2002 Higher School Certificate Solutions

GENERAL MATHEMATICS

SECTION I

SUMMARY

1. D	**7.** C	**13.** D	**18.** C
2. B	**8.** D	**14.** A	**19.** A
3. C	**9.** D	**15.** B	**20.** B
4. B	**10.** B	**16.** C	**21.** D
5. A	**11.** C	**17.** B	**22.** C
6. A	**12.** D		

1. (D) Range $= 54 - 23 = 31$.

2. (B) $8x^3 - 5x^3 = 3x^3$.

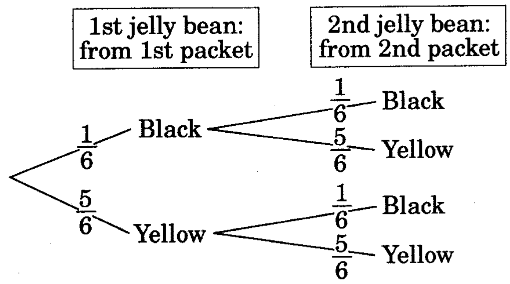
3. (C) $A = 230 \times 230$ (square base)
$= 52\,900 \text{ m}^2$.

$V = \frac{1}{3}Ah$ (volume of a pyramid)
$= \frac{1}{3} \times 52\,900 \times 135$
$= 2\,380\,500 \text{ m}^3$.

4. (B) Residential amount
$= 377\,000 \times 0{\cdot}272\,950\text{c}$
$= 102\,902{\cdot}15\text{c}$
$= \$1029{\cdot}0215$
$\doteqdot \$1029{\cdot}02$.

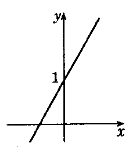
Total payable $= \$1029{\cdot}02 + \$195{\cdot}00$
$= \$1224{\cdot}02$.

5. (A) In each packet, 1 black + 5 yellow
= 6 jelly beans.



P(both black) $= \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$.

6. (A) $y = 3x + 1$ has gradient 3, y intercept 1. The gradient is positive and steep, and the line cuts the y axis at 1.



7. (C) Market value of all shares $= 2000 \times \$4{\cdot}80$
$= \$9600$.

Dividend yield $= \frac{240}{9600} \times 100\%$
$= 2{\cdot}5\%$.

8. (D) z-score of 3 means 3 standard deviations above the mean.

$\therefore$ Di's mark $= 55 + 6 \times 3 = 73$.

OR $z = \frac{x - \bar{x}}{s}$.

$z = 3,\ \bar{x} = 55,\ s = 6,$

$\therefore\quad 3 = \frac{x - 55}{6}$

$18 = x - 55$

$x = 73$.

9. (D) From the table, the monthly repayment for a loan of \$200 000 for 30 years at 6·5% pa interest is \$1265.

Total repayments
$= \$1265 \times 12 \times 30$
$= \$455\,400$.

10. (B) Radius of ball $r = \frac{1}{2} \times 1{\cdot}2 = 0{\cdot}6$ m.

Surface area of ball $= 4\pi r^2$
$= 4 \times \pi \times (0{\cdot}6)^2$
$= 4{\cdot}5238\ldots \text{ m}^2$.

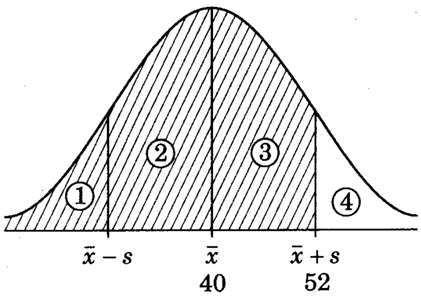
Cost of vinyl $= 4{\cdot}5238\ldots \times \$32$
$= \$144{\cdot}764\ldots$
$\doteqdot \$145$.

11. (C) $V_0 = \$4999,\ r = 40\% = 0{\cdot}4,\ n = 2$.

$S = V_0(1 - r)^n$ (Declining balance formula)
$= 4999(1 - 0{\cdot}4)^2$
$= 4999(0{\cdot}6)^2$
$= 1799{\cdot}64$
$\doteqdot \$1800$.

12. (D) 52 is one standard deviation above the mean.

It is necessary to find the percentage represented by the shaded area in the following diagram.



METHOD 1

Areas ① + ② = 50% (half of scores are below the mean).

Areas ② + ③ = 68% (within 1 standard deviation of the mean).

$\therefore$ Area ③ $= \frac{1}{2} \times 68\%$
$= 34\%$.

$\therefore$ Shaded area $= 50\% + 34\% = 84\%$.

METHOD 2

Areas ② + ③ = 68% (within 1 standard deviation of the mean).

Areas ① + ④ $= 100\% - 68\%$
$= 32\%$.

$\therefore$ Area ④ $= \frac{1}{2} \times 32\%$
$= 16\%$.

$\therefore$ Shaded area $= 100\% -$ Area ④
$= 100\% - 16\%$
$= 84\%$.

13. (D) Using the arc length of a circle formula with $\ell = 12$, $r = 10$:

$$\begin{aligned} \ell &= \frac{\theta}{360} \times 2\pi r \\ 12 &= \frac{\theta}{360} \times 2 \times \pi \times 10. \\ 12 \times 360 &= \theta \times 20\pi \\ 4320 &= \theta \times 20\pi \\ \frac{4320}{20\pi} &= \theta \\ \theta &= 68{\cdot}754\ldots^\circ \\ &\doteqdot 69^\circ. \end{aligned}$$

14. (A) The smallest numbers have the lowest powers of 10:
$5{\cdot}6 \times 10^{-2}, 7{\cdot}2 \times 10^{-2}, 4{\cdot}8 \times 10^{-1}$

15. (B) Using the present value formula with $M = \$1200$, $r = 0{\cdot}05$, $n = 10$:

$$\begin{aligned} N &= M\left[\frac{(1+r)^n - 1}{r(1+r)^n}\right] \\ &= 1200\left[\frac{(1+0{\cdot}05)^{10} - 1}{0{\cdot}05(1+0{\cdot}05)^{10}}\right] \\ &= 1200\left[\frac{(1{\cdot}05)^{10} - 1}{0{\cdot}05(1{\cdot}05)^{10}}\right] \\ &= \$9266{\cdot}0819\ldots \\ &\doteqdot \$9266. \end{aligned}$$

16. (C)

$$\begin{aligned} w &= 2y^3 - 1 \\ 13 &= 2y^3 - 1 \\ 14 &= 2y^3 \\ \frac{14}{2} &= y^3 \\ 7 &= y^3 \\ y^3 &= 7 \\ \therefore \quad y &= \sqrt[3]{7}. \end{aligned}$$

17. (B) From the graph,
cumulative frequency for 3 movies = 25,
cumulative frequency for 4 movies = 35.
$\therefore$ Frequency for 4 movies $= 35 - 25$
$= 10$.

18. (C) *METHOD 1*

$$\begin{aligned} \text{P(win 1st prize)} &= \frac{1}{200}, \\ \text{return} &= \$100 - \$1 \\ &= \$99. \\ \text{P(win 2nd prize)} &= \frac{1}{200}, \\ \text{return} &= \$50 - \$1 \\ &= \$49. \\ \text{P(not winning prize)} &= \frac{198}{200}, \\ \text{return} &= \$0 - \$1 \\ &= -\$1. \end{aligned}$$

Financial expectation
$= \frac{1}{200}(\$99) + \frac{1}{200}(\$49) + \frac{198}{200}(-\$1)$
$= -\$0{\cdot}25$.

METHOD 2

Ticket sales $= 200 \times \$1 = \$200$.
Total prizes $= 100 + 50 = \$150$.
Raffle makes a profit of \$50.
Financial expectation for all tickets is $-\$50$.
Financial expectation for each ticket
$= \frac{-50}{200} = -\$0{\cdot}25$.

19. (A) *METHOD 1*

Let the original population be P.
After the first year, the population is
$P + 20\%P = P + 0{\cdot}2P = 1{\cdot}2P$.
After the second year, the population
$= 1{\cdot}2P - 10\%(1{\cdot}2P)$
$= 1{\cdot}2P - 0{\cdot}1(1{\cdot}2P)$
$= 1{\cdot}2P - 0{\cdot}12P$
$= 1{\cdot}08P$.
$\therefore$ P increasing to $1{\cdot}08P$ is a $0{\cdot}08$ increase or an 8% increase.

METHOD 2

Let the original population be, say, 10 000.

After the first year, the population is

$$10\,000 + 20\% \times 10\,000 = 10\,000 + 2000 = 12\,000.$$

After the second year, the population is

$$12\,000 - 10\% \times 12\,000 = 12\,000 - 1200 = 10\,800.$$

$$\therefore \text{ Total increase} = \$10\,800 - 10\,000 = 800.$$

$$\text{Hence \% increase} = \frac{800}{10\,000} \times 100\% = 8\%.$$

METHOD 3

Let the original population be P.

After the first year, the population is

$$P(1 + 0{\cdot}2) = 1{\cdot}2P.$$

After the second year, the population is

$$1{\cdot}2P(1 - 0{\cdot}1) = 1{\cdot}2P(0{\cdot}9) = 1{\cdot}08P.$$

$\Rightarrow$ An increase of 0·08 or 8%.

20. (B) Rob (R) and Alex (A) each have twice the chance of Tan (T) to win the race. Sample space is R, R, A, A, T.

$$\therefore \text{ P(Tan winning)} = \frac{1}{5}.$$

21. (D)

$$\text{Area of one sheet} = 0{\cdot}21 \times 0{\cdot}3 = 0{\cdot}063 \text{ m}^2.$$

$$\therefore \text{ Mass of one sheet} = 0{\cdot}063 \times 80 \text{ g} = 5{\cdot}04 \text{ g}.$$

$$\text{Mass of pile of paper} = 25{\cdot}2 \text{ kg} = 25{\cdot}2 \times 1000 \text{ g} = 25\,200 \text{ g}.$$

$$\therefore \text{ No. of sheets} = 25\,200 \text{ g} \div 5{\cdot}04 \text{ g} = 5000.$$

22. (C) Tax per dollar earned over \$30 000 is equal to the gradient of the line between (30 000, 1000) and (60 000, 7000).

$$\text{Gradient} = \frac{7000 - 1000}{60\,000 - 30\,000} = \frac{6000}{30\,000} = \$\frac{1}{5} = 20 \text{ cents per dollar.}$$

SECTION II

Question 23

(a) (i) Fortnightly net pay

$$= \$1500 - \$269{\cdot}17 - \$7{\cdot}88 - \$16 = \$1206{\cdot}70.$$

(ii)

$$\text{4 weeks' gross pay} = 2 \times \$1500 = \$3000.$$

$$\text{Annual leave loading} = 17\tfrac{1}{2}\% \times \$3000 = \$525.$$

(iii) (1) *METHOD 1*

$$120\% \text{ of original price} = €180.$$

$$1\% \text{ of original price} = €180 \div 120 = €1{\cdot}5.$$

$$\therefore \text{ Original price (100\%)} = €1{\cdot}5 \times 100 = €150.$$

METHOD 2

$$1{\cdot}2 \times \text{original price} = €180.$$

$$\text{Original price} = €180 \div 1{\cdot}2 = €150.$$

(2) *METHOD 1*

Let \$Ax = €180.

By comparing ratios of \$A amounts and € amounts:

$$\frac{x}{1} = \frac{180}{0{\cdot}58}$$

$$x = 310{\cdot}3448\ldots \doteqdot 310{\cdot}34.$$

$$\therefore \quad €180 \doteqdot \$A310{\cdot}34.$$

METHOD 2

$$\$A1 = €0{\cdot}58$$

Divide both sides by 0·58:

$$\$A1{\cdot}7241\ldots = €1$$

Multiply both sides by 180:

$$\$A1{\cdot}7241\ldots \times 180 = €180$$

$$\$A310{\cdot}3448\ldots = €180$$

$$\therefore €180 \doteqdot \$A310{\cdot}34.$$

(b) (i) *Katherine*

Using the compound interest formula with

$$P = \$50\,000, \quad n = 5, \quad r = 3{\cdot}1\% = \frac{3{\cdot}1}{100} = 0{\cdot}031.$$

$$\begin{aligned} A &= P(1+r)^n \\ &= \$50\,000(1 + 0{\cdot}031)^5 \\ &= \$50\,000(1{\cdot}031)^5 \\ &= \$58\,245{\cdot}627\,81\ldots \\ &\doteqdot \$58\,246 \text{ (to the nearest dollar).} \end{aligned}$$

(ii) *Liz*

Using the compound interest formula with

$$P = \$50\,000,$$

$$r = \frac{0{\cdot}03}{12} = 0{\cdot}0025 \text{ (per month),}$$

$$n = 5 \times 12 = 60 \text{ (months).}$$

$$\begin{aligned} A &= P(1+r)^n \\ &= \$50\,000(1+0{\cdot}0025)^{60} \\ &= \$50\,000(1{\cdot}0025)^{60} \\ &= \$58\,080{\cdot}839\ldots \\ &\doteqdot \$50\,081. \end{aligned}$$

$\therefore$ Katherine makes the better investment.

(c) (i) 10% deposit = $10\% \times \$5000 = \$500$.
Remaining balance = $\$5000 - \$500$
= $4500.

Interest on balance
= Prn (Simple interest formula)
= $\$4500 \times 0{\cdot}15 \times 3$
= 2025.

Total owing = $\$4500 + \$2025$
= $6525.

Monthly repayment
= $\$6525 \div 36$
= $181·25.

(ii) (1) 2 years = 24 months.
On the graph, each unit on the vertical axis represents $100. The balance owing after 24 months = $3400.

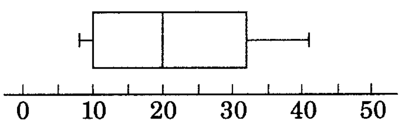
(2) Half-paid loan = $\frac{1}{2} \times \$5000 = \$2500$.
On the graph, each unit on the horizontal axis represents 1 month. The loan is half-paid at 35 months (or 2 years 11 months).

Question 24

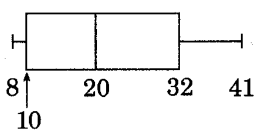
(a) (i) Bags more than 30 kg = 3.

$\therefore$ P(more than 30) = $\frac{3}{12} = \frac{1}{4}$.

(ii) *METHOD 1*



METHOD 2



(iii) $IQR = Q_3 - Q_1$
$= 32 - 10$
$= 22$.

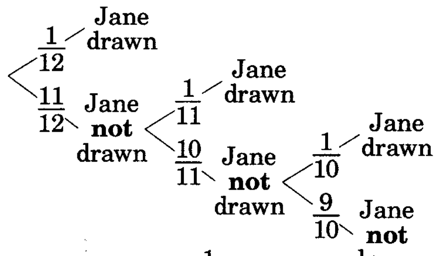
(b) (i) ***A*** = 53 − 10 = 43 (**or** 105 − 62 = 43)
B = 10 + 8 = 18 (**or** 53 + 70 − 105 = 18).

(ii) 53 + 70 = 123 cars (**or** 18 + 105 = 123).

(iii) $\dfrac{\text{Number of female drivers}}{\text{Number of drivers}} = \dfrac{70}{123}$.

(iv) $\dfrac{\text{Number of female drivers, headlights on}}{\text{Number of female drivers}}$
$= \dfrac{8}{70} = \dfrac{4}{35}$.

(c)



(i) P(Jane 1st) = $\frac{1}{12}$.

(ii) *METHOD 1*
P(Jane 2nd)
= P(not drawn first, then drawn 2nd)
$= \frac{11}{12} \times \frac{1}{11}$
$= \frac{1}{12}$.

METHOD 2
Only one particular student can be drawn second.
$\therefore$ P(Jane 2nd) = $\frac{1}{12}$.

(iii) *METHOD 1*
P(Jane not drawn) = $\frac{11}{12} \times \frac{10}{11} \times \frac{9}{10}$
$= \frac{9}{12}$
$= \frac{3}{4}$.

METHOD 2
3 people are chosen,
$\therefore$ 9 are not chosen.
$\therefore$ P(Jane not chosen) = $\frac{9}{12} = \frac{3}{4}$.

Question 25

(a) (i) (1) ΔAED and ΔACB.

(2) AE = 50 cm
AC = 50 cm + 150 cm = 200 cm.
$\therefore$ Enlargement factor = $\dfrac{AC}{AE} = \dfrac{200}{50}$
= 4.

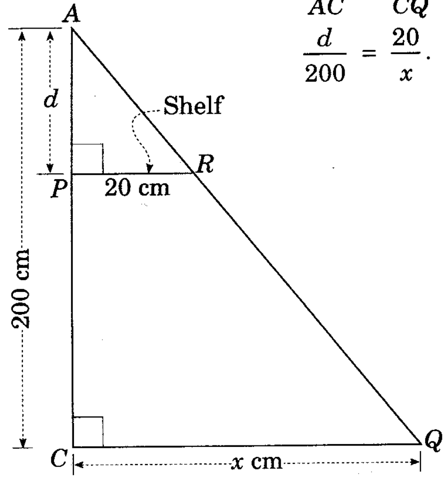
(3) *METHOD 1*
CB = 4 times larger than ED
$= 4 \times 20$
= 80 cm.

METHOD 2
By ratios of matching sides: $\dfrac{CB}{ED} = \dfrac{AC}{AE}$
$\dfrac{CB}{20} = 4$.

$\therefore$ $CB = 4 \times 20 = 80$ cm.

(ii) As triangles are similar, $\dfrac{AP}{AC} = \dfrac{PR}{CQ}$

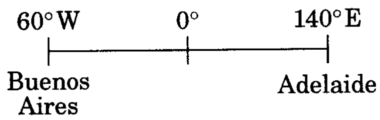
$$\frac{d}{200} = \frac{20}{x}.$$



(Or equivalent equations such as

$d = \dfrac{4000}{x}$ or $dx = 4000$.)

(b) (i)



Difference in longitude $= 60° + 140° = 200°$.

METHOD 1

$1°$ longitude $=$ 4 minutes.

$\therefore$ Difference in time $= 200 \times 4$
$= 800$ minutes
$= 13$ h 20 min.

METHOD 2

$15°$ longitude $=$ 1 hour.

$\therefore$ Difference in time $= \dfrac{200°}{15°}$
$= 13\frac{1}{3}$ hours
$= 13$ h 20 min.

ie. Adelaide is 13 h 20 min ahead of Buenos Aires.

(ii) Roy must call 13 h 20 min after 7 pm Friday. That is, at 8:20 am Saturday (Adelaide time).

(c) (i) Using the formula for the area of an annulus:

$$\text{Area} = \pi\left(R^2 - r^2\right), \text{ where } R = 1{\cdot}496 \times 10^8, \quad r = 1{\cdot}082 \times 10^8$$

$$= \pi\left[\left(1{\cdot}496 \times 10^8\right)^2 - \left(1{\cdot}082 \times 10^8\right)^2\right]$$

$$= 3{\cdot}352 \ldots \times 10^{16}$$

$$\doteqdot 3{\cdot}4 \times 10^{16} \text{ km}^2 \text{ (2 sig. figures).}$$

(ii) *METHOD 1*

$$A = \pi\left(R^2 - r^2\right).$$
$$A = \pi \times R^2 - \pi \times r^2$$
$$A + \pi \times r^2 = \pi \times R^2$$
$$\therefore \quad \pi \times R^2 = A + \pi \times r^2$$
$$R^2 = \frac{A + \pi \times r^2}{\pi}$$
$$R = \pm\sqrt{\frac{A + \pi r^2}{\pi}},$$

but since R is the length of the radius, it must be positive,

$$\therefore \quad R = \sqrt{\frac{A + \pi r^2}{\pi}}.$$

METHOD 2

$$A = \pi\left(R^2 - r^2\right).$$
$$\frac{A}{\pi} = R^2 - r^2$$
$$\frac{A}{\pi} + r^2 = R^2$$
$$R^2 = \frac{A}{\pi} + r^2$$
$$R = \pm\sqrt{\frac{A}{\pi} + r^2}.$$

But since R is the length of the radius, it must be positive.

$$\therefore \quad R = \sqrt{\frac{A}{\pi} + r^2}.$$

(iii) $A = 6{\cdot}79 \text{ mm}^2$, $r = 0{\cdot}75$ mm.

METHOD 1

Using the 1st formula from (ii),

$$R = \sqrt{\frac{A + \pi r^2}{\pi}}$$
$$= \sqrt{\frac{6{\cdot}79 + \pi \times (0{\cdot}75)^2}{\pi}}$$
$$= 1{\cdot}650\,401 \ldots$$
$$\doteqdot 1{\cdot}65 \text{ (correct to 2 d.p.).}$$

METHOD 2

Using the 2nd formula from (ii),

$$R = \sqrt{\frac{A}{\pi} + r^2}$$
$$= \sqrt{\frac{6{\cdot}79}{\pi} + 0{\cdot}75^2}$$
$$= \sqrt{2{\cdot}7238 \ldots}$$
$$= 1{\cdot}650\,401 \ldots$$
$$\doteqdot 1{\cdot}65 \text{ mm (correct to 2 d.p.).}$$

METHOD 3

Using $A = \pi\left(R^2 - r^2\right)$.

$$6{\cdot}79 = \pi\left(R^2 - 0{\cdot}75^2\right)$$
$$= \pi\left(R^2 - 0{\cdot}5625\right)$$
$$= \pi R^2 - 1{\cdot}7671 \ldots$$
$$6{\cdot}79 + 1{\cdot}7671 \ldots = \pi R^2$$
$$8{\cdot}557\,14 \ldots = \pi R^2$$

$$R^2 = \frac{8{\cdot}557\,14\ldots}{\pi}$$
$$= 2{\cdot}7238\ldots$$
$$R = \sqrt{2{\cdot}7238\ldots}$$
$$= 1{\cdot}650\,401\ldots$$
$$\doteqdot 1{\cdot}65 \text{ mm (correct to 2 d.p.).}$$

Question 26

(a) *METHOD 1*

For the average to be 6 after four quizzes, the total marks must be $6 \times 4 = 24$.
Vicki's marks so far $= 3 \times 5 = 15$.
$\therefore$ Vicki needs 9 marks in the next quiz.

METHOD 2

$$\bar{x} = \frac{\text{sum of scores}}{\text{no. of scores}}$$

After 3 quizzes, $\bar{x} = 5$,

$$\therefore \quad 5 = \frac{\text{sum of scores}}{3}$$

$\therefore$ sum of scores is 15.
Let the required mark be x.
After the next quiz, Vicki wants an average $\bar{x} = 6$.

$$\therefore \quad 6 = \frac{\text{sum of scores}}{4}$$
$$6 = \frac{15+x}{4}$$
$$24 = 15 + x$$
$$x = 24 - 15$$
$$= 9.$$

$\therefore$ Vicki needs 9 marks in the next quiz.

(b) (i) $\bar{x} = 3{\cdot}1\dot{3}$ (Sum of scores = 94, number of scores = 30.)
$\doteqdot 3{\cdot}13$ (correct to 2 d.p.).

(ii) Sample s.d. $(\sigma_{n-1}) = 1{\cdot}66$ (correct to 2 d.p.).

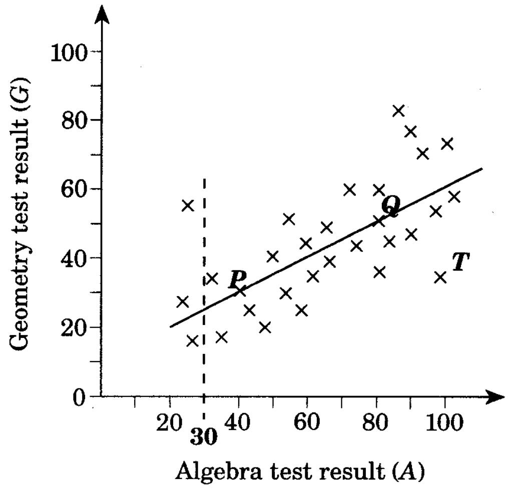
(iii) There are 30 scores.
$\therefore$ Median is average of 15th and 16th scores

$$= \frac{4+4}{2}$$
$$= 4.$$

(iv) As the scores are bunched at the higher end (4 or 5) the data are negatively skewed.

(v) In the sample, $9 + 7 = 16$ students sent more than 3 text messages.
P(student sending more than 3 messages)
$= \frac{16}{30}$.
Estimate $= \frac{16}{30} \times 150 = 80$.
$\therefore$ The number is approximately 80 students.

(c)



(i) By counting the crosses on the left of the vertical line $A = 30$, three students scored less than 30 for algebra.

(ii) Using points $P(40, 30)$ and $Q(80, 50)$:
- vertical change $= 50 - 30 = 20$.
- horizontal change $= 80 - 40 = 40$.

$\therefore$ Gradient $= \frac{20}{40} = \frac{1}{2}$.

(iii) *METHOD 1*

Extending the line, y intercept $= 10$.
Equation $y = mx + b$
$$\Rightarrow y = \frac{1}{2}x + 10.$$

METHOD 2

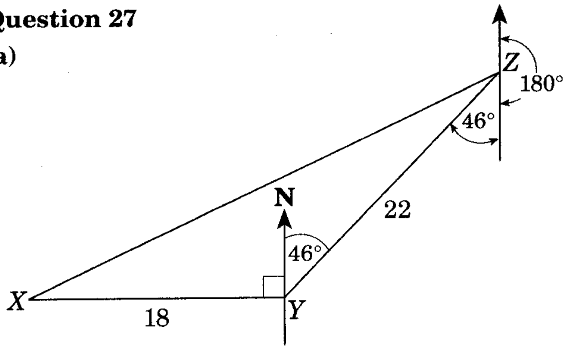
Equation of the line is $y = mx + b$,
where $m = \frac{1}{2}$,
$$\therefore \quad y = \frac{1}{2}x + b.$$
Substitute $P(40, 30)$ to find b:
$$30 = \frac{1}{2}(40) + b$$
$$= 20 + b$$
$$\therefore \quad b = 10.$$
$\therefore$ Equation of the line is $y = \frac{1}{2}x + 10$.

(iv) There is a strong positive correlation between the two sets of results.

(v) This statement is true for **some** students, but not **all**. For example, the student marked T on the graph came second (about 96) in algebra, but below half-way (about 32) in geometry.

Question 27

(a)



(i) $\angle XYZ = 90 + 46 = 136°$.

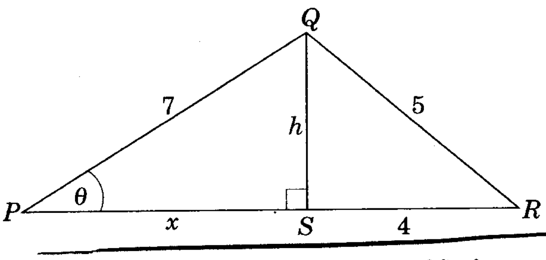
(ii) Using the cosine rule:

$c^2 = a^2 + b^2 - 2ab\cos c$,
with $a = 18$, $b = 22$, $c = 136°$.

$$XZ^2 = 18^2 + 22^2 - 2 \times 18 \times 22 \times \cos 136°$$
$$XZ = 1377{\cdot}71\ldots$$
$$X = \sqrt{1377{\cdot}71\ldots}$$
$$= 37{\cdot}117\ldots$$
$$\doteqdot 37{\cdot}1 \text{ km (1 d.p.)}.$$

(iii) See diagram in (i).
The marked angle at $Z = 46°$ by alternate angles between parallel lines.
Bearing of Y from $Z = 180 + 46 = 226°$.

(b)



(i) Need to know PS. Let $PS = x$ and $QS = h$.
Use Pythagoras's theorem twice.
In ΔQRS:

$$h^2 + 4^2 = 5^2$$
$$h^2 = 25 - 16$$
$$= 9$$
$$h = 3.$$

In ΔQPS:

$$x^2 + h^2 = 7^2$$
$$x^2 = 49 - 9$$
$$= 40$$
$$x = \sqrt{40}$$
$$= 6.3 \quad (1 \text{ d.p.})$$

$\therefore$ Perimeter of ΔPQR

$$= 7 + 5 + 4 + 6.3$$
$$= 22.3 \text{ cm} \quad (1 \text{ d.p.})$$

(ii) Let $\theta = \angle QPS$.

Use ΔQPS: $\sin\theta = \dfrac{3}{7}$

$$\theta = 25{\cdot}37\ldots$$

$\therefore \quad \angle QPS = 25°$ (nearest degree).

(c) (i) Shaded area (area $ABFE$):
Using Simpson's rule,

$$A \doteqdot \frac{h}{3}(d_f + 4d_m + d_l), \text{ with } h = 150, \; d_f = 120, \; d_m = 37, \; d_l = 40$$
$$= \frac{150}{3}(120 + 4 \times 37 + 40)$$
$$= 15\,400 \text{ m}^2.$$

(ii) Area of lake = area of rectangle − (area $ABFE$ + area $DCFE$)

Area of rectangle $= 300 \times 200$
$= 60\,000 \text{ m}^2$.

Area $ABFE$ = 15 400, from (i).

Area $DCFE$ $= \dfrac{150}{3}(80 + 4 \times 77 + 160)$ (Simpson's rule)
$= 27\,400$.

$\therefore$ Area of lake $= 60\,000 - (15\,400 + 27\,400)$
$= 17\,200 \text{ m}^2$.

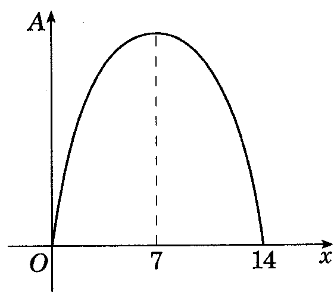
Question 28

(a) (i)

$A = bx$ (area of a rectangle).
Now $b + 2x = 28$
so $b = 28 - 2x$.
$\therefore \quad A = (28 - 2x)x$
$= 28x - 2x^2$.

(ii) x must be greater than zero in order to have a gutter, and x must be less than 14 (half of 28) because when $x = 14$ the base is zero.

(iii)

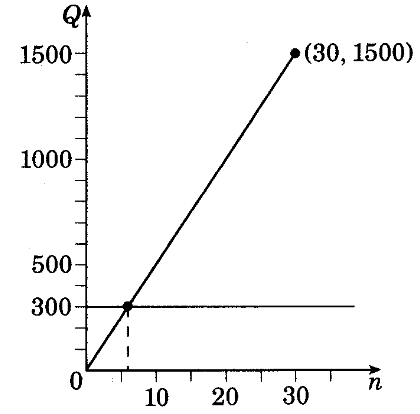


Since the parabola is symmetrical, the maximum occurs at the middle, where $x = 7$.

When $x = 7$, $A = 28 \times 7 - 2 \times 7^2$
$= 98 \text{ cm}^2$.

(b) (i) $Q = 50n$.

(ii)



(iii) On the graph, the point where the lines meet is the break-even point where Toby has earned the amount he spent on equipment. This occurs where $n = 6$, which means that after 6 parties he starts making a profit.

(iv) *Amount saved in each year* $

2001, Year 9: 900

2002, Year 10:

$900 + 30\% \times 900 = 1170$

2003, Year 11:

$1170 + 30\% \times 1170 = 1521$

2004, Year 12:

$1521 + 30\% \times 1510 = 1977{\cdot}30$

$\therefore$ Total saved = \$5568·30.

METHOD 1

Savings account

	Balance at start of year	*Interest earned (4%)*	*Amount added*	*Balance at end of year*
2005, 1st yr	5568·30	222·73	2500	8291·03
2006, 2nd yr	8291·03	331·64	2500	11 122·67
2007, 3rd yr	11 122·67	444·91	2500	14 067·58

With \$14 067·58, Toby does not quite reach his goal of \$15 000.

METHOD 2

After 2004, the investment can be thought of as a combination of:

- an annuity of \$2500 invested yearly for 3 years;
- \$5568·30 invested for 3 years;

These calculations are:

- \$2500 invested yearly at 4% pa compounded annually for 3 years. Using the future value formula, with $M = \$2500,\ r = 4\% = 0{\cdot}04,\ n = 3$:

$$A = \frac{M\left[(1+r)^n - 1\right]}{r} = \frac{2500\left[(1+0{\cdot}04)^3 - 1\right]}{0{\cdot}04} = \$7804;$$

- \$5568·30 invested for 3 years at 4% p.a. Using the compound interest formula, with $P = \$5568{\cdot}30,\ r = 4\% = 0{\cdot}04,\ n = 3$:

$$A = P(1+r)^n = 5568{\cdot}30(1+0{\cdot}04)^3 = \$6263{\cdot}58 \text{ (nearest cent).}$$

$\therefore$ Total value of investment
$= \$7804 + \$6263{\cdot}58$
$= \$14\,067{\cdot}58.$

With \$14 067·58, Toby will not reach his goal of \$15 000. (He will be short by \$932·42.)

END OF GENERAL MATHEMATICS SOLUTIONS