

Question 1 (15 Marks)**Marks**

(a) Find:

(i) $\int \frac{e^x}{\sqrt{e^{2x}-1}} dx$ 2

(ii) $\int \frac{1}{x^2-5x+6} dx$ 2

(iii) $\int \frac{d\theta}{2+\cos\theta}$ 3

(b) Evaluate: $\int_{-1}^1 \frac{x}{x^2+2x+5} dx$ 4

(c) If $I = \int_0^{\frac{\pi}{2}} \frac{\cos x}{\cos x + 2\sin x} dx$ and $J = \int_0^{\frac{\pi}{2}} \frac{\sin x}{\cos x + 2\sin x} dx$,

(i) Show that $2I - J = \ln 2$. 1

(ii) Evaluate $I + 2J$. 1

(iii) Hence, find the exact values of I and J . 2

Question 2 (15 Marks) START A NEW PAGE(a) Plot neatly on an Argand diagram the points A , B and C corresponding to the complex numbers w , w^2 and $w\bar{w}$ respectively where $w = \sqrt{3} + i$. 3(b) Let $z = x + iy$ be a complex number satisfying the inequality 4

$$z\bar{z} + (1-2i)z + (1+2i)\bar{z} \leq 4 \quad \text{where } x \text{ and } y \text{ are real.}$$

Sketch the locus of z on an Argand diagram.(c) (i) Solve the equation for w : 2

$$w^2 = -11 - 60i.$$

Write your answer in the form $w = x + yi$, where x and $y \in \mathbb{R}$ (ii) Hence, or otherwise, solve the equation: 3

$$z^2 - (1+4i)z - (1-17i) = 0$$

(d) Five girls and three boys are seated at random around a circular table. What is the probability that at least two boys are sitting next to each other? 3

Question 3 (15 Marks) START A NEW PAGE

Marks

- (a) $ABCD$ is a cyclic quadrilateral. Chords BE and DF bisect $\angle ABC$ and $\angle ADC$ respectively.

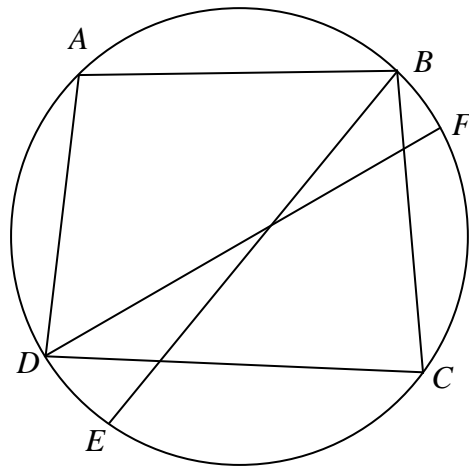


Diagram not to scale

Copy the diagram and prove that EF is a diameter of the circle.

3

- (b) (i) Show whether the function $f(x) = 2|x-1| - |x| + 2|x+1|$ is even, odd or neither, giving reasons. **2**
- (ii) Sketch the graph of the function $f(x) = 2|x-1| - |x| + 2|x+1|$, clearly showing all intercepts with the coordinate axes and critical points. Label all branches with the relevant equations. **3**
- (c) $P(x_1, y_1)$ is a point on the rectangular hyperbola $xy = 9$.
- (i) Show that the Cartesian equation of the tangent at P is $y_1x + x_1y = 18$. **2**
- (ii) Hence, or otherwise, derive the equation of the chord of contact from an external point $T(x_0, y_0)$ to the hyperbola $xy = 9$. **2**
- (iii) Prove that the chord of contact is a focal chord when T is a point on the directrix. **3**

Question 4 (15 Marks) START A NEW PAGE**Marks**

- (a) (i) Find all stationary points for the curve $y^2 = x(3-x)^2$. **3**
- (ii) Sketch the curve $y^2 = x(3-x)^2$, showing all stationary points and the intercepts with the coordinate axes. **3**
- (b) A particle of mass 2kg is projected vertically upwards with a velocity of $U \text{ ms}^{-1}$ in a medium which exerts a resistive force of $\frac{v}{10}$ Newtons.

- (i) Show that the maximum height H metres reached by the particle is given by: **3**

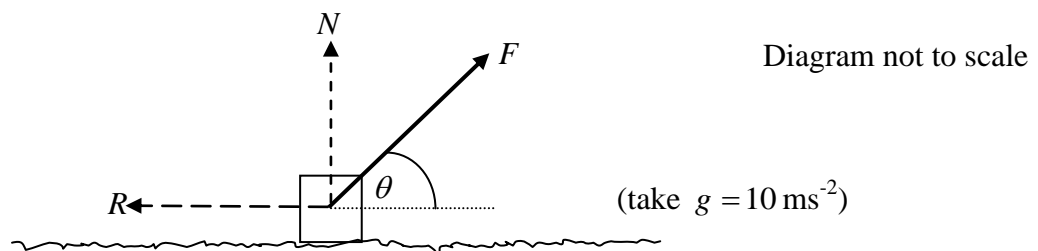
$$H = 20U + 4000 \ln\left(\frac{200}{200+U}\right) \quad (\text{take } g = 10 \text{ ms}^{-2})$$

- (ii) Find the time taken for the particle to reach the maximum height H . **3**
- (iii) If $U = 400$, show that the average speed during the ascent is: **3**

$$200\left(\frac{2}{\ln 3} - 1\right) \text{ ms}^{-1} .$$

Question 5 (15 Marks) START A NEW PAGE

- (a) A block of mass 5 kg is to be moved along a rough horizontal surface by a force (F Newtons) inclined at an angle of θ with the direction of motion where $0 \leq \theta \leq \frac{\pi}{2}$.



The motion is resisted by a frictional force (R Newtons) which is proportional to the normal reaction force (N Newtons) exerted on the block by the surface, such that $R = 0.2N$.

- (i) Show that $F = \frac{50}{5 \cos \theta + \sin \theta}$ Newtons, when the block is about to move. **3**
- (ii) Calculate the minimum value of F needed to overcome the frictional resistance between the block and the surface. **3**

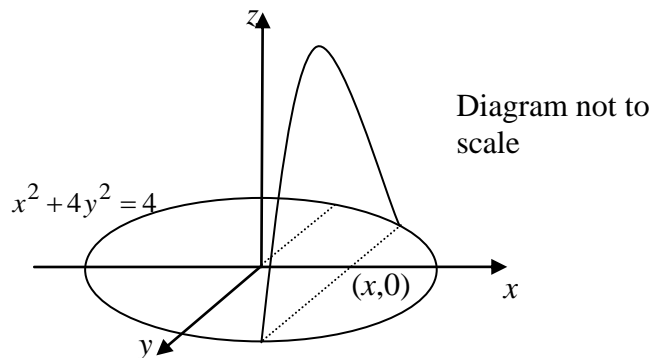
Question 5 continued over page

Question 5 continued

Marks

- (b) (i) A parabola has the equation $x^2 = 4ay$. Show that the area bounded by this parabola and the focal chord perpendicular to the axis is equal to $\frac{8a^2}{3}$ units². **3**

- (ii) A solid has an elliptical base whose equation is $x^2 + 4y^2 = 4$ and each cross-section perpendicular to the major axis of the base is a parabola with its focus on the major axis.



- (α) Show that the area of the parabolic cross-section, x units from the origin, is given by the formula

$$A(x) = \frac{4 - x^2}{6} \quad \mathbf{3}$$

- (β) Hence, find the volume of the resultant solid. **3**

Question 6 (15 Marks) START A NEW PAGE

- (a) The points $P(a \cos \theta, b \sin \theta)$ and $Q(a \cos \phi, b \sin \phi)$ lie on the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \text{ where } 0 < \theta < \phi \text{ and } a > b.$$

The points $P'(a \cos \theta, a \sin \theta)$ and $Q'(a \cos \phi, a \sin \phi)$ lie on the auxiliary circle and subtend a right angle at the origin.

- (i) Draw a neat sketch of the above information showing the relative positions of the points P, Q, P' and Q' . **2**
- (ii) Express the coordinates of Q in terms of θ . **1**
- (iii) The tangents at P and Q meet in point R . Find the coordinates of R in terms of θ . **4**
- (iv) Show that R lies on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$ **1**

Question 6 continued over page

Question 6 continued

Marks

- (b) (i) If $\tan(x)\tan(\theta-x) = k$ prove that:

4

$$\frac{1+k}{1-k} = \frac{\cos(2x-\theta)}{\cos\theta}$$

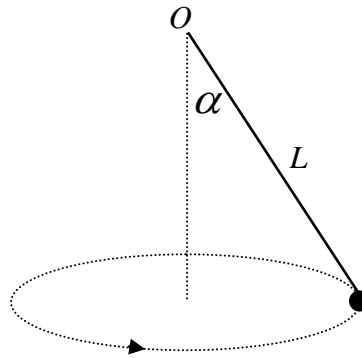
- (ii) Hence, or otherwise, solve the equation for all x .

3

$$\tan x \tan\left(\frac{\pi}{3} - x\right) = 2 + \sqrt{3}$$

Question 7 (15 Marks) START A NEW PAGE

- (a) A particle of mass m kg is fastened to one end of a light inextensible string of length L metres and the other end is attached to a fixed point O . The particle rotates with a uniform angular velocity ω rad/s about a vertical line through O .



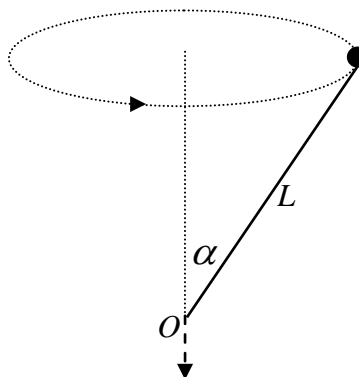
- (i) Show that if α is the angle of inclination of the string to the downward vertical, then $\alpha = \cos^{-1}\left(\frac{g}{L\omega^2}\right)$.

4

- (ii) Explain why steady circular motion is only possible when $\omega^2 > \frac{g}{L}$.

2

- (iii) The point O is now made to descend with an acceleration of f ms⁻², whilst the particle continues to rotate with uniform angular velocity ω .



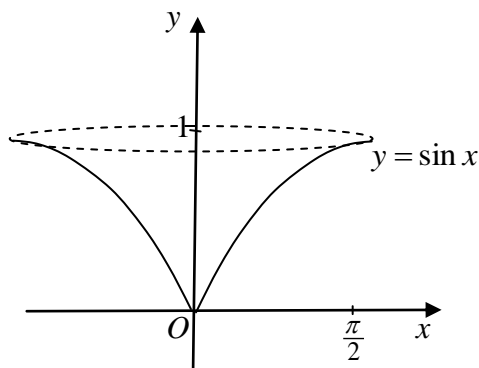
Find the value of f when the string again makes a constant angle of α with the upward vertical.

3

Question 7 continued

Marks

- (b) The area between the curve $y = \sin x$, from $x = 0$ to $x = \frac{\pi}{2}$, the y -axis and the line $y = 1$ is rotated about the y -axis.



- (i) Show that the volume of the solid formed can be found by using the formula 3

$$V = \pi \int_0^{\frac{\pi}{2}} x^2 \cos x \, dx$$

- (ii) Hence, calculate the volume of the solid. 3

Question 8 (15 Marks) START A NEW PAGE

- (a) The total number of different groups with 4 members which can be chosen from a group of n people is five times as many as the total number of different groups with 3 members which can be chosen from a group of $n - 2$ people. 3

Find all possible values of n .

- (b) Prove that $\tan^{-1}(5) + \tan^{-1}(3) + \tan^{-1}\left(\frac{4}{7}\right) = \pi$ 4

- (c) A curve, defined by the equation $x^2 + 2xy + y^5 = 4$, has a horizontal tangent at the point $P(X, Y)$.

- (i) Show that X is a root to the equation $x^5 + x^2 + 4 = 0$. 3

- (ii) Show that there is root to the equation $x^5 + x^2 + 4 = 0$ between -2 and -1 . 1

- (iii) With the use of a graph, or otherwise, show that X is the only real root to the equation $x^5 + x^2 + 4 = 0$. 4

End of Examination

