HIGHER SCHOOL CERTIFICATE
GENERAL MATHEMATICS
SOLUTIONS TO SPECIMEN PAPER

SECTION I

SUMMARY

1. (B)	7. (B)	13. (C)	18. (A)
2. (C)	8. (C)	14. (D)	19. (D)
3. (A)	9. (A)	15. (B)	20. (B)
4. (D)	10. (A)	16. (B)	21. (B)
5. (C)	11. (A)	17. (B)	22. (C)
6. (C)	12. (D)		

1. (B) $v = 8 + 10 \times 5 = 58.$

2. (C) $d^2 = 45^2 + 24^2 = 2601.$
$d = \sqrt{2601} = 51.$

3. (A) Data are discrete since numbers can only be whole numbers.

4. (D) Longitude (148°E) is the same, and latitude is further south.

5. (C) From the graph, reading taxable income on the horizontal axis, and tax payable on the vertical axis.

6. (C) Separately: $4 \times 1{\cdot}50 = \$6{\cdot}00$
Together: $1600\text{ g} = 1{\cdot}6\text{ kg}$
Cost $= \$2{\cdot}50.$
Amount saved $= \$3{\cdot}50.$

7. (B) $\dfrac{120}{600} \times 100 = 20.$

8. (C) Height of pole (on diagram) $= 4{\cdot}5$ cm.
$\dfrac{25}{15} \times 4{\cdot}5 = 7{\cdot}5$ cm.

9. (A) The angle at the centre for blue is twice that for green.

10. (A) Passes through (0, 3) and (4, 4).
Use $y = mx + b.$
Gradient: $m = \dfrac{\text{rise}}{\text{run}} = \dfrac{1}{4}$
y intercept: $b = 3$
Equation: $y = \dfrac{1}{4}x + 3$

11. (A) Using Simpson's rule formula:
$A \approx \dfrac{130}{3}(90 + 4 \times 100 + 30)$

12. (D) Sine rule: $\dfrac{x}{\sin 19°} = \dfrac{15}{\sin 31°}$

13. (C) Monthly payment $= \$107{\cdot}46.$
Total repaid $= 107{\cdot}46 \times 12 \times 20$
$= 25\,790{\cdot}40.$
Interest $= 25\,790{\cdot}40 - 15\,000$
$= \$10\,790{\cdot}40.$

14. (D) All scores increase by 4, so the mean does also. The spread does not alter, so neither does the standard deviation.

15. (B) $10(x+3) - 2(4x+2) = 10x + 30 - 8x - 4$
$= 2x + 26.$

16. (B) Deposit $= \dfrac{1}{3} \times 1494 = 498.$
Loan $= 1494 - 498 = 996.$
Monthly payment $= 996 \div 24 = \$41{\cdot}50.$

17. (B) Using formula for area of an annulus, where $R = 10$ and $r = 7$.

18. (A) Since the points are clustered around a straight line that has a positive gradient.

19. (B) P(wins both)
$= \text{P(wins 1st)} \times \text{P(wins 2nd)}$
$= \dfrac{5}{50} \times \dfrac{4}{49}.$

20. (B) 500 g is two standard deviations below the mean. In a normal distribution, 95% of scores are within two standard deviations of the mean.
This means that $\frac{1}{2}$ of 5% $= 2{\cdot}5\%$ of scores are smaller than two standard deviations below the mean.

21. (B) $\dfrac{6\times5}{2} = 15$

(Divide by 2 because the order of selection is not important.)

22. (A) Let distance be d and speed be s.

Then $d = ks^2$.

When $s = 60$, $d = 40$:

$$40 = k\times60^2$$
$$k = \frac{40}{3600}.$$

If $d = 80$: $80 = \dfrac{40}{3600}\times s^2$

$$s^2 = 80\times\frac{3600}{40} = 7200$$
$$s = \sqrt{7200} = 84{\cdot}85\ldots$$
$$\doteqdot 85 \text{ km/h}.$$

SECTION II

Question 23

(a) 3×10^5 km/s $= 3\times10^5\times1000$ m/s $= 3\times10^8$ m/s.

(b) $\dfrac{h}{3} = \sin55°$.

$h = 3\times\sin55°$
$= 2{\cdot}4574\ldots$ m
$\approx 2{\cdot}46$ m or 246 cm (nearest cm).

(c) Volume $= V_{\text{block}} - V_{\text{groove}}$.

$V_{\text{block}} = 78\times15\times25 = 29\,250$.

$V_{\text{groove}} = \frac{1}{2}\times10\times10\times15 = 750$.

Volume $= 29\,250 - 750 = 28\,500$ cm^3.

(d) (i) 30 games

(ii) $P(\text{A or E}) = \dfrac{18}{30}$.

(iii) $10\times9 = 90$ games.

(iv) $n(n-1)$ games.

(c) (i) $35\times9{\cdot}50 + 1{\cdot}5\times3\times9{\cdot}50 = \$375{\cdot}25$.

(ii) $35\times11{\cdot}30 = 395{\cdot}50$,
$412{\cdot}45 - 395{\cdot}50 = 16{\cdot}95$,
$16{\cdot}95\div11{\cdot}30 = 1{\cdot}5$.
So, 1 overtime hour.

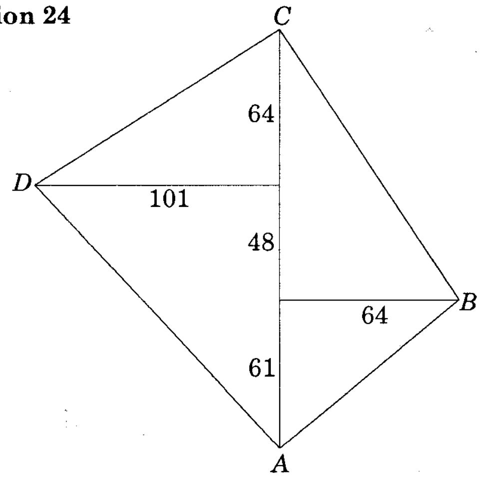
(iii) $W = N\times R + 1{\cdot}5\times V\times R$.

$1{\cdot}5\times V\times R = W - N\times R$

$$V = \frac{W - N\times R}{1{\cdot}5\times R}.$$

Question 24

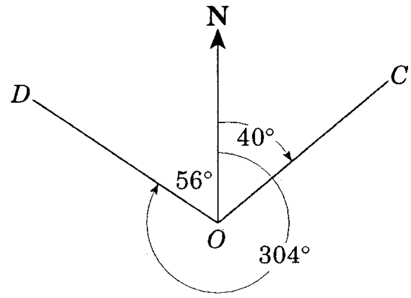
(a) (i)



(ii) $x^2 = 101^2 + 64^2 = 14\,297$.

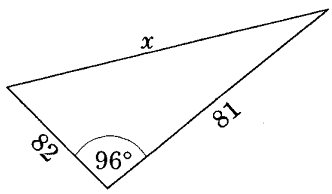
$x = \sqrt{14\,297}$
$= 119{\cdot}57\ldots$
≈ 120 m (nearest m).

(iii)



$\angle DOC = 56° + 40° = 96°$.

(iv)



$x^2 = 81^2 + 82^2 - 2\times81\times82\times\cos96°$
$= 14\,673{\cdot}55\ldots$.

$x = \sqrt{14\,673{\cdot}55\ldots}$
$= 121{\cdot}1344\ldots$
≈ 121 m (nearest m).

(v) Assuming all of the given measurements are correct, the difference between the two answers is the result of round-off errors. Distances have been rounded to the nearest metre, and angles to the nearest degree (including, presumably, right angles in the traverse survey).

(b) (i) $C = \pi\times d$
$= \pi\times20$
$= 62{\cdot}831\ldots$ cm
$\approx 62{\cdot}8$ cm or 628 mm (nearest mm).

(ii) $V = \pi r^2 h$
$= \pi \times 10^2 \times 15$
$= 4712{\cdot}388 \ldots \text{ cm}^3$
$= 4712{\cdot}388 \ldots \div 1000 \text{ L}$
$= 4{\cdot}712 \ldots \text{ L}$
$\approx 4{\cdot}7 \text{ L}$ (2 sig. figs).

Question 25

(a) (i) $F = 2C + 30.$

(ii) $2C = F - 30$
$C = \frac{1}{2}(F - 30).$

(b) (i) $A = 3{\cdot}18 + 0{\cdot}19 = \$3{\cdot}37.$
$B = 3{\cdot}17 + 1{\cdot}00 = \$4{\cdot}37.$
$C = 6\% + 4{\cdot}37 = \$0{\cdot}26.$

(ii) $500 \times 2{\cdot}18 = \$1090;$
or $(500 \times 1{\cdot}06 + 500) \times 1{\cdot}06 = \$1091{\cdot}80.$

(iii) Let investment be P.

Then, $P(1{\cdot}06)^5 = 5{\cdot}97,$
$P = 5{\cdot}97 \div (1{\cdot}06)^5$
$= 4{\cdot}4611\ldots$
$\approx \$4{\cdot}46$ (nearest cent).

(c) (i) 9 L / 100 km.

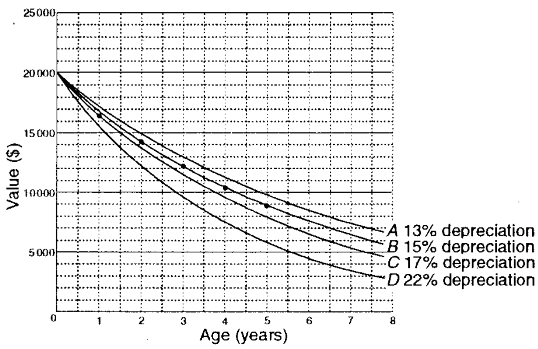
(ii) At 30 km/h, consumption is 12 L / 100 km.
So, in 20 km, $12 \div 5 = 2{\cdot}4$ L used.

(iii) $S = 80$: $C = 0{\cdot}01 \times 80^2 - 80 + 33$
$= 17$ L / 100 km.

(iv) For $S = 0$, the formula gives a consumption of 33 L / 100 km, but if $S = 0$, the car is stationary, so you cannot work out fuel consumption in litres per 100 km.

Question 26

(a) (i)



(ii) B is best because all the points plotted lie very close to this curve.

(iii) Use $S = V_0(1 - r)^n$.
$V_0 = 20\,000,$
$r = 15\%$ (from (ii), curve B represents 15% depreciation),
$n = 10.$
$S = 20\,000(1 - 0{\cdot}15)^{10}$
$= 20\,000 \times 0{\cdot}85^{10}$
$= 3937{\cdot}488\ldots$
$\doteqdot \$3900$ (nearest \$100).

OR

Taking the starting point at the end of the 5th year: $V_0 = 9000$, $r = 15\%$ and $n = 5$.
$S = 9000(1 - 0{\cdot}15)^5$
$= 9000 \times 0{\cdot}85^5$
$= 3993{\cdot}347\ldots$
$\doteqdot \$4000$ (nearest \$100).

(b) (i) Interquartile range $= 38 - 27 = 11$ years.

(ii) The distributions for actors and actresses are both strongly positively skewed, with the median only 12 years above the lowest age. They have similar ranges (M: 47, F: 53) and interquartile ranges (M: 10, F: 11). The overall shapes of the distributions are similar, apart from the dip in the frequency for actresses in the 50–59 age group. The striking difference between the two distributions is the fact that the bulk of the acrtesses are about 10 years younger than the corresponding actors.

(iii) As indicated in part (ii), the shapes and spreads of the distributions are similar, but the locations are quite different, so that the statement is incorrect.

Question 27

(a) (i) Range $= 39 - 3 = 36$ minutes.

(ii) 11 minutes (or 12 or 13).

(iii) Mean $= 16{\cdot}6.$
Sample standard deviation (σ_{n-1})
$= 7{\cdot}836\ldots$
$\doteqdot 7{\cdot}8$ (1 d.p.).

(iv) $\bar{x} + \sigma_{n-1} = 16{\cdot}6 + 7{\cdot}8 = 24{\cdot}4.$
$\bar{x} + \sigma_{n-1} = 16{\cdot}6 - 7{\cdot}8 = 8{\cdot}8.$
So, times from 9 min to 24 min are within one standard deviation of the mean.
There are 15 times in this range, out of a total of 20.
Probability $= \frac{15}{20} = 0{\cdot}75.$

(v) The mean is greater for Centre X because more of the times are towards the bottom of the stem than for Centre Y. Also, for Centre Y, the times are more bunched towards the centre, so its standard deviation will be smaller than that of Centre X.

(b) (i) 200 people.

(ii) $\frac{190}{200} = 95\%$.

(iii) Number of people who tested positive
$= 19 + 9 = 28$.

(iv) Number of people who tested positive that had the disease $= 19$.
So, using (iii), the probability that a person who tested positive had the disease
$= \frac{19}{25} \approx 68\%$.

Question 28

(a) (i) Nathan's mark is 2·2 standard deviations above the mean.

(ii) $61 + 2{\cdot}2 \times 12{\cdot}7 = 88{\cdot}94$
≈ 89 (nearest mark).

(b) (i) \$10 000

(ii) Using the formula for the future value of an annuity, with $M = 1000$, $r = 0{\cdot}08$, $n = 10$:

$$A = 1000\left(\frac{1{\cdot}08^{10} - 1}{0{\cdot}08}\right)$$
$$= 14\,486{\cdot}562\ldots$$
$$\approx \$14\,486{\cdot}56 \text{ (nearest cent).}$$

(iii) After a further 30 years,
value $= 14\,486{\cdot}56 \times 1{\cdot}08^{30}$
$= 145\,773{\cdot}2828\ldots$
$\approx \$145\,773{\cdot}28$ (nearest cent).

(iv) After 10 years, Gemma's investment is worth \$14 486·56. In every other year, this amount adds 8% of its value to Gemma's investment.
Now, $0{\cdot}08 \times 14\,486{\cdot}56 \approx \$1158{\cdot}92$.
That is, each year this initial investment contributes more to Gemma's total investment than the \$1000 that Clare invests contributes to hers, so Clare can never catch up.

(v) If the person wanted to save for a long-term goal, such as their retirement, they would do better by starting saving an amount they could afford right from the start, and gradually increase this amount as their earnings increased.
However, this can be difficult to do because people want to spend money on short-term goals, and also because extra responsibilities such as children and a home loan can mean that even though earnings may increase, the amount of money available for saving does not increase.

END OF HSC SPECIMEN PAPER SOLUTIONS