SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2011
Trial Higher School Certificate

Mathematics Extension 1

General Instructions

- Reading Time – 5 Minutes
- Working time – 2 Hours

- Write using black or blue pen. Pencil may be used for diagrams.
- Board approved calculators maybe used.
- All necessary working should be shown in every question, if full marks are to be awarded.
- Marks may NOT be awarded for messy or badly arranged work.
- All answers to be given in simplified exact form unless otherwise stated.

Total Marks – 84

- Attempt all questions 1-7
- All questions are of equal value.
- Start each new question in a separate answer booklet.
- Hand in your answers in 7 separate bundles: Questions 1, 2, 3, 4, 5 , 6 and 7.

- The mark value of each question is given in the right margin.

Examiner: *R Boros*

This is an assessment task only and does not necessarily reflect the content or format of the Higher School Certificate

Start a new booklet.

Question 1 (12 marks). **Marks.**

a) Solve $x(3-2x)>0$. 2

b) Find $\frac{d}{dx}\left(e^{-x}\cos^{-1}x\right)$. 2

c) The remainder when x^3+ax^2-3x+5 is divided by $(x+2)$ is 11. Find the value of a. 2

d) Using the table of standard integrals, find the exact value of: 2

$$\int_0^{\frac{\pi}{8}} \sec 2x \tan 2x \, dx.$$

e) Solve for x, $\frac{x^2-9}{x} \geq 0$. 2

f) Find $\int_0^2 \frac{1}{4+x^2}\,dx$, leaving your answer in exact form. 2

End of Question 1.

Start a new booklet.

Marks.

Question 2 (12 marks).

a) Use the substitution $x = \ln u$ to find: 3

$$\int \frac{e^x}{\sqrt{1-e^{2x}}}\,dx.$$

b) Use one application of Newton's method to find an approximation to the root of the equation $\cos x = x$ near $x = \frac{1}{2}$. Give your answer correct to 2 decimal places. 2

c) The curves $y = e^{2x}$ and $y = 1 + 4x - x^2$ intersect at the point $(0,1)$. Find the angle, to the nearest minute, between the 2 curves at this point of intersection. 3

d) Prove that $\dfrac{2}{\tan A + \cot A} = \sin 2A$. 2

e) Find the derivative of $\cos^3 x^\circ$. 2

End of Question 2.

Start a new booklet.

Marks.

Question 3 (12 marks).

a) (i) Expand $\cos(\alpha+\beta)$ — 1

(ii) Show that $\cos 2\alpha = 1-2\sin^2\alpha$ — 1

(iii) Evaluate $\lim_{x\to 0}\frac{1-\cos 2x}{x^2}$ — 1

b) If $\alpha = \tan^{-1}\left(\frac{5}{12}\right)$ and $\beta = \cos^{-1}\left(\frac{4}{5}\right)$, calculate the exact value of $\tan(\alpha-\beta)$. — 2

internal

c) $A(-1,7)$ and $B(5,-2)$ are 2 points. Point P divides AB in the ratio $k:1$.

(i) Write down the coordinates of P in terms of k. — 2

(ii) If P lies on the line $5x-4y-1=0$, find the ratio of $AP:PB$. — 1

d) Use mathematical induction to prove that: — 3

$$1\times 1! + 2\times 2! + 3\times 3! + \ldots + n\times n! = (n+1)! - 1$$

where n is a positive integer.

e) State the domain of $y = 2\sin^{-1}(1-x)$. — 1

End of Question 3.

Start a new booklet.

Marks.

Question 4 (12 marks).

a) $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ are 2 points on the parabola $x^2 = 4ay$.

(i) Show that the equation of PQ is given by: $y - \frac{1}{2}(p+q)x + apq = 0$ 2

(ii) Find the condition that PQ passes through the point $(0, -a)$. 1

(iii) If the focus of the parabola is S and PQ passes through $(0, -a)$, 2

prove that $\frac{1}{SP} + \frac{1}{SQ} = \frac{1}{a}$.

b) A family consists of a father, mother, 3 girls and 4 boys.

(i) If the family is seated at random along a bench, find the probability that the parents are at the ends, and the boys and girls are seated alternately between them. 1

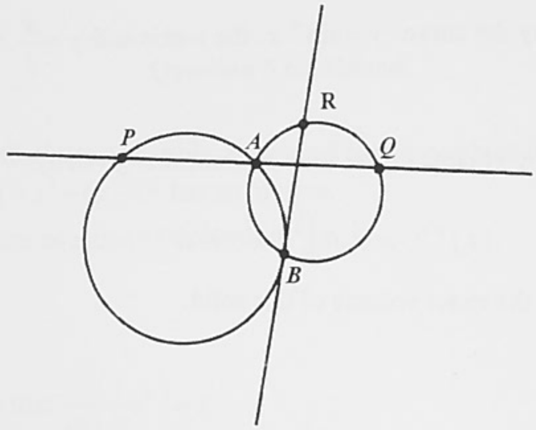
(ii) If the same family is seated randomly at a round table, find the probability that the parents are separated by exactly two seats, and these seats are both occupied by boys. 1

Question 4 continued on next page.

Question 4 continued.

Marks.

c) Two circles cut at A and B. A line through A meets one circle at P. Also, BR is a tangent to the circle ABP and R lies on the circle ABQ.



(i) Copy the diagram showing the above information.

(ii) Prove that $PB \parallel QR$. 2

d) One root of $x^3 + px^2 + qx + r = 0$ equals the sum of the two other roots. 3

Prove that $p^3 + 8r = 4pq$.

End of Question 4.

Start a new booklet.

Marks.

Question 5 (12 marks).

a) The area bounded by the curve $y = \sin^{-1} x$, the y-axis and $y = \frac{\pi}{2}$ is rotated about the y-axis.

(i) Show that the volume of the solid so formed is given by: 1

$$\pi \int_0^{\frac{\pi}{2}} \sin^2 y \, dy.$$

(ii) Hence, find the exact volume of this solid. 2

b) The area of an equilateral triangle is increasing at the rate of 4 cm^2/s.

(i) If x is the length of the side of the triangle, find an expression for the area of the triangle. 1

(ii) Find the exact rate of increase of the side of the triangle, when it has a side length of 2 cm. 2

Question 5 continued on next page.

Question 5 continued. **Marks.**

c) (i) Find the largest possible domain of positive values for which $f(x) = x^2 - 6x + 13$ has an inverse. 1

(ii) Find the equation of the inverse function $f^{-1}(x)$. 2

d) (i) Prove that $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = \ddot{x}$. 1

(ii) A cat moving in a straight line has an acceleration given by $\ddot{x} = x(8-3x)$, where x is the displacement in metres from a fixed point O. Initially the cat is at the origin, O, and has a speed of 4 m/s. Find the cat's speed when it is 1 m on the positive side of O. 2

End of Question 5.

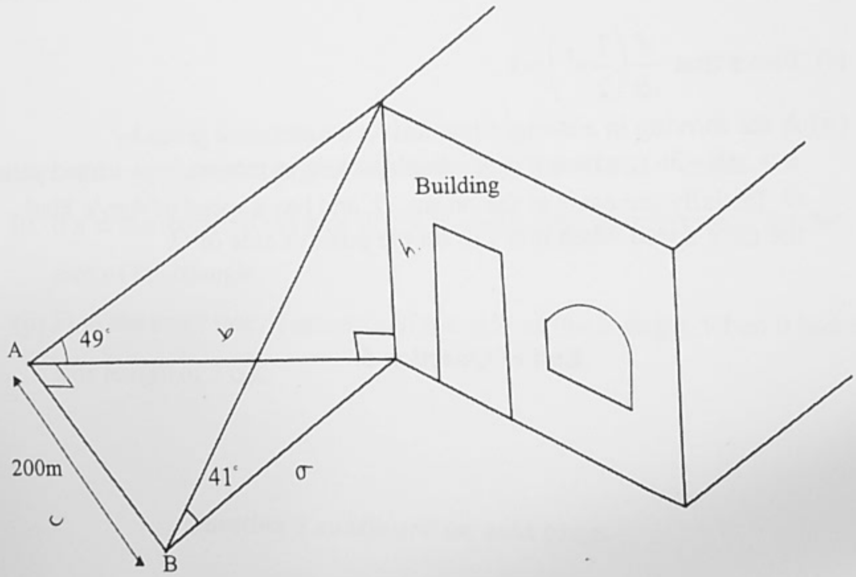
Start a new booklet.

Marks.

Question 6 (12 marks).

a) Given $f(x) = 3\cos^{-1}(\sin 2x) - 2\sin^{-1}(\cos 3x)$, show that $f(x)$ is a constant function by finding $f'(x)$. 2

b) From point *A*, due south of a building, the angle of elevation to the top of the building is 49°. Two hundred metres due east of *A*, at point *B*, the angle of elevation is found to be 41° as shown in the diagram below. Determine, to the nearest metre, the height of the building. 3



Question 6 continued on next page.

Question 6 continued. **Marks.**

c) A particle is oscillating in simple harmonic motion such that its displacement x metres from a given origin O satisfies the equation

$\frac{d^2x}{dt^2} = -4x$, where t is the time in seconds.

(i) Show that $x = a\cos(2t + \beta)$ is a possible equation of motion for the particle, where a and β are constants. **1**

(ii) The particle is observed at time $t = 0$ to have a velocity of 2 m/s and a displacement from the origin of 4 m. Find the amplitude of the oscillation. **2**

(iii) Determine the maximum velocity of the particle. **1**

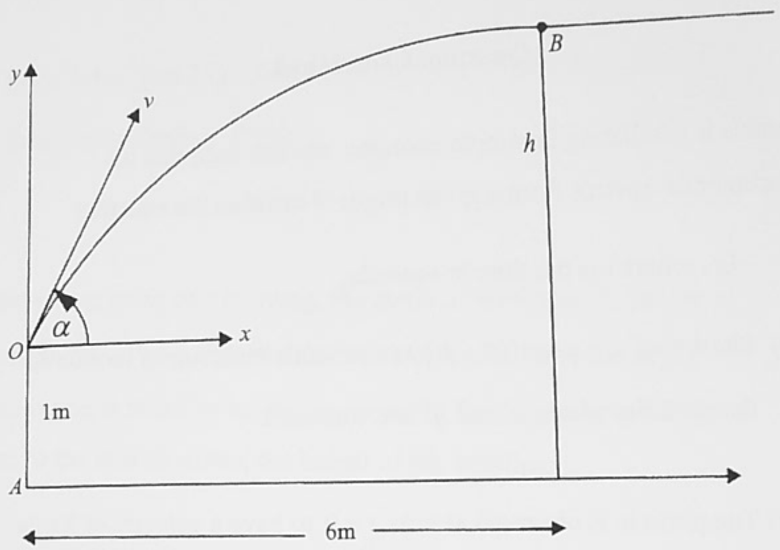
d) Using $t = \tan\frac{\theta}{2}$, find the general solution in radians to $\sin\theta - \cos\theta = 1$. **3**

End of Question 6.

Start a new booklet.

Marks.

Question 7 (12 marks).



a) A girl, 1 metre tall, throws a frisbee from a point O, with velocity v m/s at an angle α with the horizontal. It strikes the wall of a building at the highest point B of its trajectory. It takes $1\frac{1}{2}$ seconds to travel from O to B and the wall is 6 metres away from the girl. Taking the coordinate axes from O as shown and $g \doteqdot 10$ m/s^2.

(i) Prove that at any time t, the position of the frisbee is given by: **2**

$$x = vt\cos\alpha \text{ and } y = vt\sin\alpha - 5t^2.$$

(ii) Show that $v\cos\alpha = 4$ and $v\sin\alpha = 15$. **2**

(iii) Determine the initial velocity v, in exact form, and the angle of projection α to the nearest degree. **2**

(iv) Find the height h of the building, correct to 2 decimal places. **1**

Question 7 continued on next page.

Question 7 continued. **Marks.**

b) $P(x,y)$ is a point on the curve $y = e^{-x^2}$, where $x > 0$, O is the origin, and the perpendiculars from P to the x-axis and y-axis meet at A and B respectively.

(i) Show that the maximum area of the rectangle $OAPB$ is $\dfrac{1}{\sqrt{2e}}$. 2

(ii) Show that the minimum length of OP is $\sqrt{\dfrac{1+\ln 2}{2}}$. 3

End of Question 7.

End of Examination.