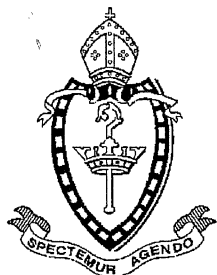


# NEWCASTLE GRAMMAR SCHOOL

Student Number: \_\_\_\_\_



2011

## TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

# Mathematics

*Examination Date: 17<sup>th</sup> August 2011*

Examiner: K Armstrong

### General Instructions

- Reading Time - 5 minutes
- Working Time - 3 hours
- Write using a blue or black pen
- Write your student number on every booklet
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question
- Each question attempted is to be returned in a separate Writing Booklet clearly marked Questions 1 etc.
- If required, additional booklets may be requested

### Total marks - 120

- Attempt Questions 1-10
- All questions are of equal value

**Question 1** (12 Marks) Use a SEPARATE writing booklet

**Marks**

a) If  $a = 0.45$ , evaluate  $\frac{1-a^2}{1+a^2}$  correct to 2 decimal places

2

b) Simplify  $\frac{4x-1}{2} - \frac{x+3}{3}$

2

c) Find a primitive of  $2x^4 + e^{2x}$

2

d) Factorise completely  $6x^3 - 48$

2

e) Solve for  $x$ :  $|2x-1| \leq 5$

2

f) Write  $\frac{2}{\sqrt{5}-1}$  with a rational denominator

2

**Question 2** (12 Marks) Use a SEPARATE writing booklet

a) Differentiate with respect to  $x$

i)  $x^2 e^{-x}$

2

ii)  $\frac{x^2}{3x+1}$

2

b) Solve  $\cos \theta = \frac{1}{\sqrt{2}}$  for  $0 \leq \theta \leq 2\pi$

2

c) Find  $\int \frac{2x^2}{2x^3-3} dx$

2

d) Evaluate  $\int_0^{\frac{\pi}{3}} \sin 2x dx$

2

e) Find the equation of the normal to  $y = \log_e x$  at the point  $(e, 1)$

2

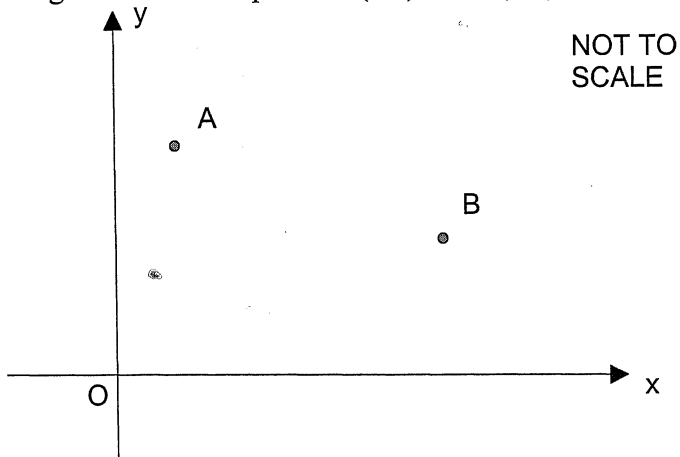
Question 3 (12 Marks) Use a SEPARATE writing booklet

Marks

a) Evaluate  $\sum_{n=2}^{20} 6n - 2$

2

b) The diagram shows the points A(1,3) and B(4,2)



i) Calculate the length of the interval AB.

2

ii) Find the gradient of AB.

1

iii) Find the equation of the line through A perpendicular to AB.

2

iv) Show that the line in part (iii) passes through O.

1

v) Calculate the length of AO and hence find the size of  $\angle ABO$ .

2

vi) Find the coordinates of point C so that ABCO is a square.

2

**Question 4 (12 Marks)** Use a SEPARATE writing booklet

- a) A game is played in which two coloured dice are thrown once. The six faces of the red die are numbered 2, 4, 6, 8, 10 and 12. The six faces of the black die are numbered 1, 3, 5, 7, 9 and 11. The player wins if the number on the red die is larger than the number on the black die.
- i) By drawing up a table of outcomes, or otherwise, calculate the probability of a player winning a game. 2
- ii) Calculate the probability that the player wins at least once in two successive games. 2
- b) If  $\frac{d^2y}{dx^2} = 2 - \frac{4}{x^2}$  and at  $(1, 6)$   $\frac{dy}{dx} = 3$ , find  $y$  in terms of  $x$  3
- c) Find the value of  $\log_{16} 128$  1
- d) The sides of a triangle are 8cm, 10cm and 12cm
- i) Calculate the size of the smallest angle (Answer to the nearest minute) 2
- ii) Calculate the area of the triangle (Answer correct to 2 decimal places) 2

**Question 5** (12 Marks) Use a SEPARATE writing booklet

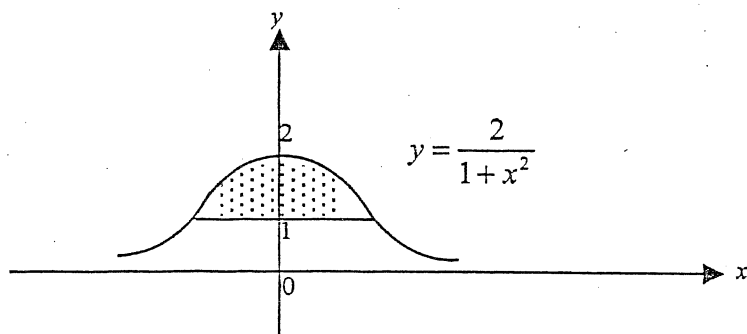
Marks

- a) If  $\alpha$  and  $\beta$  are roots of a quadratic equation  $3x^2 - 4x - 1 = 0$  find
- i)  $\alpha + \beta$  1
  - ii)  $\alpha\beta$  1
  - iii)  $\alpha^{-1} + \beta^{-1}$  2
  - iv)  $\alpha\beta^3 + \beta\alpha^3$  3
- b) i) Sketch the curve  $y = \tan \frac{x}{2}$  for  $-2\pi \leq x \leq 2\pi$  2
- ii) State the period of  $y = \tan \frac{x}{2}$  1
- iii) Solve the equation  $\tan \frac{x}{2} = 1$  for  $-2\pi \leq x \leq 2\pi$  2

**Question 6** (12 Marks) Use a SEPARATE writing booklet

- a) State the domain and range for the curve  $y = \frac{4}{x-3}$  2

b)



Consider the region bounded by the curve  $y = \frac{2}{1+x^2}$  and the line  $y = 1$  as shown in the diagram.

The region is revolved about the  $y$ -axis. Find the volume of the solid of revolution generated

4

**Question 6 continues next page**

**Question 6(continued)****Marks**

- c) A chord AB of a circle with centre O has length 16cm. If the radius of the circle is 10cm, calculate
- i) the size of angle AOB (Answer to the nearest degree) 2
  - ii) the length of the minor arc AB (Answer correct to 1 decimal place) 2
  - iii) the area of the minor segment formed by the chord AB (Answer to the nearest  $\text{cm}^2$ ) 2

**Question 7 (12 Marks) Use a SEPARATE writing booklet**

- a) A particle moves in a straight line. At time  $t$  seconds, its displacement  $x$  metres from the origin is given by
- $$x = 8t - 2t^2$$
- i) Sketch the graph of  $x$  as a function of  $t$ , showing the vertex 1
  - ii) Find the distance the particle travels in the first three seconds 2
- b) Find the coordinates of the point P on the curve  $y = 3e^x - 2x$  at which the tangent to the curve is parallel to the line  $y = 2x - 1$  3
- c) For the curve  $y = 2x^3 - 6x^2$
- i) Determine all the stationary points and their nature. 2
  - ii) Find what values of  $x$  the curve is concave down and decreasing. 2
  - iii) Sketch the curve showing **all** main features in the domain  $-2 \leq x \leq 4$  2

Question 8 (12 Marks) Use a SEPARATE writing booklet

Marks

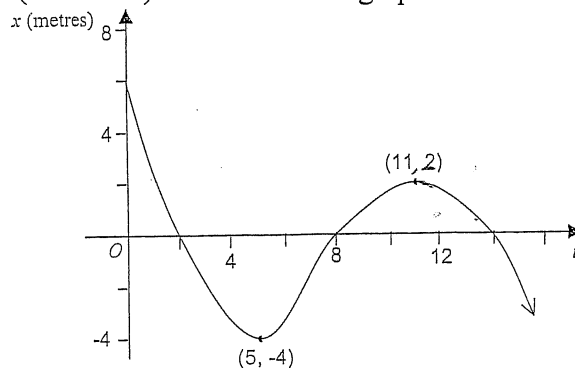
- a) The population  $P$  of a growing town satisfies the equation

$$P = P_0 e^{kt} \quad \text{where } t \text{ is time in years}$$

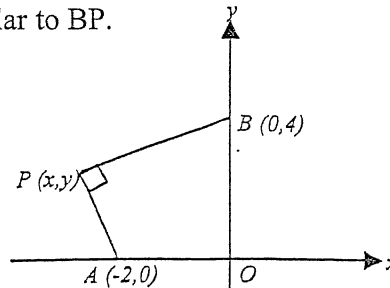
The initial population is 22 000 and five years later the population is 27 000

- i) Find  $P_0$  and  $k$  3  
 ii) When does the population reach 35 000? Give your answer correct to three significant figures. 2

- b) The displacement,  $x$  metres, from the origin of a particle moving in a straight line At any time ( $t$  seconds) is shown in the graph.



- i) When was the particle at the origin? 1  
 ii) How far did the particle travel in the first 11 seconds? (Show working) 2
- c)  $A(-2,0)$  and  $B(0,4)$  are two points in the number plane.  $P(x,y)$  is any point such that  $AP$  is perpendicular to  $BP$ .

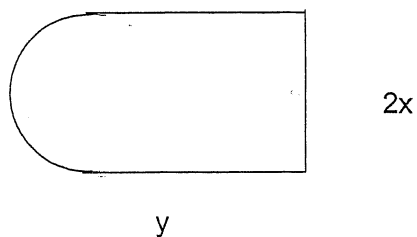


- i) Prove that the equation of the locus of  $P(x,y)$  is  $x(x+2) + y(y+4) = 0$  2  
 ii) Deduce that the equation in (i) represents a circle and find its centre and radius. 2

Question 9 (12 Marks) Use a SEPARATE writing booklet

Marks

- a) An artist's work is in the shape of a rectangle and a semi-circle as shown. The perimeter is to be 4 metres.



- i) Show that  $y = 2 - x - \frac{\pi x}{2}$  2
- ii) Show that the area of the work can be expressed as  $A = 4x - 2x^2 - \frac{\pi x^2}{2}$  2
- iii) Find the maximum area of the artwork. 3
- b) i) Show that  $\sin^2 x \cos x = \cos x - \cos^3 x$  1
- ii) Hence show that  $\frac{d}{dx} \left( \sin x - \frac{1}{3} \sin^3 x \right) = \cos^3 x$  2
- iii) Hence find  $\int 3 \cos^3 x \, dx$  2

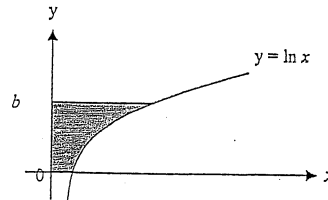


Question 10 (12 Marks) Use a SEPARATE writing booklet

Marks

- a) If the shaded area is one square unit, find the value of  $b$  as an irrational number.

3



- b) Solve  $e^{6x} - 7e^{3x} + 6 = 0$

3

- c) When Suzie was born, her grandparents set up a trust fund for her to receive on her 18th birthday. The conditions agreed upon for this trust fund involved investing \$100 every four months. Interest is calculated on the balance at the end of each year and is credited at the rate of 8%p.a.

- i) Show that after two years the trust fund will amount to  $\$(300 \times 1.08^2 + 300 \times 1.08)$

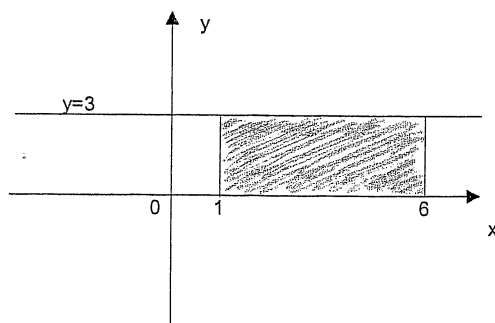
2

- ii) How much will Suzie receive on her 18th birthday from this trust fund?

2

- d)

2



State two methods to find the volume generated by rotating the area shown in the diagram about the  $x$ -axis.

## STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln \left( x + \sqrt{x^2 - a^2} \right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln \left( x + \sqrt{x^2 + a^2} \right)$$

NOTE :  $\ln x = \log_e x, \quad x > 0$