<u>Question 1</u> (12 Marks) **Marks**

(a) Find $\frac{d}{dx}(\tan 4x)$. **2**

(b) Find the co-ordinates of the point that divides the interval joining $A(7,2)$ and $B(11,6)$ externally in the ratio 3:5. **2**

(c) Evaluate $\lim_{x\to 0}\frac{3\sin x\cos x}{4x}$. **2**

(d) Solve $\cos 2x = -\frac{1}{2}$ for $0 \le x \le 2\pi$. **2**

(e) If $x = 1+\cos\theta$ and $y = 2-\sin\theta$ find a relationship between x and y only. **2**

(f) Evaluate $\int_0^{2\sqrt{3}} \frac{dx}{4+x^2}$. **2**

<u>Question 2</u> **START A NEW PAGE** (12 Marks) **Marks**

(a) Using all the letters of the word MATHEMATICS, how many different arrangements can be made. **2**

(b) The temperature, $T°$ centigrade, of a pie t minutes after being placed in an oven is given by the formula $T = 180 + Be^{kt}$. Initially the temperature of the pie is $5°\ C$ and after 15 minutes the temperature has risen to $40°\ C$.

(i) Find the value of the constant *B*. **1**

(ii) Find the exact value of the constant *k*. **2**

(iii) Find the temperature of the pie one hour after being placed in the oven. Give your answer correct to the nearest degree. **3**

(c) (i) On the same set of co-ordinate axes draw neat sketches of the graphs $y = x$ and $y = \frac{2}{x-1}$. **2**

(ii) Hence or otherwise solve $x > \frac{2}{x-1}$. **2**

Question 3 **START A NEW PAGE** (12 Marks) **Marks**

(a) A district squad of 9 netball players is chosen from 3 netball teams (*A*, *B* and *C*). There are 8 players in each of the teams *A*, *B* and *C*.

(i) If 4 players are chosen at random from team *A*, 3 from team *B* and 2 from team *C*, in how many ways can the district squad be formed? **2**

(ii) Find the probability that Janice from team *B* and Sarah from team *C* will be chosen as members of the district squad. **2**

(b) Solve $\sec^2 x + \tan x - 7 = 0$ for $0° \le x \le 360°$. Give your answers correct to the nearest minute. **3**

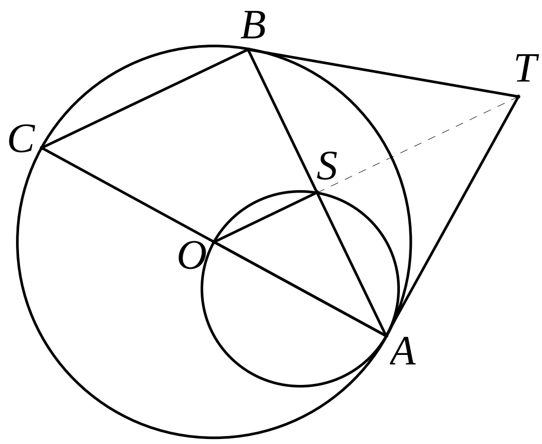
(c) (i) By equating coefficients, find the values of P and Q in the identity $P(2\sin x + \cos x) + Q(2\cos x - \sin x) \equiv 7\sin x + 11\cos x$. **2**

(ii) Hence, or otherwise, evaluate $\int_0^{\frac{\pi}{2}} \frac{7\sin x + 11\cos x}{2\sin x + \cos x}\, dx$. **3**

Question 4 **START A NEW PAGE** (12 Marks) **Marks**

(a) Evaluate $\int_0^1 \frac{x}{(2x+1)^2}\, dx$ using the substitution $u = 2x + 1$. **4**

(b) Two circles touch at point *A*. The small circle passes through the centre *O* of the large circle. *AB* is a chord of the large circle and cuts the small circle at *S*. *AC* is a diameter of the large circle. *AT* and *BT* are tangents to the large circle. (See diagram)



(i) Copy the diagram into your book and prove that *CB* is parallel to *OS*. **2**

(ii) Hence prove that *BS* = *SA*. **2**

(iii) Find the size of $\angle OSA$. **1**

(iv) Prove that the points *O*, *S* and *T* are collinear. **3**

Question 5 **START A NEW PAGE** (12 Marks) **Marks**

(a) Given that A, B, C and D are the vertices of a cyclic quadrilateral, find the value of $\cos A+\cos B+\cos C+\cos D$. **2**

(b) Use the Principle of Mathematical Induction to prove that 11^n-2^{2n} is divisible by 7 for all integers $n\geq 1$. **4**

(c) The arc of the curve $y=\sin^{-1}x$ that lies in the positive quadrant is rotated one revolution about the y-axis to form the surface of a container.

(i) If the container is filled to a depth of h metres, show that the volume, $V\ m^3$, of water in the container is given by: $V=\frac{\pi}{4}\left(2h-\sin 2h\right)$. **3**

(ii) The container is being filled at a rate of $6m^3/hr$. Calculate the rate at which the depth of water is increasing when the depth is $\frac{\pi}{6}m$. **3**

Question 6 **START A NEW PAGE** (12 Marks) **Marks**

(a) In a small rural community two hobby farms provide eggs for the local grocer. The grocer makes up cartons containing one dozen eggs, always using 8 eggs from farm *A* and 4 eggs from farm *B*. Some of the eggs contain two yolks (called a "double-yolker" egg). Eggs from farm *A* have an 18% probability of being a double-yolker while the probability for farm *B* is 24%.

(i) If an egg is chosen at random from one of the cartons, show that there is a 20% probability that it will be a double-yolker. **2**

(ii) Find the probability that a carton chosen at random will have exactly three double-yolker eggs. Give your answer correct to the nearest percent. **2**

(iii) Find the probability that a carton chosen at random will have at least three double-yolker eggs. Give your answer correct to the nearest percent. **2**

(b) Masses are placed at two points *A* and *B* which are 1 metre apart. A 1 kg mass (*M*) is placed at a point *P* between *A* and *B*. The mass *M* experiences forces of attraction towards both the points *A* and *B*. The force (in Newtons) of the attraction towards *A* is equal to four times the distance *AP* while the force of attraction towards point *B* is equal to the square of the distance *PB*.
Take the origin of the motion at point *A* and the positive direction of motion in the direction of the ray *AB*.

(i) The mass *M* at point *P* is initially x metres from the origin *A*. Briefly explain why the acceleration, $\ddot{x}$ m/s, of the mass *M* is given by: $\ddot{x}=x^2-6x+1$. **1**

(ii) If the mass *M* now starts from rest halfway between *A* and *B*, in which direction will it begin to move? Briefly explain you answer. **2**

(iii) Find the speed of the mass *M* when it first reaches point *A*. **3**

Question 7 **START A NEW PAGE** (12 Marks) **Marks**

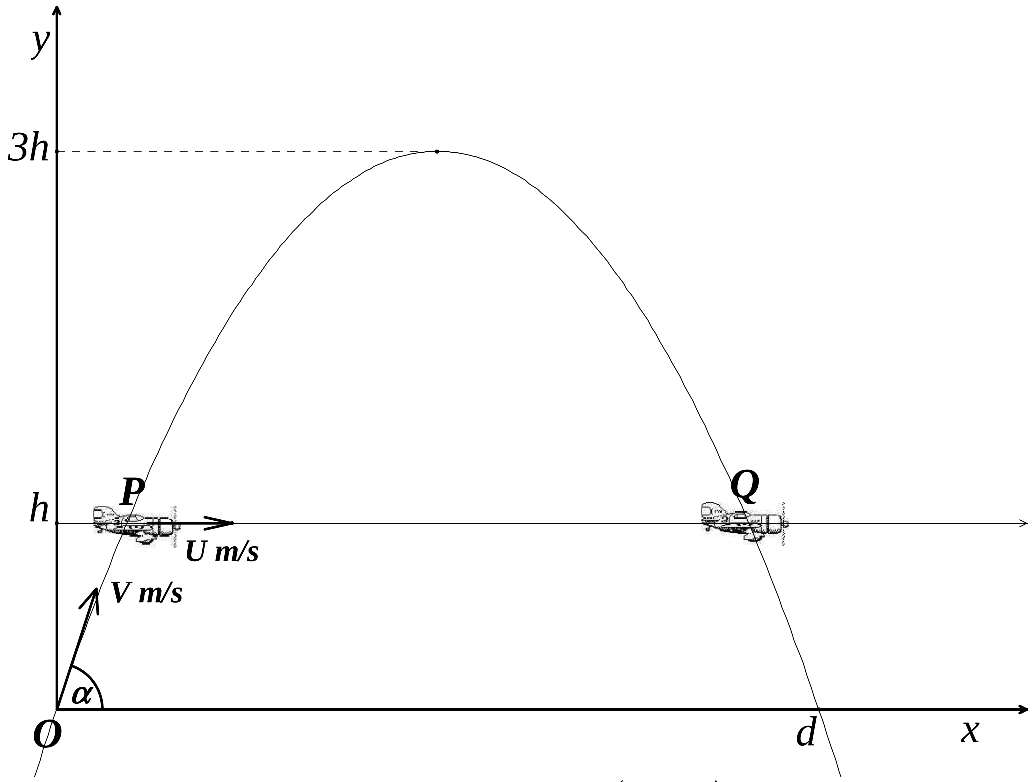
(a) Find the value of the constant term in the expansion of $\left(2y-\frac{1}{y^3}\right)^{20}$. **3**

(b) An enemy plane is flying horizontally at height h metres with speed U m/s.

When it is at point P a ground rocket is fired towards it from the origin O with speed V m/s and angle of elevation α.

The rocket misses the plane, passing too late through the point P. However, it goes on to reach a maximum height of $3h$ metres and then on its descent strikes the plane at Q.

With the axes as shown in the diagram, you may assume that the position of the rocket is given by: $x = Vt\cos\alpha$ and $y = -\frac{1}{2}gt^2 + Vt\sin\alpha$, where t is the time in seconds after firing and g is the acceleration due to gravity.



(i) Show that the initial vertical velocity component $(V\sin\alpha)$ of the rocket's speed equals $\sqrt{6gh}$. **2**

(ii) If the rocket had not struck the plane at Q, it would have returned to the x-axis at a distance d metres from O. **2**

Show that the horizontal component $(V\cos\alpha)$ of the rocket's speed equals $\frac{gd}{2\sqrt{6gh}}$.

(iii) Show that the equation of the path of the rocket is $y = \frac{12hx}{d}\left(1-\frac{x}{d}\right)$. **2**

(iv) If the horizontal component of the rocket's speed is $100\left(3+\sqrt{6}\right)$ m/s, find the time taken by the rocket to strike the plane at Q, in terms of d. **2**

(v) Find the speed of the enemy plane. **1**