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QUESTION ONE (12 marks)

a) Evaluate **2**

$$\int_0^3 \frac{dx}{x^2+9}.$$

b) Find the acute angle between lines $2x - y = 0$ and $x + 3y = 0$. **3**
Give your answer correct to the nearest minute.

c) Solve $\dfrac{x}{2x-1} \le 5$. **3**

d) Differentiate $\cos^4 x$ with respect to x and hence find $\int \sin x \cos^3 x \, dx$. **2**

e) A is the point (–2,–1) and B is the point (1,5). **2**
Find the coordinates of Q which divides AB internally in the ratio 5:3.

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QUESTION TWO (12 marks)

a) Evaluate $\displaystyle\int_0^{\frac{\pi}{3}} \sin^2(2x)\, dx$. **3**

b) Use the substitution $u = \tan x$, to evaluate $\displaystyle\int_0^{\frac{\pi}{3}} \tan^2 x \sec^2 x \, dx$. **3**

c) A biased coin has a probability of 0.6 of showing a head when it is tossed.
The biased coin is tossed five times.

(i) What is the probability of tossing exactly three heads and two tails? **2**

(ii) What is the probability of getting less than three heads? **2**

(iii) What is the most likely outcome and what is the probability that this event will occur? **2**

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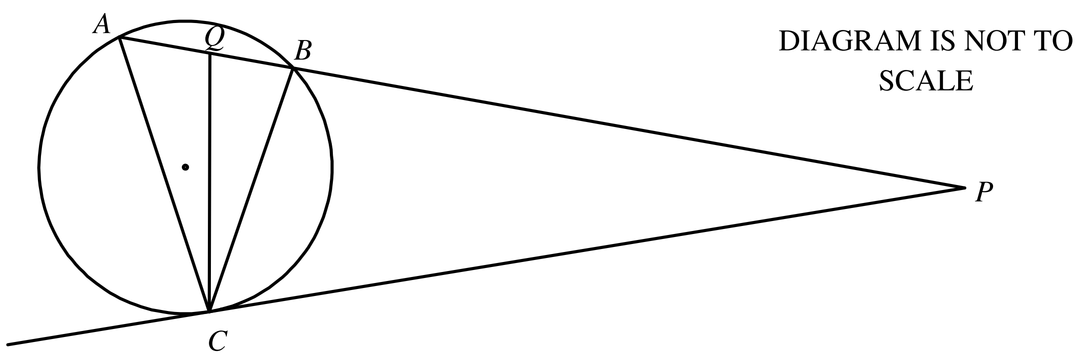
QUESTION THREE (12 marks)

a) Find the exact value of $\sin\left(\sin^{-1}\left(\frac{4}{5}\right)-\tan^{-1}\left(\frac{-5}{12}\right)\right)$. **3**

b) In the diagram below PC is a tangent to the circle at C and QC bisects $\angle ACB$.



(i) Copy the diagram into your answer booklet and prove that $PC = PQ$. **2**

(ii) If $PC = 10$cm and $AQ = 2$cm, Find the length of QB. **2**

c) Molten plastic at a temperature of 250° C, is poured into a mould to form a car part. After 20 minutes the plastic has cooled to 150° C . If the temperature after t minutes, is T ° C, and the surrounding temperature is 30° C, then the rate of cooling is given by:

$$\frac{dT}{dt} = -k\left(T - 30\right), \text{ where } k \text{ is constant.}$$

(i) Show that $T = 30 + Ae^{-kt}$, where A is a constant, satisfies this equation. **1**

(ii) Show that the value of A is 220° C . **1**

(iii) Find the value of k to 2 decimal places. **1**

(iv) The plastic can be taken out of the mould when the temperature drops below 80° C. How long after the plastic has been poured will this temperature be reached? Give your answer to the nearest minute. **2**

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QUESTION FOUR (12 marks)

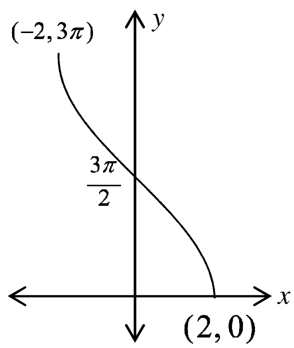
a) Prove by mathematical induction that:
$5^n - 4n - 1 \geq 0,$ for all $n \geq 1.$ **4**

b) John contributes $800 every quarter into a superannuation fund earning 6% pa, compounded quarterly. Contributions are made for 10 years and then John stops making contributions for 3 years whilst unemployed. He is re-employed, and contributions resume under the previous conditions. John works for a further 7 years until retirement.

(i) Show that on retirement the contributions for the first 10 years can be expressed as: **1**
$800(1.015)^{80} + 800(1.015)^{79} + ... + 800(1.015)^{41}.$

(ii) Hence, calculate the total amount in the superannuation fund on retirement. **3**

c) Below is the graph $y = a\cos^{-1} bx$.



(i) Show that the value of a is 3 and that b is $\frac{1}{2}$. **2**

(ii) Find the exact value of the area enclosed by the curve $y = a\cos^{-1} bx$, the x axis and the line $y = \frac{\pi}{2}$. **2**

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QUESTION FIVE (12 marks)

a) A particle moves in a straight line so that its acceleration, a, is given by $a = 4x$. The displacement, x, of the particle is initially 1 metre to the right of the origin with a velocity of $-2ms^{-1}$.

(i) Show that $v = -2x\ ms^{-1}$. **2**

(ii) Express x as a function of t. **2**

(iii) Hence, find the displacement when $t = 2$ seconds to 3 decimal places. **1**

b) (i) Write an expression for $(1+x)^{20}+(1-x)^{20}$. **2**

Leave your answer in ${}^{n}C_{k}$ notation.

(ii) Show that $\sum_{k=0}^{10} {}^{20}C_{2k}\, x^{2k} = \frac{1}{2}\left[(1+x)^{20}+(1-x)^{20}\right]$. **1**

(iii) Integrate both sides of the equation in part (ii). **2**

(iv) Hence, or otherwise show: **2**

$${}^{20}C_0 + \frac{1}{3}\,{}^{20}C_2 + \frac{1}{5}\,{}^{20}C_4 + \ldots\ldots + \frac{1}{21}\,{}^{20}C_{20} = \frac{2^{20}}{21}.$$

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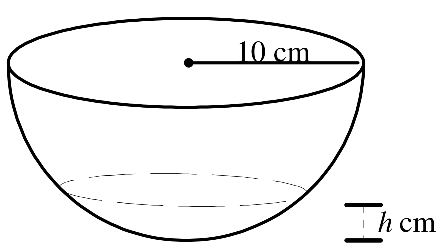
QUESTION SIX (12 marks)

a) (i) The diagram below represents a hemispherical bowl of radius 10 cm. It is filled with water to a depth of h cm. By finding the volume generated by rotating $x^2 + y^2 = 100$ between $y = 10$ and $y = 10 - h$ about the y axis, show that the volume of water in the bowl is given by:

$$V = \frac{\pi h^2}{3}(30-h)$$ **2**



(ii) The hemispherical bowl referred to above is being filled with water at a constant rate of $2\pi\ cm^3\ \min^{-1}$. Find the rate of increase of the depth of the water when the depth is 2 cm. **3**

b) Consider the function $f(x)=\sqrt{x+3}$

(i) Find an expression for $y=f^{-1}(x)$. **1**

(ii) State the domain for $y=f^{-1}(x)$. **1**

(iii) $f(x)$ and $f^{-1}(x)$ intersect at exactly one point P. Let α be the x coordinate of P. **2**

Explain why α is a root of the equation $x-\sqrt{x+3}=0$.

(iv) Show that the value of α occurs when $2<\alpha<3$. **1**

(v) Take $x=2.5$ as the first approximation of α, use one application of Newton's method to find a second approximation for α correct to 3 decimal places. **2**

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QUESTION SEVEN (12 marks)

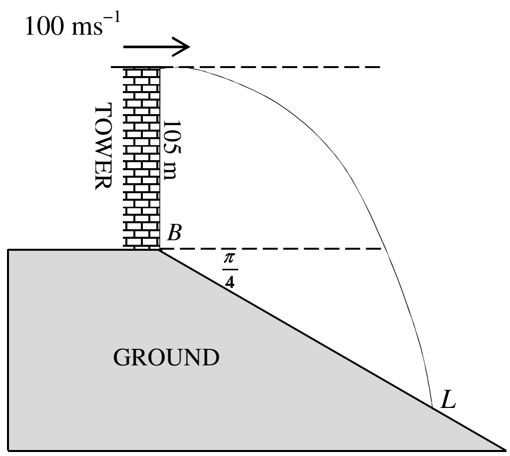
a) On a certain day the depth of water in a harbour is 7 metres at low tide (9:25am) and $10\frac{2}{3}$ metres at high tide (3:40pm). Assume the rise and fall of the surface of the water to be in simple harmonic motion in the form $\ddot{x}=-n^2(x-b)$ where $x=b$ is the centre of motion and $x=a$ is the amplitude.

(i) Show that $x=b-a\cos nt$ satisfies $\ddot{x}=-n^2(x-b)$. **1**

(ii) Find the values of a, b and n. **3**

(iii) Hence, find the earliest time before 3:40pm on this day, a boat may safely enter the harbour if the minimum depth of $9\frac{1}{2}$ metres of water is required. **2**

b) A bullet is fired horizontally with a velocity of 100ms^{-1} from the top of a tower 105 metres high.The tower is at the top of a hill, which slopes downwards at an angle of depression of $\frac{\pi}{4}$. The bullet lands at *L*, on the ground as represented on the diagram below.



(i) Consider *B*, the base of the tower, as the origin, and using acceleration due to gravity as $10\ ms^{-2}$, show that the expressions for the *x* and *y* coordinates of the position of the bullet at time *t* seconds are: **2**

$x = 100t$ and $y = 105 - 5t^2$.

(ii) Show that the equation of the line *BL* is $y = -x$. **1**

(iii) Find the time taken for the bullet to hit the ground at *L*. **2**

(iv) Find the distance *BL* to the nearest metre. **1**

END OF THE EXAMINATION