamma} \gamma^{-1}(\rho_m - eP_0 + \rho_r)(1 - e)^{-e}]^{1/(\delta + \gamma + 1)} = \pi_m^*(e),
\]

\[
\bar{\pi}_r^*(e) = \pi(\rho_m - eP_0 + \rho_r)(1 - e)^{-e} - \left(1 - \bar{t}(e)\gamma + \delta + \frac{\rho_r}{\rho_m - eP_0 + \rho_r}\right)[\beta \delta^{-\delta \gamma} \gamma^{-1}(\rho_m - eP_0 + \rho_r)(1 - e)^{-e}]^{1/(\delta + \gamma + 1)}
\]

\[
\geq \pi(\rho_m - eP_0 + \rho_r)(1 - e)^{-e} - (\gamma + 1)\frac{\rho_r}{(\rho_m - eP_0 - \gamma \rho_r)}[\beta \delta^{-\delta \gamma} \gamma^{-1}(\rho_m - eP_0 - \gamma \rho_r)(1 - e)^{-e}]^{1/(\delta + \gamma + 1)} = \pi_r^*(e),
\]

and of course

\[
\bar{\pi}(e) = \bar{\pi}_m^*(e) + \bar{\pi}_r^*(e).
\]
The range of $\bar{t}(\varepsilon)$ is

$$\bar{t}(\varepsilon) \leq \bar{t}_{\max}(\varepsilon) = \frac{1}{\gamma} \left[ (\delta + \gamma + 1) \left( \frac{\rho_m - \varepsilon P_0 - \gamma \rho_t}{\rho_m - \varepsilon P_0 + \rho_t} \right)^{1/(\delta + \gamma + 1)} - \delta - \frac{\rho_m - \varepsilon P_0}{\rho_m - \varepsilon P_0 + \rho_t} \right],$$

and

$$\bar{t}(\varepsilon) \geq \bar{t}_{\min}(\varepsilon) = \frac{1}{\gamma} \left[ \gamma \left( \frac{\rho_m - \varepsilon P_0 - (\gamma + 1) \rho_t}{\rho_m - \varepsilon P_0 + \rho_t} - (\gamma + 1) \rho_t \right) \left( \frac{\rho_m - \varepsilon P_0 - \gamma \rho_t}{\rho_m - \varepsilon P_0 + \rho_t} \right)^{1/(\delta + \gamma + 1)} \right].$$

(24)

It is easy to show that $\bar{t}_{\max}(\varepsilon) \geq \bar{t}_{\min}(\varepsilon)$. With no further assumption, $\bar{t}_{\max}(\varepsilon)$ could be higher than 1. If this happens, the implication is that if both sides follow a partnership scheme and the retailer has manipulative power, then the retailer can force the manufacturer to pay all local advertisement plus additional compensation. On the other hand, $\bar{t}_{\min}(\varepsilon)$ could be lower than zero. This implies that the manufacturer can pay no local advertising allowance, and squeeze more profit from the retailer if the manufacturer dominates the supply chain. As long as the retailer still earns profit and such profit is no less than the case when the manufacturer is a leader, the manufacturer can always persuade the retailer (even without providing a local advertising allowance) to follow through the partnership scheme. Therefore, we conclude that, in general, the manufacturer’s local advertising allowance is determined by the two parties’ bargaining powers. If we do know the bargaining powers of both sides, then further bargaining results as given by Huang et al. (2002) can also be applicable to determine the optimal manufacturer’s local advertising allowance for the price deduction model.

4.2. Partnership price deduction determination

If a price deduction can increase the profit of the whole chain in the partnership scheme, the manufacturer needs to decide the optimal price deduction percentage to maximize the total profit. Here again, the price deduction cannot only be the response to competitors’ actions, but also will obtain the maximum profit for the whole supply chain. We also assume that the brand name investment and local advertisement are determined by (19) and (20), respectively.

**Theorem 2.** The optimal price deduction percentage offered by the manufacturer to customer is independent of the local advertisement and brand name investment. This optimal value is purely determined by the price elasticity, full price (MSRP) and its associated marginal profits of the manufacturer and the retailer.

Furthermore, the optimal price deduction percentage is

$$\varepsilon^* = \begin{cases} 
\frac{[e(\rho_m + \rho_t) - P_0]}{[(e - 1)P_0]} & \text{when } (e - 1) > 0, \\
0 & \text{otherwise}.
\end{cases}$$

(25)

See the proof in Appendix D.

Eq. (25) shows that the price deduction will only be applied for the merchandise with price “elastic” demand ($\varepsilon > 1$). In practice, many types of merchandise satisfy this condition. For example, the long run price elasticity of motion pictures is 3.67, of China and glassware is 2.55, of restaurant meals is 2.3, and of electricity (household) is 1.89 (Houthakker and Taylor, 1970), etc. The long-term price deductions and therefore the long run retailing prices can be determined by (25). For some other types of merchandise, long run elasticities may be less than 1 (like automobiles), but the short-run price elasticities are higher than 1 (the short-run price elasticity of Chevrolet is 4). Eq. (25) can help to determine the short-run sale price deductions.
If \( [e(\rho_m + \rho_t) - P_0] \) in (25) is greater than zero, reducing MSRP will increase the total profit in the whole supply chain. However, if \( [e(\rho_m + \rho_t) - P_0] < 0 \), the manufacturer should not reduce the price. Instead, charging higher than the initial full price will increase the total profit in the whole supply chain! This rarely happens in practice, therefore we only discuss the condition that \( [e(\rho_m + \rho_t) - P_0] \) is greater than zero in the rest of the paper. From (25), the optimal retailing price for customers, denoted as \( \bar{P}^r \), is

\[
\bar{P}^r = P_0(1 - \bar{e}^*) = \frac{e}{(e - 1)}[P_0 - (\rho_m + \rho_t)] = \frac{e}{(e - 1)} VC_w,
\]

and which is purely determined by price elasticity \( e \) and variable costs in the whole supply chain.

With optimal price deduction percentage \( \bar{e}^* \), the total profit in the supply chain (21) is

\[
\bar{\pi}^*(\bar{e}^*) = \alpha \left( \frac{P_0}{e} \right)^e \left( \frac{e - 1}{P_0 - \rho_m - \rho_t} \right)^{(e-1)} - (\delta + \gamma + 1)
\times \left[ \beta \delta^{-\delta} \gamma^{-\gamma} \right]^{1/(\delta + \gamma + 1)} \left[ \left( \frac{P_0}{e} \right)^e \left( \frac{e - 1}{P_0 - \rho_m - \rho_t} \right)^{(e-1)} \right]^{1/(\delta + \gamma + 1)}.
\]  

(26)

It is the largest possible total profit of the whole supply chain. The associated local advertisement and brand name investment are

\[
\bar{a}^*(\bar{e}^*) = [\beta \delta^{-\delta} \gamma^{\delta+1}]^{1/(\delta + \gamma + 1)} \left[ \left( \frac{P_0}{e} \right)^e \left( \frac{e - 1}{P_0 - \rho_m - \rho_t} \right)^{(e-1)} \right]^{1/(\delta + \gamma + 1)},
\]  

(27)

and

\[
\bar{q}^*(\bar{e}^*) = [\beta \delta^{\delta+1} \gamma^{-\gamma}]^{1/(\delta + \gamma + 1)} \left[ \left( \frac{P_0}{e} \right)^e \left( \frac{e - 1}{P_0 - \rho_m - \rho_t} \right)^{(e-1)} \right]^{1/(\delta + \gamma + 1)}.
\]  

(28)

Using \( \bar{e}^* \) in (25) instead of \( e \) in (22) and (23), we can also obtain the manufacturer’s profit \( \bar{\pi}_m^*(\bar{e}^*) \), the retailer’s profit \( \bar{\pi}_r^*(\bar{e}^*) \) when optimal price deduction is offered by the manufacturer. The partnership game results are summarized in Table 2.

There is an interesting observation here. If the optimal price deduction percentage with a partnership scheme \( (\bar{e}^*) \) is very large (much larger than the optimal price deduction percentage \( e^* \) with the scheme when the manufacturer is a leader), the manufacturer’s profit with a partnership scheme could be lower than that with a manufacturer leader scheme even if the manufacturer provides no local advertising allowance. It is an obstacle for the manufacturer to provide the optimal price deduction in partnership scheme. However, since the total profit in the whole supply chain will be maximized, instead of providing whole optimal price deduction percentage with a partnership scheme \( (\bar{e}^*) \), the manufacturer may convince the retailer to share some of the price deduction. It means that the manufacturer and the retailer will share the total price deduction percentage of \( \bar{e}^* \). This strategy will make sure that each party will obtain the profit no less than that in the optimal situation of the manufacturer leader scheme.

5. Bargaining results

In the two-stage game when the manufacturer is a leader and the retailer is a follower, the manufacturer offers the optimal price deduction percentage of \( e^* \) determined by the Searching Algorithm (A). The associated national brand name investment, local advertisement, and the manufacturer’s local allowance are
Optimal price deduction

The manufacturer’s local allowance is between

\[ \tilde{t}(\epsilon) \leq t_{\text{max}}(\epsilon) = \frac{1}{\gamma} \left( -\frac{\rho_m - \epsilon P_0}{\rho_m - \epsilon P_0 + \rho_t} + \frac{\delta + \gamma + 1}{\rho_m - \epsilon P_0 + \rho_t} \left( \rho_m - \epsilon P_0 - \gamma \rho_t \right)^{1/(\delta + \gamma + 1)} \right), \]

and

\[ \tilde{t}(\epsilon) \geq t_{\text{min}}(\epsilon) = \frac{1}{\gamma} \left( -\frac{\rho_m - \epsilon P_0}{\rho_m - \epsilon P_0 + \rho_t} + \frac{\delta + \gamma + 1}{\rho_m - \epsilon P_0 + \rho_t} \left( \rho_m - \epsilon P_0 - \gamma \rho_t \right)^{1/(\delta + \gamma + 1)} \right). \]

the whole supply chain profit is

\[ \bar{\pi}^*(\epsilon) = \alpha (\rho_m - \epsilon P_0 + \rho_t) (1 - \epsilon)^{-\epsilon} - (\delta + \gamma + 1)]/b \delta^{1/(\delta + \gamma + 1)} (\rho_m - \epsilon P_0 - \gamma \rho_t) (1 - \epsilon)^{-\epsilon} \right]. \]

Optimal price deduction \( \bar{\epsilon}^* \) is

\[ \bar{\epsilon}^* = \left\{ \begin{array}{ll} \epsilon (\rho_m + \rho_t) - P_0 / ((\epsilon - 1)P_0) & \text{when } (\epsilon - 1) > 0, \\ 0 & \text{otherwise}. \end{array} \right. \]

Optimal price deduction percentages \( q^*(\epsilon^*), a^*(\epsilon^*), \) and \( t^*(\epsilon^*) \), respectively, by using \( \epsilon^* \) instead of \( \epsilon \) in \( q^*(\epsilon), a^*(\epsilon), \) and \( t^*(\epsilon) \) in (11). Under this scheme, the profit of the whole chain is

\[ \pi^*(\epsilon^*) = \alpha (\rho_m + \rho_t - \epsilon^* P_0) (1 - \epsilon^*)^{-\epsilon} - [(\delta + \gamma + 1) - 1/\gamma + 1)P_0 - \gamma \rho_t] + (1 + \gamma)P_0] \times [\beta \delta^{1/(\delta + \gamma + 1)} (\rho_m - \epsilon^* P_0 - \gamma \rho_t)^{-\gamma} (1 - \epsilon^*)^{-\epsilon}]. \]

When the manufacturer and the retailer finally agree to use the partnership scheme, a new optimal price deduction percentage \( \bar{\epsilon}^* \) in (25) will be offered to the customers. As the consequence of using the partnership scheme, the total profit in the supply chain will increase to \( \bar{\pi}^*(\epsilon^*) \) in (26). The whole chain profit gain when the scheme changes from a two-stage game to a partnership game is

\[ \Delta \pi = \bar{\pi}^*(\epsilon^*) - \pi^*(\epsilon^*) = \Delta \pi_m + \Delta \pi_r, \]

where \( \Delta \pi_m \) and \( \Delta \pi_r \) are the profit gains of the manufacturer and the retailer, respectively. How to share the whole chain profit gain between the manufacturer and the retailer depends on the bargaining powers of two parties.

As mentioned in the previous section, if the optimal price deduction in partnership scheme \( \bar{\epsilon}^* \) is very large and the manufacturer is not profitable, compared with the two-stage game, instead of providing
the whole price deduction by himself, the manufacturer might force the retailer to share some of the price deduction. Therefore, both parties will negotiate based on their powers and their utility functions, to determine not only the manufacturer’s local allowance, but the price deduction share also.

Assume that each of the manufacturer’s and the retailer’s utility functions is purely determined by its own profit gain with no correlation effects, and has a constant risk aversion function. These assumptions are valid for the manufacturer and the retailer in a two-level supply chain. In such a supply chain, the manufacturer and the retailer are not competitors. The manufacturer in the chain might pay attention to the profits of its competitive manufacturers rather than that of its retailer. Similarly, the retailer in the chain might be concerned with its competitive retailers’ profit gains, but not its manufacturer’s. Therefore, the utility function of each party in the chain will only consider its own profit gain, not the other party’s. The utility functions of the manufacturer and the retailer are

\[ u_m(\Delta \pi_m) = 1 - \exp(-b_m \Delta \pi_m), \]
\[ u_r(\Delta \pi_r) = 1 - \exp(-b_r \Delta \pi_r), \]  
respectively. In (31), \( u_m(\Delta \pi_m) \) and \( u_r(\Delta \pi_r) \) are the utility functions of the manufacturer with the manufacturer’s profit gain \( \Delta \pi_m \) and the utility function of the retailer with the retailer’s profit gain \( \Delta \pi_r \), respectively. The \( b_m \) and \( b_r \) are the constant risk aversion coefficients of the manufacturer and the retailer, respectively. The similar utility functions are found in Eliashberg (1986), and Huang et al. (2002).

The utility function of the whole supply chain \( u_s(\Delta \pi_m, \Delta \pi_r) \) is obtained by linear aggregation of the utility functions of the manufacturer and the retailer, which is

\[ u_s(\Delta \pi_m, \Delta \pi_r) = \lambda_m u_m(\Delta \pi_m) + \lambda_r u_r(\Delta \pi_r), \]  
where \( \lambda = (\lambda_m, \lambda_r) \) is a vector of aggregation weights and the summation of the two weights equals one. More specifically, \( \lambda_m \) and \( \lambda_r \) represent the bargaining powers of the manufacturer and the retailer, respectively. If \( \lambda_m \) is close to one, the manufacturer has the manipulative power in the chain and will take the majority part of the whole chain profit gain and vice versa. Since \( (\lambda_m + \lambda_r) = 1 \), (32) becomes

\[ u_s(\Delta \pi_m, \Delta \pi_r) = 1 - \lambda_m \exp(-b_m \Delta \pi_m) - \lambda_r \exp(-b_r \Delta \pi_r). \]  
Considering \( \Delta \pi_s = \Delta \pi - \Delta \pi_m \), the whole supply chain profit gain when switching from a two-stage game to a partnership game becomes

\[ u_s(\Delta \pi_m) = 1 - \lambda_m \exp(-b_m \Delta \pi_m) - \lambda_r \exp(-b_r \Delta \pi + b_r \Delta \pi_m). \]  
Equaling the first derivative of \( u_s(\Delta \pi_m) \) in (34) with respect to \( \Delta \pi_m \) to zero, the manufacturer’s profit gain determined by the bargaining powers is obtained as

\[ \Delta \pi^*_m = \frac{b_r}{(b_m + b_r)} \Delta \pi + \frac{1}{(b_m + b_r)} \log \left( \frac{\lambda_m b_m}{\lambda_r b_r} \right), \]  
and the associated retailer’s profit gain is

\[ \Delta \pi^*_r = \frac{b_m}{(b_m + b_r)} \Delta \pi - \frac{1}{(b_m + b_r)} \log \left( \frac{\lambda_r b_m}{\lambda_r b_r} \right). \]  
Each party’s share of the profit gain is determined by its relative risk aversion coefficient and bargaining power in the chain. For the manufacturer, the profit gain of \( \Delta \pi^*_m \) in (35) can be fulfilled by providing the manufacturer’s local allowance \( t \) and offering the price deduction percentage \( \epsilon_m \). Different combination of \( t \) and \( \epsilon_m \) may get the same fixed \( \Delta \pi^*_m \). In this research, we propose a way by which the manufacturer can determine a reasonable combination of \( t \) and \( \epsilon_m \) and obtain the profit gain in (35).

Suppose that the change from the optimal leader follower two-stage game to the optimal partnership game is accomplished in two steps: In the first step, both parties agree to switch from the two-stage game
scheme to the partnership game scheme, but keep the price deduction percentage unchanged at the two-stage game optimal level of \( \epsilon^* \). As we observed in the previous section, the whole chain profit will increase. We call this increase as the step one profit gain and denote it as \((\Delta \pi)_1\), where \((\Delta \pi)_1\) can be obtained as following:

\[
(\Delta \pi)_1 = \pi^*(\epsilon^*) - \pi^*(\epsilon^*)
\]

\[
= \{x(\rho_m - \epsilon^*P_0 + \rho_r) (1 - \epsilon^*)^{-\gamma} - (\delta + \gamma + 1)|\beta\delta^{-\delta} \gamma^{-\gamma} (\rho_m - \epsilon^*P_0 + \rho_r)(1 - \epsilon^*)^{-\gamma} |^{1/(\delta + \gamma + 1)}
\]

\[
- \{x(\rho_m + \rho_r - \epsilon^*P_0) (1 - \epsilon^*)^{-\gamma} - [(\delta + \gamma + 1)(\rho_m - \epsilon^*P_0 - \gamma\rho_r) + (1 + \gamma)\rho_r]
\]

\[
\times [\beta\delta^{-\delta} \gamma^{-\gamma} (\rho_m - \epsilon^*P_0 - \gamma\rho_r)^{(\delta + \gamma)/(\delta + \gamma + 1)}] > 0.
\]

In the second step, we change the price deduction percentage from \( \epsilon^* \) to \( \tilde{\epsilon}^* \) under the partnership scheme. Again, the profit in the chain will increase by this change. We call the profit gain here the step two profit gain and denote it as \((\Delta \pi)_2\). We have

\[
(\Delta \pi)_2 = \pi^*(\tilde{\epsilon}^*) - \pi^*(\epsilon^*)
\]

\[
= \{x(P_0^{1/\gamma}) \left( \frac{e - 1}{P_0 - \rho_m - \rho_r} \right) (e - 1) - (\delta + \gamma + 1)
\]

\[
\times [\beta\delta^{-\delta} \gamma^{-\gamma}]^{1/(\delta + \gamma + 1)} \left[ \frac{P_0}{e} \left( \frac{e - 1}{P_0 - \rho_m - \rho_r} \right) (e - 1) \right]^{1/(\delta + \gamma + 1)}
\]

\[
- \{x(\rho_m - \epsilon^*P_0 + \rho_r)(1 - \epsilon^*)^{-\gamma} - (\delta + \gamma + 1)|\beta\delta^{-\delta} \gamma^{-\gamma} (\rho_m - \epsilon^*P_0 + \rho_r)(1 - \epsilon^*)^{-\gamma} |^{1/(\delta + \gamma + 1)}
\].

Of course \( \Delta \pi \) in (30) is the sum of \((\Delta \pi)_1\) and \((\Delta \pi)_2\), which is

\[
\Delta \pi = (\Delta \pi)_1 + (\Delta \pi)_2.
\]

We can use the above two-step profit gain structure to determine a reasonable manufacturer’s local allowance and associated price deduction percentage for each party.

In the two-stage game, the optimal price deduction percentage \( \epsilon^* \) is obtained based on the manufacturer’s best interest and is provided by the manufacturer only. Therefore, if only switching from the two-stage game to the partnership game without changing price deduction percentage, the assumption that the manufacturer is still willing to provide all price deduction percentage \( \epsilon^* \) to customers is reasonable. In the step one profit gain of \((\Delta \pi)_1\), the manufacturer’s share is also determined by manufacturer’s relative risk aversion coefficient and bargaining power. By using \((\Delta \pi)_1\) (37) instead of \((\Delta \pi)\) in (35), the manufacturer’s share of step one profit gain is

\[
(\bar{\Delta} \pi_{m1})_1 = \frac{b_t}{(b_m + b_t)} (\Delta \pi)_1 + \frac{1}{(b_m + b_t)} \log \left( \frac{\lambda_m^* b_m}{\lambda_t^* b_t} \right).
\]

The associated retailer’s share of step one profit gain is

\[
(\bar{\Delta} \pi_{r1})_1 = [(\Delta \pi)_1 - (\bar{\Delta} \pi_{m1})_1] = \frac{b_m}{(b_m + b_t)} (\Delta \pi)_1 - \frac{1}{(b_m + b_t)} \log \left( \frac{\lambda_m^* b_m}{\lambda_t^* b_t} \right).
\]

On the other hand, the manufacturer’s share in the step one profit gain is also determined by the manufacturer’s local allowance. We have
\( \overline{\Pi_m}^* = \pi_m^* (\epsilon^*) - \pi_m^* (\epsilon^*) \\
= (\delta + \gamma + 1) [\beta \delta^{-\delta} \gamma^{-\gamma} (\rho_m - \epsilon^* P_0 - \gamma \rho_t) (1 - \epsilon^*)^{-\epsilon} 1^{\delta + \gamma + 1}] \\
- \left( \bar{t}(\epsilon^*) \gamma + \delta + \frac{\rho_m - \epsilon^* P_0}{\rho_m - \epsilon^* P_0 + \rho_t} \right) \times [\beta \delta^{-\delta} \gamma^{-\gamma} (\rho_m - \epsilon^* P_0 + \rho_t) (1 - \epsilon^*)^{-\epsilon} 1^{\delta + \gamma + 1}]. \) (41)

Equaling the right side of (40) to the right side of (41) and let
\[
\bar{a}^* (\epsilon^*) = [\beta \delta^{-\delta} \gamma^{(\delta + 1)} (\rho_m - \epsilon^* P_0 + \rho_t) (1 - \epsilon^*)^{-\epsilon} 1^{(\delta + \gamma + 1)}], \\
\bar{a}_0 (\epsilon^*) = \left( \delta + \gamma + 1 \right) \left( \frac{\rho_m - \epsilon^* P_0 - \gamma \rho_t}{\rho_m - \epsilon^* P_0 + \rho_t} \right)^{1/(\delta + \gamma + 1)} - \delta - \frac{\rho_m - \epsilon^* P_0}{\rho_m - \epsilon^* P_0 + \rho_t},
\]
and
\[
\bar{a}_1 (\epsilon^*) = \frac{1}{\gamma} \left[ \gamma (\rho_m - \epsilon^* P_0) + (\gamma + 1) \rho_t \right] - \frac{(\gamma + 1) \rho_t}{\rho_m - \epsilon^* P_0 + \rho_t} \left( \frac{\rho_m - \epsilon^* P_0 - \gamma \rho_t}{\rho_m - \epsilon^* P_0 + \rho_t} \right)^{1/(\delta + \gamma + 1)},
\]
the manufacturer’s local allowance is obtained as
\[
\bar{t}(\epsilon^*) = \bar{a}_0 (\epsilon^*) - (\overline{\Pi_m}^*)_1/\bar{a}_0 (\epsilon^*) = \bar{a}_0 (\epsilon^*) + [(\Delta \pi)_1 - (\overline{\Pi_m}^*)_1]/\bar{a}_0 (\epsilon^*). \] (42)

In the second step, the price deduction percentage is switched from \( \epsilon^* \) to \( \bar{\epsilon}^* \) under the partnership game scheme with the step two profit gain of (\( \Delta \pi)_2 \) in (38). Denote the manufacturer’s share of the step two profit gain as (\( \Pi_m^* \))\(_2\). From (35) and (40), we have
\[
(\overline{\Pi_m}^*)_2 = (\overline{\Pi_m}^*)_1 - (\Delta \pi)_2 = \frac{b_t}{b_m + b_t} (\Delta \pi)_2. \] (43)

Assume that the manufacturer’s local allowance stays unchanged as \( \bar{t}(\epsilon^*) \) in the second step. This assumption is also reasonable since once the manufacturer’s local allowance is established, both parties prefer to keep it constant for a while for management consistency purposes. However, the manufacturer may not provide the whole price deduction percentage \( \bar{\epsilon}^* \), especially when this percentage is very large. Rather, the manufacturer may force the retailer to share some of the price deduction. Assume the manufacturer provides price deduction percentage \( \bar{\epsilon}^*_m \), the retailer provides \( \bar{\epsilon}^*_r \), and the customers receive the total price deduction percentage \( \bar{\epsilon}^* \). We have
\[
\bar{\epsilon}^* = \bar{\epsilon}^*_m + \bar{\epsilon}^*_r.
\]

The manufacturer’s share in the step two profit gain is
\[
(\overline{\Pi_m}^*)_2 = (\rho_m - \bar{\epsilon}^*_m P_0) \{ x - \beta [\bar{a}^*(\bar{\epsilon}^*) \bar{q}^*(\bar{\epsilon}^*)]^{-1} \} (1 - \bar{\epsilon}^*)^{-\epsilon} - \bar{t}(\bar{\epsilon}^*) \bar{a}^*(\bar{\epsilon}^*) - \bar{q}^*(\bar{\epsilon}^*) - \bar{\Pi}^*_m (\bar{\epsilon}^*). \] (44)

Here \( \bar{\Pi}^*_m (\bar{\epsilon}^*) \) is
\[
\bar{\Pi}^*_m (\bar{\epsilon}^*) = \left\{ x (\rho_m - \bar{\epsilon}^* P_0) (1 - \epsilon^*)^{-\epsilon} - \left[ \bar{t}(\bar{\epsilon}^*) \gamma + \delta + \frac{\rho_m - \epsilon^* P_0}{\rho_m - \epsilon^* P_0 + \rho_t} \right] \times [\beta \delta^{-\delta} \gamma^{-\gamma} (\rho_m - \epsilon^* P_0 + \rho_t) (1 - \epsilon^*)^{-\epsilon} 1^{(\delta + \gamma + 1)}] \right\},
\]
and \( \bar{\epsilon}^* \), \( \bar{a}^*(\bar{\epsilon}^*) \), \( \bar{q}^*(\bar{\epsilon}^*) \), and \( \bar{t}(\bar{\epsilon}^*) \) are obtained in (25), (27), (28) and (42), respectively. Equaling the right side of (43) to the right side of (44), the manufacturer’s price deduction percentage \( \bar{\epsilon}^*_m \) is determined as
\[
\bar{\epsilon}^*_m = \frac{\rho_m}{P_0} \left[ \frac{b_t (\Delta \pi)_2}{b_m + b_t} + \bar{t}(\bar{\epsilon}^*) \bar{a}^*(\bar{\epsilon}^*) + \bar{q}^*(\bar{\epsilon}^*) + \bar{\Pi}^*_m (\bar{\epsilon}^*) \right] \frac{1}{P_0 \{ x - \beta [\bar{a}^*(\bar{\epsilon}^*) \bar{q}^*(\bar{\epsilon}^*)]^{-1} \} (1 - \bar{\epsilon}^*)^{-\epsilon}.} \] (45)
Therefore, in a partnership game, the manufacturer will provide a reasonable combination of the retailer local allowance of $\hat{\pi}_m$ in (42) and price deduction percentage of $\tilde{e}_m^*$ in (45), to obtain the share profit gain $\Delta \pi_m^*$ in (35). In the mean time, the retailer will be forced to provide price deduction of $\tilde{e}_r^* = \hat{\pi}^* - \tilde{e}_m^*$, and obtain the share of profit gain of $\Delta \pi_r^*$ in (36). The national brand name investment and the local advertisement are still $\tilde{a}^*(\tilde{e}^*)$ in (28) and $\tilde{a}^*(\tilde{e}^*)$ in (27), respectively. This partnership scheme is what we recommend to the manufacturer and retailer supply chain.

6. Concluding remarks

Most studies of cooperative advertising in the literature have focused on a relationship of where the manufacturer is a leader and retailers are followers. The market structure of recent years, however, has shown that a shift of the retailing power from manufacturers to retailers has occurred. The conventional models are therefore no longer appropriate to handle this new situation. The paper investigates a number of issues under this new environment. The major contributions of the paper include: (1) It develops a model that reveals the relationship between the expected market demand and a number of important factors. The factors include the price discount, local advertisement level and national advertisement level. (2) The paper studies the cooperative advertisement and price deduction decision when manufacturer is a leader. The Stackelberg equilibrium is obtained for the optimal manufacturer’s decisions on national advertisement, local advertisement, and manufacturer’s share of local advertising allowance. (3) It also studies the partnership advertisement scheme and the manufacturer’s optimal price deduction is determined. (4) Finally, the paper suggests a two-step bargaining model that describes how to share the profit based on bargaining pow- ers. Further, it determines manufacturer’s share of local advertisement and the associated price level.

Utilizing the Eliashberg’s (1986) cooperative bargaining technique, we further address the issue of how to implement the partnership model in co-op advertising with manufacturer’s price deductions to customers. It has been noticed in the literature that partnership itself is a complex system. There are many factors that play significant roles in a successful partnership; one of those is the element of trust. Because of shifts of competitiveness from local to global markets, many companies are growing increasingly concerned about the level of trust they have with their supply chain partners. Trust facilitates the realization of the full potential of a manufacturer–retailer relationship. When both members trust each other, they are able to share confidential information, to invest in understanding each other’s business, and to customize their information systems or to dedicate people and resources to better serving each other. A trusting party normally does not feel the need to monitor the partner’s behavior; thus monitoring costs are reduced. Trust allows a company to capture the hearts and minds of partners who are then willing to “go the extra mile.” The relationship between P&G and Wal-Mart illustrates that even powerful adversaries can benefit from establishing a relationship based on trust. Wal-Mart trusts P&G enough to share sales and price data, and to relinquish control of the order process and inventory management to P&G. P&G in turn trusts Wal-Mart enough to dedicate a large cross-functional team to the Wal-Mart account, to adopt everyday low prices, and to invest in a customized information link. Rather than focus on increasing its sales to Wal-Mart, P&G concentrates on finding ways to increase sales of P&G products to consumers, through Wal-Mart, and to maximize both companies’ profits (Walton and Huey, 1992). Incorporating the trust issue into the co-op advertising model could be a very interesting topic for future research.

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Appendix A. Proof of Theorem 1

Proof. At $\epsilon = 0$, set the first partial derivative of $\pi_m^*(\epsilon)$ in (15) with respect to $\epsilon$ to be greater than or equal to zero, we obtain (16). □

Appendix B. Proof of Proposition 2

Proof. It is straightforward to obtain the upper bound $e_u$. In fact, the manufacturer’s gross marginal profit must be positive, we have $\rho_m - \epsilon P_0 > 0$, thus, $e_u = \rho_m / P_0$.

To determine the lower bound, $\pi_m^*(\epsilon)$ in (15) can be rewritten as

$$
\pi_m^*(\epsilon) = (\rho_m - \epsilon P_0)(1 - \epsilon)^{-e_u} \times \left\{ a - (\delta + \gamma + 1)(\beta + \gamma + 1)1/(\delta + \gamma + 1) \times [(\rho_m - \epsilon P_0)(1 - \epsilon)^{-e_u} - \gamma \rho_T]^{1/(\delta + \gamma + 1)} \right\}.
$$

(B.1)

Note that $\pi_m^*(\epsilon)$ in (B.1) is still an increasing function when setting the first partial derivative of $(\rho_m - \epsilon P_0)(1 - \epsilon)^{-e_u}$ with respect to $\epsilon$ as zero. Therefore, the true optimal $\epsilon$ should be no less than the one that is obtained by setting the first partial derivative of $(\rho_m - \epsilon P_0)(1 - \epsilon)^{-e_u}$ with respect to $\epsilon$ to zero, which is $\epsilon_1$ in (17). □

Appendix C. Proof of Proposition 3

Proof. We can prove that $\pi^*(\epsilon)$ is the maximum profit of the whole chain. Let

$$
A = \frac{\partial^2 \pi^*(\epsilon)}{\partial a^2} = -(1 - \epsilon)^{-e_u} (\rho_m - \epsilon P_0 + \rho_T) \beta \gamma (\gamma + 1) a^{-(\gamma + 2)} q^{-\delta},
$$

$$
B = \frac{\partial^2 \pi^*(\epsilon)}{\partial a \partial q} = -(1 - \epsilon)^{-e_u} (\rho_m - \epsilon P_0 + \rho_T) \beta \gamma \delta a^{-(\gamma + 1)} q^{-(\delta + 1)},
$$

and

$$
C = \frac{\partial^3 \pi^*(\epsilon)}{\partial q^2} = -(1 - \epsilon)^{-e_u} (\rho_m - \epsilon P_0 + \rho_T) \beta \delta (\delta + 1) a^{-\gamma} q^{-(\delta + 2)}.
$$

We have $(A C - B^2) = (1 - \epsilon)^2 (\rho_m - \epsilon P_0 + \rho_T) \beta \gamma (\gamma + 1) a^{-(\gamma + 2)} q^{-\delta} > 0$, and $A < 0$. From calculus, we know that there is a single global maximum point. Since $\tilde{a}^*(\epsilon)$ and $\tilde{q}^*(\epsilon)$ are obtained by setting the first partial derivatives of $\pi(\epsilon)$ with respect to $a$ and $q$ to zero, respectively, $\pi^*(\epsilon)$ determined by $\tilde{a}^*(\epsilon)$ and $\tilde{q}^*(\epsilon)$ in (21) is the global maximum and larger than $\pi^*(\epsilon)$ in (15).

For local advertisement and brand name investment, since

$$
\frac{\tilde{a}^*(\epsilon)}{a^*(\epsilon)} = \frac{\tilde{q}^*(\epsilon)}{q^*(\epsilon)} = \frac{(\rho_m - \epsilon P_0 + \rho_T)}{(\rho_m - \epsilon P_0 - \gamma \rho_T)}^{1/(\delta + \gamma + 1)} > 1,
$$
\( \bar{a}^*(\epsilon) \) is larger than \( a^*(\epsilon) \) and \( \bar{q}^*(\epsilon) \) is larger than \( q^*(\epsilon) \). \( \square \)

**Appendix D. Proof of Theorem 2**

**Proof.** We need to find a price deduction percentage \( \bar{c}^* \) which will generate the largest profit in whole supply chain for all possible price deductions. For any given price deduction percentage \( \epsilon \), the maximum profit of the whole supply chain in (21) can be rewritten as

\[
\pi^*(\epsilon) = (\rho_m - \epsilon P_0 + \rho_r)(1 - \epsilon)^{-(\delta + \gamma + 1)}
\times \left\{ \frac{2}{(1 - \omega) \left( \rho_m - \epsilon P_0 + \rho_r \right)} (1 - \epsilon)^{-(\delta + \gamma + 1)} + (\delta + \gamma + 1) \left[ \beta \delta^{-\delta - \gamma} \right]^{1/(\delta + \gamma + 1)} \right\}.
\]

If \( (\rho_m - \epsilon P_0 + \rho_r)(1 - \epsilon)^{-\epsilon} \) is maximized, \( \pi^*(\epsilon) \) also achieves its maximum. Setting the first derivative of \( (\rho_m - \epsilon P_0 + \rho_r)(1 - \epsilon)^{-\epsilon} \) with respect to \( \epsilon \) to zero, we have

\[
\frac{d(\rho_m - \epsilon P_0 + \rho_r)(1 - \epsilon)^{-\epsilon}}{d\epsilon} = -P_0(1 - \epsilon)^{-\epsilon} + e(\rho_m - \epsilon P_0 + \rho_r)(1 - \epsilon)^{-(e+1)}
= (\rho_m - \epsilon P_0 + \rho_r)(1 - \epsilon)^{-\epsilon} \left[ \frac{e}{(1 - \epsilon)^{\epsilon}} - \frac{P_0}{(\rho_m - \epsilon P_0 + \rho_r)} \right] = 0. \tag{D.1}
\]

Since \( (\rho_m - \epsilon P_0 + \rho_r)(1 - \epsilon)^{-\epsilon} \) cannot be zero, otherwise total profit \( \pi^*(\epsilon) \) will be zero, we can obtain the optimal \( \bar{c}^* \) in (25) by solving

\[
\left[ \frac{e}{(1 - \epsilon)^{\epsilon}} - \frac{P_0}{(\rho_m - \epsilon P_0 + \rho_r)} \right] = 0.
\]

The condition that profit \( \pi^*(\epsilon) \) with price deduction percentage \( \bar{c}^* \) is maximized is that the second derivative of \( (\rho_m - \epsilon P_0 + \rho_r)(1 - \epsilon)^{-\epsilon} \) with respect to \( \epsilon \) at \( \bar{c}^* \) is less than zero, which is

\[
\left. \frac{d^2[(\rho_m - \epsilon P_0 + \rho_r)(1 - \epsilon)^{-\epsilon}]}{d\epsilon^2} \right|_{\bar{c}^*} < 0.
\]

Above the second derivative can be rewritten as

\[
\left\{ -2eP_0(1 - \epsilon)^{-(e+1)} + e(e + 1)(\rho_m - \epsilon P_0 + \rho_r)(1 - \epsilon)^{-(e+2)} \right\}_{\bar{c}^*} = e(1 - \epsilon)^{-(e+1)}[-P_0(1 - \epsilon)^{-\epsilon} + e(\rho_m - \epsilon P_0 + \rho_r)(1 - \epsilon)^{-(e+1)}]_{\bar{c}^*} \\
+ e(1 - \epsilon)^{-(e+2)}[-P_0(1 - \epsilon) + (\rho_m - \epsilon P_0 + \rho_r)]_{\bar{c}^*}.
\]

Since \( [-P_0(1 - \epsilon)^{-\epsilon} + e(\rho_m - \epsilon P_0 + \rho_r)(1 - \epsilon)^{-(e+1)}]_{\bar{c}^*} = 0 \), according to (D.1) and optimal price deduction percentage \( \bar{c}^* \) equals \( e\rho_m + e\rho_r - \rho_m \rho_r)/(e - 1)P_0 \), above the second derivative becomes

\[
e(1 - \epsilon)^{-(e+2)}[-P_0(1 - \epsilon) + (\rho_m - \epsilon P_0 + \rho_r)]_{\bar{c}^*} = -e\left[ \frac{e(P_0 - \rho_m - \rho_r)}{(e - 1)P_0} \right]^{-(e+2)}(P_0 - \rho_m - \rho_r).
\]

If \( e - 1 > 0 \), above second derivative is less than zero and \( \pi^*(\epsilon) \) at \( \bar{c}^* \) is maximized. \( \square \)

**References**