

2003 HIGHER SCHOOL CERTIFICATE SOLUTIONS GENERAL MATHEMATICS

SECTION I SUMMARY

1. B	7. B	13. C	18. D
2. D	8. D	14. B	19. B
3. C	9. A	15. B	20. A
4. B	10. A	16. D	21. A
5. C	11. C	17. C	22. C
6. D	12. A		

1. (B) $64 + 24 = 88$.

2. (D) $\frac{3y^3}{12y^2} = \frac{y}{4}$.

3. (C) Pay for Thursday and Friday
 $= (8 \times \$9.60) \times 2$
 $= \$153.60$.
 Pay for Saturday $= 6 \times 1.5 \times \$9.60$
 $= \$86.40$.
 Total pay $= \$153.60 + \86.40
 $= \$240.00$.

4. (B) $d = \sqrt{\frac{h}{5}}$
 $= \sqrt{\frac{28}{5}}$
 $= 2.366\ 431\ 913 \dots$
 $\div 2.4$ (1 decimal place).

5. (B) **METHOD 1**
 Value of car $= \$40\ 000 \times 0.7 \times 0.75$
 $= \$21\ 000$.

METHOD 2
 Value of the car at the end of 2001
 $= \$40\ 000 - 30\% \times \$40\ 000$
 $= \$40\ 000 - \$12\ 000$
 $= \$28\ 000$.
 Value of the car at the end of 2002
 $= \$28\ 000 - 25\% \times \$28\ 000$
 $= \$28\ 000 - \$7\ 000$
 $= \$21\ 000$.

6. (D) *Note:* After the first child is chosen, there are only 11 children left. If the first child is a boy, there are only 4 boys left.

7. (B) From the graph, Alex and Bryan met when they were 12 km from town (at time 20 minutes).
 Alex had then travelled
 $12\text{ km} - 4\text{ km} = 8\text{ km}$.

8. (D) The graphs show:
 (A) high positive correlation
 (B) low negative correlation
 (C) high negative correlation
 (D) low positive correlation

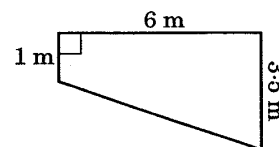
9. (A) Pool has the shape of a trapezoidal prism. That is, the uniform cross-section is a trapezium.

Area of trapezium

$$= \frac{h}{2}(a + b)$$

$$= \frac{6}{2}(1 + 3.5)$$

$$= 13.5\text{ m}^2.$$



Note:

Alternatively, the area of the trapezium could be found by adding the area of the rectangle ($1 \times 6 = 6\text{ m}^2$) to the area of the triangle ($\frac{1}{2} \times 6 \times 2.5 = 7.5\text{ m}^2$).

$$V = Ah$$

$$= 13.5 \times 5$$

$$= 67.5\text{ m}^3.$$

10. (A) **METHOD 1**
 Time difference $= 30^\circ \div 15$
 $= 2\text{ hours}$.

Kathmandu is west of Perth, so Kathmandu is 2 hours *behind* Perth time.
 When Perth is 12 noon, time in Kathmandu is 10:00 am.

METHOD 2
 Time difference $= 30^\circ \times 4\text{ min}$
 $= 120\text{ min}$
 $= 2\text{ hours}$.

Kathmandu is west of Perth, so Kathmandu is 2 hours *behind* Perth time.
 When Perth is 12 noon, time in Kathmandu is 10:00 am.

11. (C) $a = \frac{1}{2} \times 120 = 60$

$b = \frac{1}{2} \times 80 = 40$

$A = \pi ab$
 $= \pi \times 60 \times 40.$

Area = 7539.822 369 ...

Cost = 7539.822 369 ... \times \$7.50
 $= \$56\,548.667 \dots$
 $\div \$56\,549.$

12. (A) **METHOD 1**

x	f	fx
0	5	0
1	10	10
2	3	6
3	1	3
4	1	4
Totals	20	23

Mean = $\frac{\sum fx}{\sum f}$
 $= \frac{23}{20}$
 $= 1.15.$

METHOD 2

Using the statistics mode on a calculator,
 $\bar{x} = 1.15.$

13. (C) **METHOD 1**

Number of students surveyed
 $= 5 + 10 + 3 + 1 + 1$
 $= 20.$

Number with at least 2 brothers
 $= 3 + 1 + 1$
 $= 5.$

$P(\text{student has at least 2 brothers}) = \frac{5}{20}$
 $= 0.25.$

METHOD 2

$P(\text{student has at least 2 brothers})$
 $= P(2 \text{ brothers}) \text{ or } P(3 \text{ brothers})$
 $\text{or } P(4 \text{ brothers})$
 $= \frac{3}{20} + \frac{1}{20} + \frac{1}{20}$
 $= \frac{5}{20}$
 $= \frac{1}{4}$
 $= 0.25.$

14. (B) Use the cosine rule when given 2 sides and the angle included by them.

$x^2 = 10^2 + 7^2 - 2 \times 10 \times 7 \times \cos 60^\circ.$

15. (B) **METHOD 1**

From the formula $W = 0.75n + 50$,
 an employee earns \$0.75 per CD sold.
 If Kylie sells 2 more CDs than Danny,
 she will earn $2 \times \$0.75 = \1.50 more.

METHOD 2

Let Kylie sell 2 CDs	Let Danny sell 0 CDs
Kylie's weekly pay	Danny's weekly pay
$= 0.75 \times 2 + 50$	$= 0.75 \times 0 + 50$
$= \$51.50.$	$= \$50.$
\therefore Kylie's extra pay	$= \$51.50 - \50
	$= \$1.50.$

16. (D) $A = 100\,000$
 $r = 4\% + 12$ (compounded monthly)
 $\therefore r = 0.04 + 12$
 $n = 5 \times 12$
 $= 60$ months.

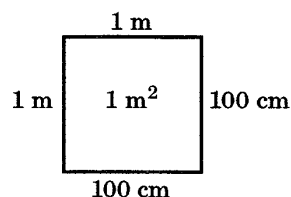
Using the present value formula:

$N = \frac{A}{(1+r)^n}$
 $= \frac{100\,000}{(1+0.04+12)^{60}}.$

17. (C) Since the current varies *inversely* with resistance, if the resistance is *doubled* (multiplied by 2), then the current must be *halved* (divided by 2).

18. (D) Measurements are correct to the nearest centimetre, so the error = ± 0.5 cm.
 True breadth is between 9.5 cm and 10.5 cm. True length is between 14.5 cm and 15.5 cm.
 \therefore Lowest possible area is $9.5 \times 14.5 \text{ cm}^2$.
 \therefore Upper possible area is $10.5 \times 15.5 \text{ cm}^2$.

19. (B) Note: $1 \text{ m}^2 = 100 \times 100 \text{ cm}^2$,
 so $1 \text{ m}^2 = 10\,000 \text{ cm}^2$.



METHOD 1

1 tile = 175 cm^2
 $= 175 \div 10\,000 \text{ m}^2$
 $= 0.0175 \text{ m}^2.$

Area of 1.056 million tiles
 $= 1.056 \times 1\,000\,000 \times 0.0175 \text{ m}^2$
 $= 18\,480 \text{ m}^2.$

METHOD 2

Area of 1.056 million tiles
 $= 1.056 \times 1\,000\,000 \times 175 \text{ cm}^2$
 $= 184\,800\,000 \text{ cm}^2$
 $= 184\,800\,000 \div 10\,000 \text{ m}^2$
 $= 18\,480 \text{ m}^2.$

20. (A) Taxable items = \$1.29 + \$7.23 + \$4.13
= \$12.65.

This total includes 10% GST on the original price, so 110% of the original price = \$12.65.

$$\therefore 10\% \text{ of original price} = \$12.65 \div 11 = \$1.15$$

$$\therefore \text{GST} = \$1.15.$$

21. (A) Total number of cameras sold in 2001
= 1070 + 200
= 1270.
Percentage of cameras that were digital
= $\frac{200}{1270} \times 100\%$
= 15.748 0315 ... %
 \div 16% (correct to nearest per cent).

22. (C) By counting the dots on the scatter graph, there were 5 students who liked milk chocolate but disliked dark chocolate.

$$\text{Estimate} = \frac{5}{12} \times 600 \\ = 250.$$

SECTION II

Question 23

- (a) (i) Water required = $13 \times 1.2 \text{ L}$
= 15.6 L.

- (ii) Water required per week = $15.6 \text{ L} \times 7$
= 109.2 L
= 0.1092 kL.

$$\text{Cost of water} = 0.1092 \times 94.22 \text{ cents} \\ = 10.288 824 \dots \text{ cents} \\ \div 10 \text{ cents.}$$

- (iii) One shrub requires 1.2 L = 1200 mL.
Number of drops = 1200×15
= 18 000.

(iv) METHOD 1

$$\text{Drip rate} = 18\,000 \text{ drops} / 10 \text{ hours} \\ = 1800 \text{ drops} / \text{h} \\ = \frac{1800}{60} \text{ drops} / \text{min} \\ = 30 \text{ drops} / \text{min.}$$

METHOD 2

$$\text{Number of minutes in 10 hours} = 10 \times 60 \\ = 600.$$

$$\text{Drip rate} = \frac{18\,000}{600} \text{ drops} / \text{min} \\ = 30 \text{ drops} / \text{min.}$$

- (b) Surface area of a sphere = $4\pi r^2$.
Radius $r = \frac{1}{2} \times 45 \text{ cm} = 22.5 \text{ cm}$.
Internal surface area = $\frac{1}{2} \times 4\pi r^2$

$$= \frac{1}{2} \times 4 \times \pi \times 22.5^2 \\ = 3180.862\,562 \dots \text{ cm}^2 \\ \div 3181 \text{ cm}^2.$$

- (c) Two applications of Simpson's rule

$$A \div \frac{h}{3} [d_f + 4d_m + d_l].$$

$$\text{Left 'half': } A_1 \div \frac{3}{3} [4 + 4(5) + 5] \\ = 29 \text{ m}^2.$$

$$\text{Right 'half': } A_2 \div \frac{3}{3} [5 + 4(4) + 0] \\ = 21 \text{ m}^2.$$

$$\therefore \text{Total area} \div 29 \text{ m}^2 + 21 \text{ m}^2 \\ = 50 \text{ m}^2.$$

- (d) (i) Volume of a cylinder $V = \pi r^2 h$.

$$r = \frac{1}{2} \times 10 = 5 \text{ cm.}$$

$$V = \pi \times 5^2 \times 7.8 \\ = 612.610\,5675 \dots \\ \div 613 \text{ cm}^3.$$

- (ii) Engine capacity

$$= 8 \times 612.610\,5675 \dots \text{ cm}^3 \\ = 4900.884\,54 \dots \text{ cm}^3 \\ = 4900.884\,54 \dots \text{ mL} \quad (1 \text{ mL} = 1 \text{ cm}^3) \\ = 4.900\,884\,54 \dots \text{ L} \\ < 5 \text{ L.}$$

\therefore Peta's engine meets the racing requirement of being under 5 litres.

Question 24

- (a) $P = 24\,000$, $r = \frac{0.0475}{12}$ (monthly),

$$n = 3 \times 12 = 36 \text{ months.}$$

$$A = P(1+r)^n \\ = 24\,000 \left(1 + \frac{0.0475}{12}\right)^{36} \\ = 27\,667.890\,17 \dots \\ \div \$27\,667.89.$$

- (b) (i) Taxable income = $\$58\,624 + 12 \times \900
= \$69 424.

- (ii) Tax payable

$$= \$17\,730 + 0.48 \times (\$69\,424 - \$66\,000) \\ = \$19\,373.52.$$

- (iii) Annual tax on second job

$$= \$19\,373.52 - \$14\,410.80 \text{ (from question)} \\ = \$4962.72.$$

METHOD 1

$$\text{Monthly tax} = \$4962.72 \div 12 \\ = \$413.56.$$

$$\text{Monthly net income} = \$900 - \$413.56 \\ = \$486.44.$$

METHOD 2

$$\text{Annual net income} = 12 \times \$900 - \$4962.72 \\ = \$5837.28.$$

$$\text{Monthly net income} = \$5837.28 \div 12 \\ = \$486.44.$$

(iv) Using future value of an annuity formula

$$A = M \left\{ \frac{(1+r)^n - 1}{r} \right\},$$

$$M = 486.44, r = \frac{0.04}{12} \text{ (monthly)}, \\ n = 24 \text{ months.}$$

$$A = 486.44 \left\{ \frac{\left(1 + \frac{0.04}{12}\right)^{24} - 1}{\frac{0.04}{12}} \right\}$$

$$= 12\,133.2183 \dots$$

$$\div \$12\,133.21$$

$$> \$12\,000.$$

\therefore Vicki will have enough in her account to pay for her holiday.

(c) Instalment = \$336, from the table.

$$\text{Total instalments} = \$336 \times 26 \times 2 \\ = \$17\,472.$$

$$\therefore \text{Interest} = \$17\,472 - \$15\,500 \\ = \$1972.$$

METHOD 1

Interest rate p.a.

$$= \frac{\$1972}{\$17\,472} \times 100\% \div 2 \text{ years}$$

$$= 6.361 \dots \%$$

$$\div 6.36\% \text{ (correct to 2 decimal places).}$$

METHOD 2

Using the simple interest formula $I = Prn$,

$$P = 15\,500, r = ?, n = 2, I = 1972.$$

$$1972 = 15\,500 \times r \times 2$$

$$= 31\,000r$$

$$r = \frac{1972}{31\,000}$$

$$= 0.063\,612\,903$$

$$= 6.361\,290\,323 \dots$$

$$\div 6.36\% \text{ (correct to 2 dec. pl.).}$$

Question 25

(a) (i) (1) Mode = 2.

(2) The most common number of cars per household, or the number of cars per household that has the highest frequency.

(ii) The number of stickers/car
 $= 0 \times 2735 + 1 \times 12\,305 + 2 \times 13\,918$
 $+ 3 \times 3980 + 4 \times 233$
 $= 53\,013.$

$$\text{(iii) Sector angle} = \frac{2735}{33\,171} \times 360^\circ \\ = 29.682\,554\,04 \dots^\circ \\ \div 30^\circ.$$

(iv) Number of hours from 6 pm to 8:30 pm
 $= 2.5$. The parking fee = \$10 (from the step graph).

(b) (i) Possible answers:

- More secondary students travel by train/bus.
- More / most primary students travel by car/walking.
- Secondary students spread more evenly across the methods of travel.

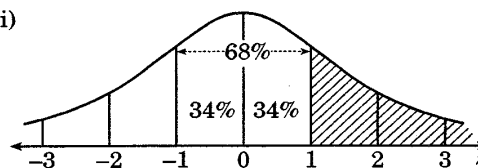
(ii) Possible answers:

- Secondary students are more mature and can take public transport such as the bus or train.
- Secondary students often need to travel further to get to school and rely on public transport.

(iii) Primary students travelling by bus
 $= 10\% \times 25\,000$
 $= 2500.$

(c) (i) Hardev's score is one standard deviation above the mean.

(ii)



Shaded region is the required percentage.

METHOD 1

$$\text{Percentage} = \frac{100\% - 68\%}{2} = 16\%.$$

METHOD 2

$$\text{Percentage} = 100\% - 34\% - 50\% \\ = 16\%.$$

\therefore 16% of people scored higher than Hardev.

Question 26

(a) (i) **METHOD 1**

$a = 60\,000$ (a is the vertical intercept of the graph of $N = a - bt$).

METHOD 2

When $t = 0$, $N = 60\,000$ (from graph).

$$N = a - bt$$

$$60\,000 = a - b(0)$$

$$60\,000 = a$$

$$a = 60\,000.$$

Explanation:

a was the number of people in the stadium immediately at the end of the game.

(ii) (1) **METHOD 1**When $t = 30$, $N = 0$.

$$N = a - bt$$

$$0 = 60\,000 - b(30)$$

$$= 60\,000 - 30b$$

$$30b = 60\,000$$

$$b = \frac{60\,000}{30}$$

$$b = 2000.$$

METHOD 2 $-b$ is the gradient of the line.

$$-b = -\frac{60\,000}{30} \left(\frac{\text{rise}}{\text{run}} \right)$$

$$-b = -2000$$

$$b = 2000.$$

(2) b is the rate at which people are leaving the stadium, in people / minute.

$$\begin{aligned} \text{(iii)} \quad N &= a - bt. & bt + N &= a \\ & & bt &= a - N \\ & & t &= \frac{a - N}{b}. \end{aligned}$$

(iv) **METHOD 1**When 10 000 people have left, there are $60\,000 - 10\,000 = 50\,000$ in the stadium.

$$\therefore N = 50\,000.$$

Using the formula from (iii):

$$\begin{aligned} t &= \frac{60\,000 - 50\,000}{2000} \\ &= \frac{10\,000}{2000} \\ &= 5 \text{ minutes.} \end{aligned}$$

METHOD 2Using $N = a - bt$:When $N = 50\,000$ (see Method 1),

$$50\,000 = 60\,000 - 2000t$$

$$2000t + 50\,000 = 60\,000$$

$$2000t = 10\,000$$

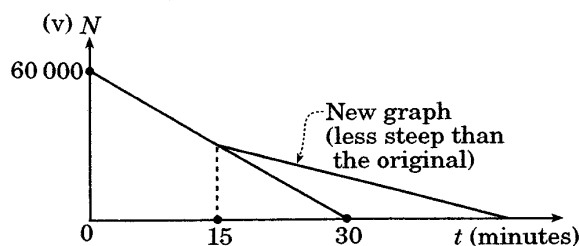
$$t = \frac{10\,000}{2000}$$

$$= 5 \text{ minutes.}$$

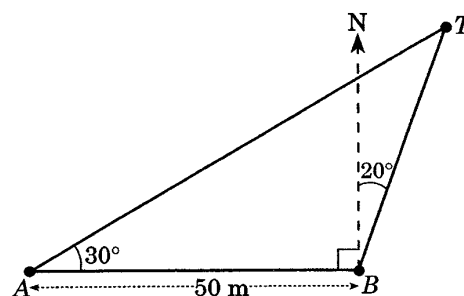
METHOD 3

2000 people / minute is the rate at which people are leaving the stadium, from (ii).

$$\therefore \text{The time it takes 10 000 people to leave} = \frac{10\,000}{2000} = 5 \text{ minutes.}$$



(b)

(i) N is north.

$$\angle NBT = 20^\circ \quad (T \text{ is } 020^\circ \text{ from } B)$$

$$\angle NBA = 90^\circ \quad (B \text{ is due east of } A)$$

$$\therefore \angle ABT = 20^\circ + 90^\circ = 110^\circ.$$

$$\text{(ii)} \quad \angle T = 180^\circ - 30^\circ - 110^\circ \quad (\angle \text{sum of } \triangle TBA) = 40^\circ.$$

By the sine rule,

$$\frac{BT}{\sin 30^\circ} = \frac{50}{\sin 40^\circ}$$

$$BT = \frac{50 \sin 30^\circ}{\sin 40^\circ}$$

$$= 38.893\,095\,67 \dots$$

$$\div 39 \text{ m.}$$

Question 27(a) (i) $P(\text{staying at hotel } X) = 50\%$.

$$\therefore P(\text{staying at hotel } Y \text{ or } Z)$$

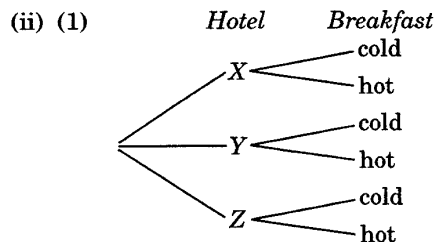
$$= 100\% - 50\%$$

$$= 50\%.$$

$$\therefore P(\text{staying at hotel } Z)$$

$$= \frac{1}{2} \times 50\% \quad (\text{equally likely})$$

$$= 25\% \quad \left(\text{or } \frac{1}{4} \right).$$



6 possible combinations:

Hotel X, cold

X, hot

Y, cold

Y, hot

Z, cold

Z, hot.

(2) These combinations are not all equally likely because the choice of hotels is not equally likely. (Karl is more likely to choose X over Y or Z.)

$$(3) P(Z, \text{hot}) = 25\% \times \frac{1}{2} = 12.5\% \quad \left(\text{or } \frac{1}{8} \right).$$

(b)

		2nd die					
		1	2	3	4	5	6
1st die	1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
	2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
	3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
	4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
	5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
	6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

There are 36 possible outcomes.

- (i) Number of outcomes that don't show 6 = 25 (as shaded on the above table).

$$\therefore P(\text{neither shows 6}) = \frac{25}{36}$$

- (ii) Number of outcomes that have both showing 6 = 1, (6, 6).

$$P(\text{both show 6}) = \frac{1}{36}$$

Number of outcomes that show only one 6 = 10 (row 6 and column 6, but not (6, 6) in the table above).

$$\therefore P(\text{only one 6}) = \frac{10}{36}$$

$$P(\text{neither shows 6}) = \frac{25}{36}, \text{ from (i).}$$

Financial expectation

$$= \$20\left(\frac{1}{36}\right) + \$2\left(\frac{10}{36}\right) + (-\$2)\left(\frac{25}{36}\right)$$

$$= -\$0.277\,777\,777\ldots$$

$$\div -\$0.28 \text{ (correct to the nearest cent).}$$

(c) (i) $\tan 65^\circ = \frac{AB}{15}$

$$\begin{aligned} AB &= 15 \tan 65^\circ \\ &= 32.167\,603\,81\ldots \\ &\div 32.2 \text{ m.} \end{aligned}$$

(ii) $CA = 2 \times 32.167\,603\,81\ldots$
 $= 64.335\,207\,62\ldots$

By Pythagoras' theorem on $\triangle CEA$:

$$\begin{aligned} CE^2 &= CA^2 + AE^2 \\ &= (64.335\,207\,62\ldots)^2 + 15^2 \\ &= 4364.018\,939\ldots \end{aligned}$$

$$\begin{aligned} CE &= \sqrt{4364.018\,939\ldots} \\ &= 66.060\,7216\ldots \\ &\div 66.1 \text{ m.} \end{aligned}$$

Note: Do not round off partial answers such as 64.3352 ... and 4364.0189 ... until the end of the calculation.

Question 28

- (a) Let C = cost of pendant, L = length of pendant,
 $C \propto L^2$, $\therefore C = kL^2$.

When $L = 30$, $C = 130$: $130 = k(30^2) = 900k$

$$k = \frac{130}{900} = \frac{13}{90}$$

$$\therefore C = \frac{13}{90} L^2$$

$$\begin{aligned} \text{When } L = 40, \quad C &= \frac{13}{90}(40^2) \\ &= 231.111\,1111\ldots \\ &\div 231 \text{ pesos.} \end{aligned}$$

- (b) (i) In $y = b(a^x)$, a is the growth constant and b is the initial population.

$$\begin{aligned} \therefore a &= 1 + 1.57\% \\ &= 1 + 0.0157 \\ &= 1.0157 \\ b &= 103\,400\,000 \end{aligned}$$

$$\therefore y = 103\,400\,000(1.0157)^x$$

(ii) When $x = 2$, $y = 103\,400\,000(1.0157)^2$
 $= 106\,672\,247.1\ldots$
 $\div 106\,672\,000.$

- (c) (i) (Celsius temp.) $\times 1.8 + 32$
 $=$ (Fahrenheit temp.).

$$A \times 1.8 + 32 = 77$$

$$1.8A + 32 = 77$$

$$1.8A = 45$$

$$\therefore A = \frac{45}{1.8} = 25.$$

- (ii) [(Celsius temp.) + 12] $\times 2$
 $=$ Fahrenheit temp.).

$$(C + 12) \times 2 = F$$

$$F = 2(C + 12) \text{ or } 2C + 24.$$

- (d) **Straight-line depreciation:**

METHOD 1

The camera depreciates \$800 in 4 years.

\therefore It depreciates \$400 in 2 years (half the time).

\therefore When $n = 2$, $S = \$800 - \$400 = \$400$.

METHOD 2

Using the formula $S = V_0 - Dn$, $V_0 = 800$.

When $n = 4$, $S = 0$: $0 = 800 - D(4)$
 $= 800 - 4D$

$$4D = 800$$

$$D = \frac{800}{4} = 200.$$

$$\therefore S = 800 - 200n.$$

When $n = 2$, $S = 800 - 200(2) = 400$.

Declining-balance depreciation:

Using the formula $S = V_0(1-r)^n$, $V_0 = 800$.

When $n = 2$,

$$\begin{aligned} S &= 400 \quad (\text{same as straight-line} \\ 400 &= 800(1-r)^2 \quad \text{depreciation}) \end{aligned}$$

$$\frac{400}{800} = (1-r)^2$$

$$0.5 = (1-r)^2$$

$$\sqrt{0.5} = 1-r$$

$$r + \sqrt{0.5} = 1$$

$$r = 1 - \sqrt{0.5}$$

$$= 0.292\,893\,218\ldots$$

$$R = 29.289\,3218\ldots\%$$

$$\div 29.3\%.$$

END OF GENERAL MATHEMATICS SOLUTIONS