Notes on Draft Syllabus[*]

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Content for 2 Unit

PMA1 Binomial theorem can be in 2 Unit. It can be proved without induction. Permutations and Combinations should however not be in the syllabus.

PMA2 Probability can be removed from the syllabus.

PMA3 A more formal treatment of functions is required, e.g., $f(x) : \mathbb{R} \to \mathbb{R} : x \mapsto x^3$.

PMA4 $A = \frac{1}{2}rl$ can be included for area of a sector. Cosec should be replaced by the more internationally standard csc.

PMA5 Formal definition of limit should be given, i.e.,
$$\lim_{x \to a} f(x) = L \iff (\forall \varepsilon > 0 \ \exists \delta > 0 : |x - a| < \delta \Rightarrow |f(x) - L| < \varepsilon).$$

PMA6 Statistics is not appropriate for his course.

PMA7 The AM-GM inequality should be proved and should be allowed for use in examinations.

MA1 The formal definition for integration <u>is</u> required. Simpson's rule is more efficient than trapezoidal rule. We could omit trapezoidal rule and only do Simpson's rule instead. Surface area of revolution can be included.

MA2 3rd derivative test for inflections can be included. The sufficiency but not necessity for 2nd and higher-order derivative tests should be discussed, as well as the sufficiency and necessity for the successive derivative test.

MA3 More interesting examples such as the following should be covered:

- Lagarias' equivalence to the Riemann hypothesis should be discussed, i.e., if $h_n := n$-th harmonic number$:= \frac{1}{1} + \frac{1}{2} + \cdots + \frac{1}{n}$, and
  $\sigma_n :=$ divisor function of $n :=$ sum of positive divisors of n,
  then if $n > 1$,
  $$h_n + e^{h_n} \ln h_n > \sigma_n.$$

  There is a \$1,000,000 prize for the proof of this at www.claymath.org

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[*]http://www.boardofstudies.nsw.edu.au/syllabus_hsc/pdf_doc/mathematics-advanced-st6-draft.pdf
http://www.boardofstudies.nsw.edu.au/syllabus_hsc/pdf_doc/mathematics-extension-1-st6-draft.pdf
http://www.boardofstudies.nsw.edu.au/syllabus_hsc/pdf_doc/mathematics-extension-2-st6-draft.pdf

- Continued fraction expansion:

$$e = 2 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{2}{3 + \cfrac{3}{\ddots}}}}$$

- Proof of irrationality of e at 2 unit level

- $e = \sum_{n=0}^{\infty} \frac{1}{n!}$

MA4 2 Unit students <u>should</u> see the formal proof that $\frac{d}{dx}\sin x = \cos x$. Derivatives and integrals for $\tan x$, $\cot x$, $\csc x$, $\sec x$ should also be given.

MA5 Students should be able to solve $\frac{dN}{dt} = kN$

MA6 Only applications of sequences and series should be included. Omit applications of probability and data analysis.

CONTENT FOR 3 UNIT

PMX1 If Year 10 do circle geometry, it is not required to repeat it in Year 11.

PMX2 The cubic formula can be included here or in the MX2 topic.

PMX3 It is good that $\csc x$ is used in this topic instead of $\operatorname{cosec} x$, but we need to be more consistent.

PMX4 Formal proofs of the existence and uniqueness of the quotient and remainder should be given before we embark on doing long division exercises.

PMX5 It is good that the mantra "True for $n = 1$ $\therefore$ true for $n = 2$ $\therefore$ true for $n = 3$, etc, $\therefore$ true for all positive integers" has been omitted. It should be made clear to students that such mantras are to be discouraged in examinations. This is because some texts and websites insist on using it.

PMX6 A more formal treatment of chaos is required. Chaos arises in many other contexts and it is misleading to suggest it only comes from the logistic equation.

MX1 This can be in the 2 Unit course. The probability section however should be omitted. Finding greatest terms and coefficients <u>should</u> be in this course (or in the 2 Unit course if it is moved to that course).

MX2 Equating a cubic in x to a quadratic in y forms an elliptic curve. All such curves over $\mathbb{Q}$ are modular, from which Fermat's Last Theorem can be proved.

MX3 Sums to products, products to sums and the tangent rule can be included here.

MX4 When the syllabus says something like "the required substitutions will be given in all cases", or "this is not part of this course", teachers attuned to the behaviour of exam committees in recent years are likely to include it, e.g., partial fractions with higher powers in the denominator, or a trigonometric integral where the substitution is <u>not</u> given. Such behaviour means we cannot trust the syllabus nor the exam committee. So we now have many 3 unit teachers who <u>don't</u> always give substitutions, and many 4 unit teachers who deliberately teach partial fractions with higher powers in the denominator. I suggest removing all such comments from the syllabus.

MX5 Inverse reciprocal trigonometric functions such as $\csc^{-1} x$, $\cot^{-1} x$, $\sec^{-1} x$ should be included. Also the derivatives and integrals of all the inverse trigonometric functions should be included.

MX6 Halving the interval is not necessary. Newton's method is better. The continuous logistic equation $P' = a(1 - P)P$ can be done after exponential growth and decay giving a continuous sigmoid graph.

<center>CONTENT FOR 4 UNIT</center>

MXX1 Don't remove conics or most of Harder 3 Unit. Restricting it to inequalities only constitutes dumbing down of 4 unit and will not be tolerated by most teachers. Some may revolt by teaching Harder 3 Unit <u>in 3 Unit</u>!

MXX2 It is good that Euler's formula $cis\,\theta = e^{i\theta}$, is in the syllabus, but it should be in MXX2, not MXX7. A general formula for $\sqrt{a + ib}$ can be proved. Euler's formula can be used to prove de Moivre's theorem, as well as many other results such as $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$. The fundamental theorem of algebra can be proved by use of the maximum-modulus theorem, well within the scope of 4 unit, and without recourse to Liouville's theorem as it is done at university.

MXX3 Results such as $x^n e^{-x} \to 0$ as $x \to \infty$ or $x^n / \ln x \to \infty$ as $x \to \infty$ can be more formally done in the 3 unit course. They don't have to be exclusively done only by 4 unit students.

MXX4 If pronouncements are made like "cases where repeated factors occur are not included in this course" and then the exam committee put it in the HSC exam anyway, teachers are going to do the exact opposite of what the syllabus says. Another example is "relations such as $\int_0^1 x^m (1 - x)^n \, dx$ which involve more than one integer parameter, are excluded". Of course some teachers will teach it anyway.

MXX5 For volume of revolution, it can be done about lines <u>not</u> parallel to a coordinate axis. The formula for cylindrical shells can be used without having to derive it every time.

MXX6 Energy and momentum should be brought back into this syllabus as well as use of **i**, **j**, **k** vectors.

MXX7 Put Euler's formula in MXX2.