2011

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

# Mathematics Extension 1

**General Instructions**

- Reading Time- 5 minutes
- Working Time – 2 hours
- Write using a black or blue pen
- Approved calculators may be used
- A table of standard integrals is provided at the back of this paper.
- All necessary working should be shown for every question.
- Begin each question in a new booklet.

Total marks (**84**)

- Attempt Questions 1-7
- All questions are of equal value

**Total Marks – 84**
**Attempt Questions 1-7**
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**QUESTION 1** (12 MARKS) Begin a NEW booklet. **Marks**

a) Calculate $\lim_{x \to 0} \frac{\tan 3x}{2x}$. **1**

b) Solve $\frac{x}{2x-1} \geq 3$. **3**

c) If $\alpha, \beta, \gamma$ are the roots of the equation $4x^3 - 6x^2 - 3x + 8 = 0$, find the value of

(i) $\alpha\beta + \alpha\gamma + \beta\gamma$ **1**

(ii) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$. **2**

d) If $\log_5 10 = 2 \cdot 48$ find the exact value of $\log_5 4$. **2**

e) Use the substitution $u = x^2 - 3$ to evaluate $\int_2^6 \frac{x}{\sqrt{x^2 - 3}}\, dx$. **3**

**QUESTION 2** (12 MARKS) Begin a NEW booklet.

a) (i) Prove that $\dfrac{1+\cos 2\theta}{\sin 2\theta}=\cot\theta$. **2**

(ii) Hence calculate the exact value of $\cot\dfrac{\pi}{12}$. **2**

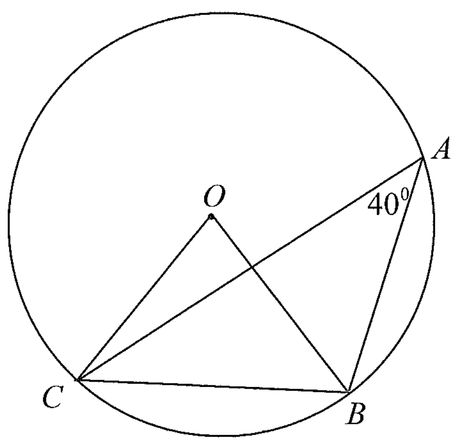
b) A polynomial is given by $P(x)=x^3+ax^2+bx+8$. **3**

Determine the values of $a$ and $b$ if $(x+4)$ is a factor of $P(x)$

and 18 is the remainder when $P(x)$ is divided by $(x+1)$.

c) Find the exact value of $\displaystyle\int_{\sqrt{2}}^{\sqrt{6}}\frac{dx}{6+3x^2}$. **3**

d) In the diagram below $A$, $B$ and $C$ are points on the circumference of a circle centre $O$. If $\angle CAB = 40^\circ$, find the size of $\angle OBC$ giving reasons for your answer. **2**



**QUESTION 3** (12 MARKS) Begin a NEW booklet. **Marks**

a) Use one application of Newton's method to find a second approximation to the root of the equation $3\sin x - 2x = 0$, by taking $1\cdot 56$ as your first approximation. **3**

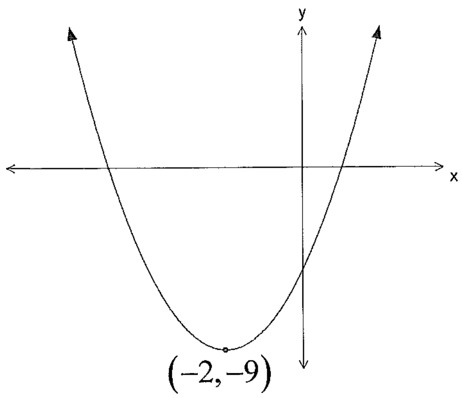
Write your answer correct to 2 decimal places.

b) Find the term independent of $x$ in the expansion of **3**

$$\left(x^2 - \frac{1}{x^3}\right)^{10}.$$

c) Show that the function $f(x) = \dfrac{e^x}{4 - e^x}$ is monotonically increasing over the domain of $x$. **2**

d) The graph of $g(x) = x^2 + 4x - 5$ is shown in the diagram.




(i) Sketch the graph of the inverse function of $g(x) = x^2 + 4x - 5$, for $x \geq -2$. **1**

(ii) State the domain of the inverse function $g^{-1}(x)$. **1**

(iii) Find an expression for $y = g^{-1}(x)$ in terms of $x$. **2**

**QUESTION 4** (12 MARKS) Begin a NEW booklet. **Marks**

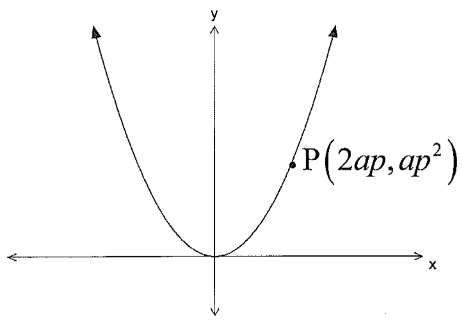
a) Find $\int \sin\theta \cos^2\theta \, d\theta$ by using the substitution $u = \cos\theta$. **2**

b) Evaluate $\sin\left[\tan^{-1}\left(-\sqrt{3}\right)\right]$. **2**

c) A spherical balloon is inflated at a constant rate of $12 \cdot 6 \text{ cm}^3 / \text{s}$. At what rate is the surface area increasing when the radius of the balloon is 12 cm? **3**

$SA = 4\pi r^2$ and $V = \frac{4}{3}\pi r^3$.

d) $P\left(2ap, ap^2\right)$ is a point on the parabola $x^2 = 4ay$ as shown in the diagram drawn below.




The equation of the normal to the curve at P is $x + py = 2ap + ap^3$. DO NOT prove this.

(i) Find the co-ordinates of the point Q where the normal at P meets the $y$–axis. **1**

(ii) Show that the co-ordinates of the point R, which divides the interval PQ externally in the ratio $1:2$ are given by $\left(4ap, ap^2 - 2a\right)$ **2**

(iii) Find the Cartesian equation of the locus of R. **2**

**QUESTION 5** (12 MARKS) Begin a NEW booklet. **Marks**

a) Consider the function $y = 4\sin^{-1}\left(\frac{x}{3}\right)$

(i) State the domain and range of the function. **2**

(ii) Sketch the graph of the function showing all essential features. **1**

(iii) Calculate the gradient of the tangent to the curve at the point where $x = \sqrt{5}$. **2**

b) The area bounded by the curve $y = \sin 2x$, the $x$-axis and the line $x = \frac{\pi}{4}$ is rotated about the $x$-axis. **3**

Calculate the exact volume of the solid of revolution.

c) The rate of growth of bacteria in a culture is given by $\frac{dN}{dt} = k(N - 800)$, where N is the number of bacteria and $t$ is time, in seconds.

(i) Show that $N = 800 + Ae^{kt}$ is a solution of this equation. **1**

(ii) Initially there are 1 000 bacteria and five seconds later there are 1 700 bacteria present in the culture. Calculate the number of bacteria present after ten seconds. **3**

**QUESTION 6** (12 MARKS) Begin a NEW booklet. **Marks**

a) (i) Express $\sqrt{3}\sin\theta - \cos\theta$ in the form $R\sin(\theta - \alpha)$ **2**

where R is positive and $\alpha$ is acute.

(ii) Hence solve $\sqrt{3}\sin\theta - \cos\theta = -1$ for $0 \le \theta \le 2\pi$. **2**

b) Write the binomial expansion of $(3a - 2b)^4$ in simplified form. **2**

c) Use the table of standard integrals to show that $\int_6^{10} \frac{dx}{\sqrt{x^2 - 36}} = \log_e 3$. **2**

d) Use the principle of Mathematical Induction to prove that for all positive integers

$$\frac{1}{1\times 3} + \frac{1}{3\times 5} + \frac{1}{5\times 7} + \dots\dots\dots\dots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}.$$ **4**

**QUESTION 7** (12 MARKS) Begin a NEW booklet. **Marks**

a) The acceleration of a particle P is given by $\ddot{x} = 4x\left(x^2 - 1\right)$, **3**

where $x$ is the displacement of the particle from the origin, in metres, after $t$ seconds. Initially the particle is at the origin, moving to the right with a velocity of $\sqrt{2}\,\text{m/s}$.

Prove that the velocity of the particle is $v = -\sqrt{2}\left(x^2 - 1\right)$.

b) Consider the expansion of $\left(x + \frac{3}{x^2}\right)^8$ with the general term $T_{k+1}$.

(i) Show that $\frac{T_{k+1}}{T_k} = \frac{9-k}{k} \times \frac{3}{x^3}$ **3**

(ii) Hence calculate the greatest co-efficient in the expansion. **2**

c) A particle is moving in simple harmonic motion about a fixed point, with a velocity measured in metres/second, given by $v^2 = 21 + 4x - x^2$.

(i) Between what two points is the particle oscillating? **1**

(ii) What is the centre of the motion? **1**

(iii) Write the amplitude of the motion. **1**

(iv) Calculate the particle's maximum speed. **1**