Question 1

a) $\int \frac{2}{\sqrt{36 - x^2}}\, dx$

$= 2\sin^{-1}\frac{x}{6} + c$ 1 r/w

b) $\frac{d}{dx}\left(\tan^{-1}(\ln x)\right)$

$= \frac{1/x}{1 + (\ln x)^2}$

$= \frac{1}{x + x(\ln x)^2}$ 1 r/w. Subsequent errors ignored

c) $A\overset{x_1\ \ y_1}{(-3, 8)}$ and $B\overset{x_2\ \ y_2}{(7, -3)}$ $\overset{m}{2} : \overset{n}{3}$

$x = \frac{nx_1 + mx_2}{m+n}$ $y = \frac{ny_1 + my_2}{m+n}$

$\therefore = \frac{3\times(-3) + 2\times 7}{2+3}$ $= \frac{3\times 8 + 2\times(-3)}{2+3}$

$= \frac{5}{5}$ 1 $= 18/5$ 1

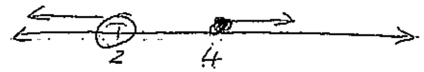
$= 1$

$\therefore$ Point is $(1, 18/5)$

1

d) $\frac{4}{x-2} \le 2$.   C.P $x = 2$.

Consider $\frac{4}{x-2} = 2$



$2x - 4 = 4$

$2x = 8$

$x = 4$

Test

$x = 1$ $\frac{4}{1-2} \le 2$ True   Testing 1

$x = 3$ $\frac{4}{3-2} \le 2$ False

$x = 5$ $\frac{4}{5-2} \le 2$ True

Sol^n $x < 2$ 1 and $x \ge 4$ 1

f) $u = x - 3$ $\int_4^5 \frac{x}{\sqrt{x-3}}\, dx$ When $x = 5$, $u = 5 - 3 = 2$; $x = 4$, $u = 4 - 3 = 1$

$= \int_1^2 \frac{u+3}{\sqrt{u}}\, du$

$= \int_1^2 u^{1/2} + 3u^{-1/2}\, du$ $u = x - 3$, $du = dx$

$= \left[\frac{2u^{3/2}}{3} + \frac{3u^{1/2}}{1/2}\right]_1^2$ 1 Correct integral and limits

$= \frac{2}{3}\times 2^{3/2} + 6\sqrt{2} - \frac{2}{3} - 6$ 1 Correct integration

$= \frac{2}{3}\times(\sqrt{2})^3 + 6\sqrt{2} - \frac{2}{3} - 6$ 2 Correct substitution and attempt at tidying answer

$= \frac{2}{3}\times 2\sqrt{2} + 6\sqrt{2} - \frac{20}{3}$

$= \frac{4}{3}\sqrt{2} + 6\sqrt{2} - \frac{20}{3}$

$= 22\sqrt{2} - 20. = 2(11\sqrt{2} - 10).$

e) $2x - y - 4 = 0 \qquad y = mx + 3 \qquad \theta = 45^\circ$

$\tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right| \qquad y = 2x - 4$

$\therefore \tan 45 = \left|\dfrac{2 - m}{1 + 2m}\right|$

$\therefore \left|\dfrac{2-m}{1+2m}\right| = 1$ (1)

$\therefore \dfrac{2-m}{1+2m} = 1$ or $\dfrac{2-m}{1+2m} = -1$

$2 - m = 1 + 2m$ or $2 - m = -1 - 2m$

$3m = 1 \qquad\qquad 3 = -m$

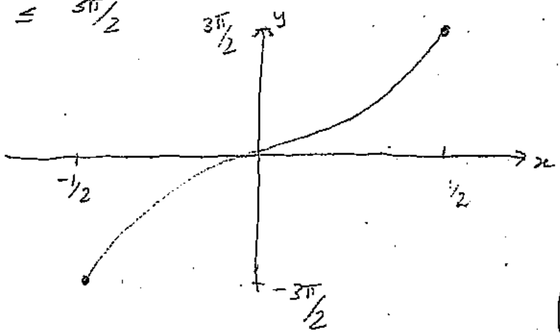
$\underline{m = 1/3}$ (1) $\qquad \underline{m = -3}$

Question 2

$f(x) = 3\sin^{-1} 2x$

a) $-\pi/2 \le \dfrac{y}{3} \le \pi/2 \qquad -1 \le 2x \le 1$

$-\dfrac{3\pi}{2} \le y \le 3\pi/2 \qquad -1/2 \le x \le 1/2$



(1) - shape

(1) - domain & range

b) i) $\dfrac{d}{dx}(x\cos^2 x)$

$= x \times 2\cos x(-\sin x) + \cos^2 x$

$= \underline{-x\sin 2x + \cos^2 x}$

(1) Correct product rule

(1) Correct derivative of $\cos^2 x$

ii) $\int x \sin 2x \, dx$

Now

$\int(\cos^2 x - x\sin 2x)\,dx = x\cos^2 x$ from i)

$\therefore \int \cos^2 x - \int x \sin 2x \, dx = x\cos^2 x$

$\therefore \int x\sin 2x\,dx = \int \cos^2 x\,dx - x\cos^2 x$ (1) Ex... for $\int x$...

$= \int \dfrac{1 + \cos 2x}{2}\,dx - x\cos^2 x$

$= \underline{\dfrac{x}{2} + \dfrac{\sin 2x}{4} - x\cos^2 x + C}$

c) $P(x) = x^3 + ax^2 - 2x + b \qquad x + 1$

$P(-1) = 0 \qquad \therefore (-1)^3 + a(-1)^2 - 2(-1) + b = 0$

$-1 + a + 2 + b = 0$

$\therefore a + b = -1$ — (1)

$P(3) = 4 \qquad \therefore (3)^3 + a(3)^2 - 2(3) + b = 4$

$\therefore 27 + 9a - 6 + b = 4$

$9a + b = -17$ — (2)

(2) − (1)

d) $f(x) = x - e^{-2x}$ $\qquad x = 0.3$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

(1) $f'(x) = 1 + 2e^{-2x}$

$$\therefore x_1 = 0.3 - \frac{0.3 - e^{-0.6}}{1 + 2e^{-0.6}}$$ (1)-sub

$f(0.3) = 0.3 - e^{-0.6}$

$f'(0.3) = 1 + 2e^{-0.6}$

$\doteq 0.418615978$

$= 0.42$ (2 dp.) (1) - correctly rounded answer

Question 3

a)
$$\lim_{x \to \infty} \frac{3x^2 + 1}{2 - 7x - 5x^2}$$

(2) Marks Correct with correct notation

$$= \lim_{x \to \infty} \frac{3x^2/x^2 + 1/x^2}{2/x^2 - 7x/x^2 - 5x^2/x^2}$$

(1) - Incorrect use of notation with correct answer

Incorrect answer with minor error

$$= \lim_{x \to \infty} \frac{3 + 1/x^2}{2/x^2 - 7/x - 5}$$

$$= -3/5$$

since $\lim_{x \to \infty} \frac{1}{x} = 0$

b) i) $\sqrt{3}\cos\theta - \sin\theta$ $\qquad r\cos(\theta + \alpha)$

$r\cos(\theta + \alpha) = r(\cos\theta\cos\alpha - \sin\theta\sin\alpha)$

$\sqrt{3}\cos\theta - \sin\theta = r(\cos\theta\cos\alpha - \sin\theta\sin\alpha)$

Equating coefficients

$r\cos\alpha = \sqrt{3}$ $\qquad r\sin\alpha = 1$

(1)

Squaring and adding

$r^2\cos^2\alpha + r^2\sin^2\alpha = 3 + 1$

$\therefore r^2 = 4$

$r = 2, \ r > 0$

$\therefore 2\cos\alpha = \sqrt{3}$ $\qquad 2\sin\alpha = 1$

$\cos\alpha = \sqrt{3}/2$ $\qquad \sin\alpha = 1/2$

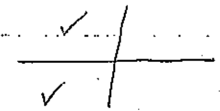
Since $0 < \alpha < \pi/2$

$\alpha = \pi/6$ (1) (1)

$\therefore \sqrt{3}\cos\theta - \sin\theta = 2\cos(\theta + \pi/6)$

ii) $2\cos(\theta + \pi/6) = -1$

$\cos(\theta + \pi/6) = -1/2$



$\therefore \theta + \pi/6 = 2\pi/3, \ 4\pi/3$

2nd and 3rd quadrants

$\therefore \theta = \pi/2, \ 7\pi/6$

(2) - Correct angles and setting out

(1) - 1 error, too many angles, one incorrect angle

c) $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

In $\triangle ADC$ $AC = 5$ by Pythagoras' Th since $\triangle ADC$ is right angled.

In $\triangle ADB$ $AB = \sqrt{2^2 + 3^2}$

$= \sqrt{13}$ also by Pythagoras' Th since $\triangle ACD$ is right angled

$\therefore \sin\alpha = 4/5$ $\qquad \cos\alpha = 3/5$ (1)

$\sin\beta = 3/\ldots$ $\qquad \cos\beta = 2/\sqrt{13}$

$$\sin(\alpha-\beta) = \frac{4}{5}\times\frac{3}{\sqrt{13}} - \frac{3}{5}\times\frac{2}{\sqrt{13}} \quad (1)$$

$$= \frac{12-6}{5\sqrt{13}}$$

$$= \frac{6}{5\sqrt{13}}\times\frac{\sqrt{13}}{\sqrt{13}}$$

$$= \frac{6\sqrt{13}}{65} \quad (1)$$

d) i. $\angle WYX = \angle WZY$ Angle between tangent and chord equals the angle at the circumference in the alternate segment.

Similarly $\angle ZWY = \angle WXY$ (1) For both angle justifications.

$\therefore \angle WYZ = \angle XWY$ angle sum triangles WXY and WZY with two equal angles in both triangles (1)

but $\angle WYZ$ and $\angle XWY$ are alternate and equal $\therefore WX \parallel YZ$ (1)

QUESTION 4

a) $$\int_0^{\ln 3} \frac{e^x}{e^x+9}\,dx$$

$$= \Big[\ln(e^x+9)\Big]_0^{\ln 3} \quad (1)$$

$$= \ln(e^{\ln 3}+9) - \ln(e^0+9)$$

$$= \ln 12 - \ln 10$$

$$= \ln 1{\cdot}2 \quad (1)$$

b). $$\binom{6}{r} 2^{6-r} x^{6-r} \cdot (-1)^r \cdot x^{-r}$$

where $6-2r=0$

$r=3$ (1)

$$\therefore {}^6C_3\, 8x^3 \times -\frac{1}{x^3}$$

$$= -160 \quad (1)$$

(c) Show statement is true for $p=1$

$\therefore 4(1)+3(1)^2+2(1)^3 = 9$

which is div by 3

$\therefore$ Statement is true for $p=1$ (1)

Assume statement is true for $n=k$

$\therefore 4k+3k^2+2k^3 = 3M$ where M is a +ve integer.

RTP the statement is true for $p=k+1$

Now $4(k+1) + 3(k+1)^2 + 2(k+1)^3$

$= (k+1)\left(4 + 3(k+1) + 2(k+1)^2\right)$

$= (k+1)(4 + 3k + 3 + 2k^2 + 4k + 2)$

$= (k+1)(2k^2 + 7k + 9)$

$= 2k^3 + 7k^2 + 9k + 2k^2 + 7k + 9$

$= 2k^3 + 9k^2 + 16k + 9$ — 1

$= (2k^3 + 3k^2 + 4k) + 6k^2 + 12k + 9$

$= 3M + 3(2k^2 + 6k + 3)$ — from assumption.

$= 3(M + 2k^2 + 6k + 3)$ — 1

where $(M + 2k^2 + 6k + 3)$ is integral.

If statement is true for $p = k$, then it is true for $p = k+1$. Since it is true for $p = 1$, then true for $p = 2$ & so on. It is true for all +ve integers p.

ii) When $t = 0$ $T = 100°c$.

$t = 15$ $T = 70°c$.

$100 = Ae^{k \times 0} + 22$

$100 = A + 22$

$A = 78$

$70 = 78e^{15k} + 22$ ← 1

$78e^{15k} = 48$

$e^{15k} = 8/13$ $15k = \ln 8/13$ ← 1

e) $LHS = \sqrt{\dfrac{1 + \sin 2\theta}{1 - \sin 2\theta}}$

$= \sqrt{\dfrac{1 + 2\cos\theta\sin\theta}{1 - 2\cos\theta\sin\theta}}$ $\div \cos^2\theta$

$= \sqrt{\dfrac{\sec^2\theta + \frac{2\cos\theta\sin\theta}{\cos^2\theta}}{\sec^2\theta - \frac{2\cos\theta\sin\theta}{\cos^2\theta}}}$

$= \sqrt{\dfrac{\sec^2\theta + 2\tan\theta}{\sec^2\theta - 2\tan\theta}}$ 1

$= \sqrt{\dfrac{1 + \tan^2\theta + 2\tan\theta}{1 + \tan^2\theta - 2\tan\theta}}$

$= \sqrt{\dfrac{(1 + \tan\theta)^2}{(1 - \tan\theta)^2}}$ 1

$= \dfrac{1 + \tan\theta}{1 - \tan\theta}$

$= RHS$.

Question 5

i) $v^2 = 27 + 18x - 9x^2$

$\frac{1}{2}v^2 = \frac{1}{2}(27 + 18x - 9x^2)$

$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = \frac{d}{dx}\left\{\frac{1}{2}(27 + 18x - 9x^2)\right\}$

$= 9 - \frac{9}{2} \times 2x$

$= 9 - 9x$

$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = 9(1 - x)$

$= -9(x - 1)$ (1)

Since $\ddot{x} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

$\ddot{x} = 9(1 - x)$ since this is a multiple of the displacement
$\ddot{x} = -3^2(x - 1)$
of the form $\ddot{x} = -n^2(x - a)$ the motion is S.H.

The centre of motion is 1 and the period is $\frac{2\pi}{3}$ (1)

When $v = 0$

$27 + 18x - 9x^2 = 0$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3$ or $x = -1$

$-1 \quad 1 \quad 3$

Particle oscillates between -1 and 3

$\therefore$ amplitude is 2 (1)

ii) $\ddot{x} = -3^2(x - 1)$

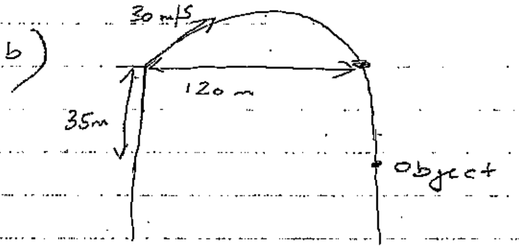
When $x = 2$ (1cm from centre of motion)

$\ddot{x} = -9(2 - 1)$
$= -9 \text{ cm/s}^2$

When $x = 0$
$\ddot{x} = -9(-1)$
$= 9 \text{ cm/s}^2$

(1) for both cases

b)



$x = 30t\cos\alpha$ — (1)

$y = -5t^2 + 30t\sin\alpha$ — (2)

From (1) $t = \frac{x}{30\cos\alpha}$ subs into (2)

$y = -5\left(\frac{x}{30\cos\alpha}\right)^2 + 30\left(\frac{x}{30\cos\alpha}\right)\sin\alpha$ (1)

$= \frac{-5x^2}{900\cos^2\alpha} + x\tan\alpha$

$y = \frac{-x^2}{180}\sec^2\alpha + x\tan\alpha$

When $x = 120$ $y = -35$

$\therefore -35 = \frac{-120^2}{180}\sec^2\alpha + 120\tan\alpha$ (1)

$-6300 = -14400(1 + \tan^2\alpha) + 21600\tan\alpha$

$14400\tan^2\alpha - 21600\tan\alpha + 8100 = 0$

$48\tan^2\alpha - 72\tan\alpha + 27 = 0$

$16\tan^2\alpha - 24\tan\alpha + 9 = 0$

$(4\tan\alpha - 3)^2 = 0$

$\therefore \tan\alpha = 3/4$ (1)

ii) When $x = 120$ $\alpha = 36°52'$

$\therefore$ $120 = 30t\cos 36°52'$

$t = \dfrac{120}{30\cos 36°52'}$

or use $\tan\alpha = 3/4$, $\cos\alpha = 4/5$ $\therefore t = \frac{4}{\cos\alpha} = 5$

$= 5$ seconds (1)

c) i) $y = x + \dfrac{4}{x}$ When $\dfrac{dy}{dx} = 0$

$dy/dx = 1 - \dfrac{4}{x^2}$ $1 - \dfrac{4}{x^2} = 0$

$\dfrac{d^2y}{dx^2} = \dfrac{8}{x^3}$ $\dfrac{-4}{x^2} = 1$

$x^2 = 4$

$x = \pm 2$

When $x = 2$ $y = 2 + \frac{4}{2} = 4$ $x = -2$ $y = -2 - \frac{4}{2} = -4$

$\therefore$ sps are at $(2, 4)$ and $(-2, -4)$

When $x = 2$ $\dfrac{d^2y}{dx^2} = \dfrac{8}{2^3} > 0$ $\therefore (2, 4)$ is a min s.p When $x = -2$ $\dfrac{d^2y}{dx^2} = \dfrac{8}{(-2)^3} < 0$ $\therefore (-2, -4)$ is a max s.p

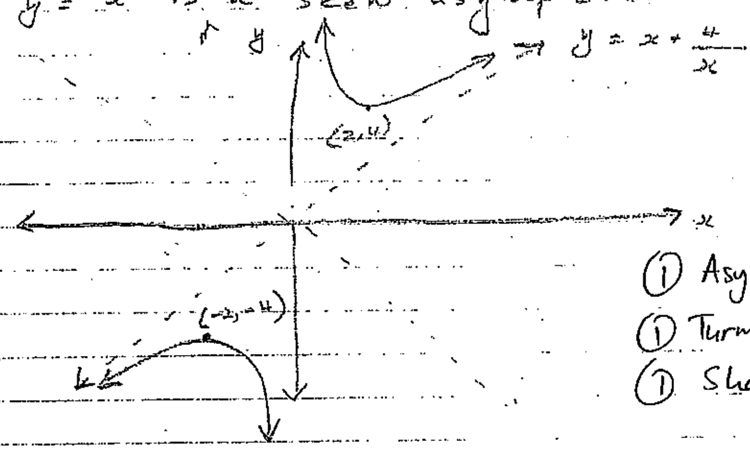
When $\dfrac{d^2y}{dx^2} = 0$ $\dfrac{8}{x^3} = 0$ which has no sol^{n}s

$\therefore$ there are no inf pts.

$\lim\limits_{x \to \infty} x + 4/x$ $\lim\limits_{x \to -\infty} x + 4/x$

$= x$ $\therefore$ $= x$

$\therefore$ $y = x$ is a skew asymptote



(1) Asymptotes
(1) Turning pts.
(1) Shape

$x = 0$ is a vertical asymptote.

ii) $x + \dfrac{4}{x} = k$ Since $y = x + \dfrac{4}{x}$ does not exist when $-4 < x < 4$ {(2, 4) is a min (−2, −4) is a max} (1)

$x + \dfrac{4}{x} = k$ will have no sol^n if $-4 < k < 4$

Question 6

a) i) In $\triangle AOT$ $\tan 45° = \dfrac{h}{AO}$

$\therefore AO = \dfrac{h}{\tan 45°}$ (1)

But $\tan 45° = 1$ $\therefore AO = h$ r/w

(0, 2)

ii) In $\triangle BOT$ $\quad \tan\alpha = \dfrac{h}{OB}$

$\therefore OB = \dfrac{h}{\tan\alpha}$

$= h\cot\alpha$ (1)

In $\triangle AOB$ by the cosine rule

$50^2 = OB^2 + OA^2 - 2\times OB\times OA\cos 60$

$50^2 = h^2\cot^2\alpha + h^2 - 2h\cot\alpha\times h\times\frac{1}{2}$ (1)

$50^2 = h^2\cot^2\alpha + h^2 - h^2\cot\alpha$

iii) When $h = 30$

$50^2 = 30^2\cot^2\alpha + 30^2 - 30^2\cot\alpha$

$900\cot^2\alpha - 900\cot\alpha + 900 - 2500 = 0$

$900\cot^2\alpha - 900\cot\alpha - 1600 = 0$

$9\cot^2\alpha - 9\cot\alpha - 16 = 0$

$\cot\alpha = \dfrac{9\pm\sqrt{81-4\times 9\times -16}}{18}$ (1)

$= \dfrac{9\pm\sqrt{657}}{18}$

* Since α is acute $\quad \cot\alpha = \dfrac{9+\sqrt{657}}{18}$ (1)

$\tan\alpha = \dfrac{18}{9+\sqrt{657}}$ (1)

$\alpha = 27°$ to nearest degree.

b) i) $\dfrac{d}{dx}\left(x\sin^{-1}x + (1-x^2)^{\frac{1}{2}}\right)$

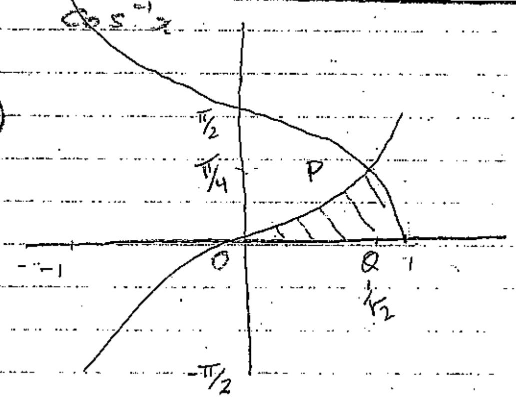
$= x\times\dfrac{1}{\sqrt{1-x^2}} + \sin^{-1}x + \frac{1}{2}(1-x^2)^{-\frac{1}{2}}\times -2x$ (1)

$= \dfrac{x}{\sqrt{1-x^2}} + \sin^{-1}x - \dfrac{x}{\sqrt{1-x^2}}$

$= \sin^{-1}x$

ii) $x\cos^{-1}x - \sqrt{1-x^2}$ is a primitive of $\cos^{-1}x$ (1)

iii)



$\sin^{-1}x = \cos^{-1}x$

when $x = 1/\sqrt{2}$

$\sin^{-1}(1/\sqrt{2}) = \pi/4$

$\cos^{-1}(1/\sqrt{2}) = \pi/4$

$\therefore \sin^{-1}x = \cos^{-1}x$ (1)

when $x = 1/\sqrt{2}$

iv) $\displaystyle\int_0^{1/\sqrt{2}}\sin^{-1}x\,dx + \int_{1/\sqrt{2}}^{1}\cos^{-1}x\,dx$

Method 1

from i) and ii)

$A = $ (1) $\left[x\sin^{-1}x + \sqrt{1-x^2}\right]_0^{1/\sqrt{2}} + \left[x\cos^{-1}x - \sqrt{1-x^2}\right]_{1/\sqrt{2}}^{1}$

(1) $= \frac{1}{\sqrt{2}}\sin^{-1}\frac{1}{\sqrt{2}} + \sqrt{1-(1/\sqrt{2})^2} - \left(0\sin^{-1}0 + \sqrt{1-0^2}\right) + 1.\cos^{-1}1$

$- \sqrt{1-(1)^2} - \left(1/\sqrt{2}\cos^{-1}1/\sqrt{2} - \sqrt{1-(1/\sqrt{2})^2}\right)$ $\quad \sqrt{2}-1$

$= \frac{2}{\sqrt{2}} - 1$

$= \sqrt{2} - 1$ sq units

OR $A = \int_0^{\pi/4} \cos y \, dy - \int_0^{\pi/4} \sin y \, dy$ (1)

$= \left[\sin y\right]_0^{\pi/4} - \left[-\cos y\right]_0^{\pi/4}$

$= \sin \pi/4 - \sin 0 + \cos \pi/4 - \cos 0$ (1)

$= 1/\sqrt{2} - 0 + 1/\sqrt{2} - 1$

$= 2/\sqrt{2} - 1$

$= \sqrt{2} - 1$ sq units (1)

Question 7

a) $\ddot{x} = 9(x-2)$ When $t = 0$ $x = 4$ $\dot{x} = 6$

i) $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = 9(x-2)$

$\frac{1}{2}v^2 = \int 9(x-2)\,dx$

$\frac{1}{2}v^2 = \frac{9x^2}{2} - 18x + C$

When $x = 4$ $v = 6$

$\therefore \frac{1}{2} \times 6^2 = \frac{9 \times 4^2}{2} - 18 \times 4 + C$

$18 = 72 - 72 + C$

$\therefore c = 18$

$\therefore \frac{1}{2}v^2 = \frac{9x^2}{2} - 18x + 18$

$v^2 = 9x^2 - 36x + 36$

$v^2 = 9(x^2 - 4x + 4)$

$v^2 = 9(x-2)^2$

Q7

a) $\ddot{x} = 9(x-2)$

$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = 9(x-2)$ 1

$\frac{1}{2}v^2 = 9\int (x-2)\,dx$

$\frac{1}{2}v^2 = \frac{9}{2}(x-2)^2 + C$ 1

When $x = 4$, $v = 6$:

$\frac{1}{2} \times 6^2 = \frac{9}{2}(4-2)^2 + C$

$18 = 18 + C$

$C = 0$ 1

$\frac{1}{2}v^2 = \frac{9}{2}(x-2)^2$

$v^2 = 9(x-2)^2$ (3)

ii) $v = \pm 3(x-2)$

But when $x = 4$, $v = +6 > 0$ 1

so $v = 3(x-2)$

$\frac{dx}{dt} = 3(x-2)$

$\frac{dt}{dx} = \frac{1}{3(x-2)}$

$t = \frac{1}{3}\int \frac{1}{x-2}\,dx$

$t = \frac{1}{3}\ln(x-2) + c$ 1

When $t = 0$, $x = 4$:

$0 = \frac{1}{3}\ln 2 + c$

$c = -\frac{1}{3}\ln 2$

$t = \frac{1}{3}\ln(x-2) - \frac{1}{3}\ln 2$

$t = \frac{1}{3}\ln\left(\frac{x-2}{2}\right)$

$\frac{x-2}{2} = e^{3t}$

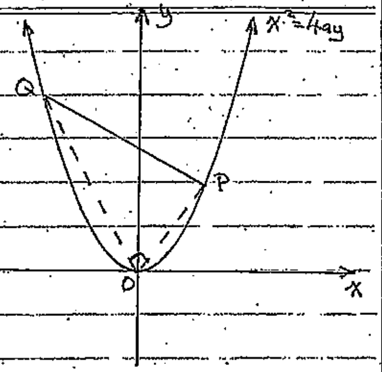
$x = 2e^{3t} + 2$ 1 (3)

b) $x^2 = 4ay$

Vertex $= (0, 0)$

$P = (2ap, ap^2)$

$Q = (2aq, aq^2)$



i) $m_{PO} = \dfrac{ap^2 - 0}{2ap - 0}$

$= \dfrac{p}{2}$ 1

$m_{QO} = \dfrac{q}{2}$

Since chord PQ subtends right angle at vertex,

$m_{PO} \times m_{QO} = -1$ 1

$\Rightarrow \dfrac{p}{2} \times \dfrac{q}{2} = -1$

$pq = -4$ (2)

ii) $x^2 = 4ay$

$y = \dfrac{x^2}{4a}$

$y' = \dfrac{x}{2a}$

$m_P = \dfrac{2ap}{2a} = p$

m of normal at $P = \dfrac{-1}{p}$ 1

Eq. of normal at P

$\Rightarrow \dfrac{-1}{p} = \dfrac{y - ap^2}{x - 2ap}$ 1

$-x + 2ap = py - ap^3$

$x + py = 2ap + ap^3$ — (1) (2)

iii) Similarly eq. of normal at Q

is $x + qy = 2aq + aq^3$ — (2)

(1) − (2) $\Rightarrow (p-q)y = 2a(p-q) + a(p^3 - q^3)$

$y = 2a + a(p^2 + pq + q^2)$ — (3)

Sub (3) into (1),

$x + 2ap + ap(p^2 + pq + q^2) = 2ap + ap^3$

$x + 2ap + ap^3 + ap^2q + apq^2 = 2ap + ap^3$

$x = -apq(p+q)$

Pt. of intersection of the normals at P & Q

$= (-apq(p+q),\ 2a + a(p^2 + pq + q^2))$ 1

Since $pq = -4$ from (i),

$x = 4a(p+q) \Rightarrow p + q = \dfrac{x}{4a}$

$y = 2a + a(p^2 + 2pq + q^2 - pq)$

$= 6a + a(p+q)^2$ 1

$y - 6a = a\left(\dfrac{x}{4a}\right)^2$

$y - 6a = \dfrac{x^2}{16a}$

$16a(y - 6a) = x^2$ (2)

(8)