

2003 HIGHER SCHOOL CERTIFICATE SOLUTIONS GENERAL MATHEMATICS

SECTION I SUMMARY

1. B	7. B	13. C	18. D
2. D	8. D	14. B	19. B
3. C	9. A	15. B	20. A
4. B	10. A	16. D	21. A
5. C	11. C	17. C	22. C
6. D	12. A		

1. (B) $64 + 24 = 88$.

2. (D) $\frac{3y^3}{12y^2} = \frac{y}{4}$.

3. (C) Pay for Thursday and Friday
 $= (8 \times \$9.60) \times 2$
 $= \$153.60$
 Pay for Saturday $= 6 \times 1.5 \times \$9.60$
 $= \$86.40$
 Total pay $= \$153.60 + \86.40
 $= \$240.00$.

4. (B) $d = \sqrt{\frac{h}{5}}$
 $= \sqrt{\frac{28}{5}}$
 $= 2.366\ 431\ 913\ \dots$
 $\doteq 2.4$ (1 decimal place).

5. (B) **METHOD 1**
 Value of car $= \$40\ 000 \times 0.7 \times 0.75$
 $= \$21\ 000$.

METHOD 2
 Value of the car at the end of 2001
 $= \$40\ 000 - 30\% \times \$40\ 000$
 $= \$40\ 000 - \$12\ 000$
 $= \$28\ 000$
 Value of the car at the end of 2002
 $= \$28\ 000 - 25\% \times \$28\ 000$
 $= \$28\ 000 - \$7\ 000$
 $= \$21\ 000$.

6. (D) *Note:* After the first child is chosen, there are only 11 children left. If the first child is a boy, there are only 4 boys left.

7. (B) From the graph, Alex and Bryan met when they were 12 km from town (at time 20 minutes).
 Alex had then travelled
 $12\ \text{km} - 4\ \text{km} = 8\ \text{km}$.

8. (D) The graphs show:
 (A) high positive correlation
 (B) low negative correlation
 (C) high negative correlation
 (D) low positive correlation

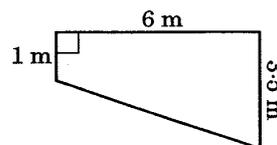
9. (A) Pool has the shape of a trapezoidal prism. That is, the uniform cross-section is a trapezium.

Area of trapezium

$$= \frac{h}{2}(a + b)$$

$$= \frac{6}{2}(1 + 3.5)$$

$$= 13.5\ \text{m}^2.$$



Note:

Alternatively, the area of the trapezium could be found by adding the area of the rectangle ($1 \times 6 = 6\ \text{m}^2$) to the area of the triangle ($\frac{1}{2} \times 6 \times 2.5 = 7.5\ \text{m}^2$).

$$V = Ah$$

$$= 13.5 \times 5$$

$$= 67.5\ \text{m}^3.$$

10. (A) **METHOD 1**
 Time difference $= 30^\circ \div 15$
 $= 2$ hours.

Kathmandu is west of Perth, so Kathmandu is 2 hours *behind* Perth time.
 When Perth is 12 noon, time in Kathmandu is 10:00 am.

METHOD 2

Time difference $= 30^\circ \times 4\ \text{min}$
 $= 120\ \text{min}$
 $= 2$ hours.

Kathmandu is west of Perth, so Kathmandu is 2 hours *behind* Perth time.
 When Perth is 12 noon, time in Kathmandu is 10:00 am.

11. (C) $a = \frac{1}{2} \times 120 = 60$

$$b = \frac{1}{2} \times 80 = 40$$

$$A = \pi ab \\ = \pi \times 60 \times 40.$$

$$\text{Area} = 7539.822\ 369 \dots$$

$$\text{Cost} = 7539.822\ 369 \dots \times \$7.50 \\ = \$56\ 548.667 \dots \\ \div \$56\ 549.$$

12. (A) **METHOD 1**

x	f	fx
0	5	0
1	10	10
2	3	6
3	1	3
4	1	4
Totals	20	23

$$\text{Mean} = \frac{\sum fx}{\sum f} \\ = \frac{23}{20} \\ = 1.15.$$

METHOD 2

Using the statistics mode on a calculator,
 $\bar{x} = 1.15.$

13. (C) **METHOD 1**

$$\text{Number of students surveyed} \\ = 5 + 10 + 3 + 1 + 1 \\ = 20.$$

$$\text{Number with at least 2 brothers} \\ = 3 + 1 + 1 \\ = 5.$$

$$P(\text{student has at least 2 brothers}) = \frac{5}{20} \\ = 0.25.$$

METHOD 2

$$P(\text{student has at least 2 brothers}) \\ = P(2 \text{ brothers}) \text{ or } P(3 \text{ brothers}) \\ \text{ or } P(4 \text{ brothers}) \\ = \frac{3}{20} + \frac{1}{20} + \frac{1}{20} \\ = \frac{5}{20} \\ = \frac{1}{4} \\ = 0.25.$$

14. (B) Use the cosine rule when given 2 sides and the angle included by them.

$$x^2 = 10^2 + 7^2 - 2 \times 10 \times 7 \times \cos 60^\circ.$$

15. (B) **METHOD 1**

From the formula $W = 0.75n + 50$, an employee earns \$0.75 per CD sold. If Kylie sells 2 more CDs than Danny, she will earn $2 \times \$0.75 = \1.50 more.

METHOD 2

Let Kylie sell 2 CDs	Let Danny sell 0 CDs
Kylie's weekly pay	Danny's weekly pay
$= 0.75 \times 2 + 50$	$= 0.75 \times 0 + 50$
$= \$51.50.$	$= \$50.$
\therefore Kylie's extra pay	$= \$51.50 - \50
	$= \$1.50.$

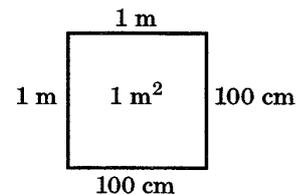
16. (D) $A = 100\ 000$
 $r = 4\% + 12$ (compounded monthly)
 $\therefore r = 0.04 + 12$
 $n = 5 \times 12$
 $= 60$ months.

Using the present value formula:

$$N = \frac{A}{(1+r)^n} \\ = \frac{100\ 000}{(1+0.04+12)^{60}}.$$

17. (C) Since the current varies *inversely* with resistance, if the resistance is *doubled* (multiplied by 2), then the current must be *halved* (divided by 2).
18. (D) Measurements are correct to the nearest centimetre, so the error = ± 0.5 cm. True breadth is between 9.5 cm and 10.5 cm. True length is between 14.5 cm and 15.5 cm.
 \therefore Lowest possible area is $9.5 \times 14.5 \text{ cm}^2$.
 \therefore Upper possible area is $10.5 \times 15.5 \text{ cm}^2$.

19. (B) Note: $1 \text{ m}^2 = 100 \times 100 \text{ cm}^2$,
so $1 \text{ m}^2 = 10\ 000 \text{ cm}^2$.



METHOD 1

$$1 \text{ tile} = 175 \text{ cm}^2 \\ = 175 \div 10\ 000 \text{ m}^2 \\ = 0.0175 \text{ m}^2.$$

$$\text{Area of 1.056 million tiles} \\ = 1.056 \times 1\ 000\ 000 \times 0.0175 \text{ m}^2 \\ = 18\ 480 \text{ m}^2.$$

METHOD 2

$$\text{Area of 1.056 million tiles} \\ = 1.056 \times 1\ 000\ 000 \times 175 \text{ cm}^2 \\ = 184\ 800\ 000 \text{ cm}^2 \\ = 184\ 800\ 000 \div 10\ 000 \text{ m}^2 \\ = 18\ 480 \text{ m}^2.$$

20. (A) Taxable items = $\$1.29 + \$7.23 + \$4.13$
 $= \$12.65.$

This total includes 10% GST on the original price, so 110% of the original price = $\$12.65.$

$$\therefore 10\% \text{ of original price} = \$12.65 \div 11$$

$$= \$1.15$$

$$\therefore \text{GST} = \$1.15.$$

21. (A) Total number of cameras sold in 2001
 $= 1070 + 200$
 $= 1270.$

Percentage of cameras that were digital

$$= \frac{200}{1270} \times 100\%$$

$$= 15.748\ 0315 \dots \%$$

$$\doteq 16\% \text{ (correct to nearest per cent).}$$

22. (C) By counting the dots on the scatter graph, there were 5 students who liked milk chocolate but disliked dark chocolate.

$$\text{Estimate} = \frac{5}{12} \times 600$$

$$= 250.$$

SECTION II

Question 23

(a) (i) Water required = $13 \times 1.2 \text{ L}$
 $= 15.6 \text{ L}.$

(ii) Water required per week = $15.6 \text{ L} \times 7$
 $= 109.2 \text{ L}$
 $= 0.1092 \text{ kL}.$

$$\text{Cost of water} = 0.1092 \times 94.22 \text{ cents}$$

$$= 10.288\ 824 \dots \text{ cents}$$

$$\doteq 10 \text{ cents}.$$

(iii) One shrub requires $1.2 \text{ L} = 1200 \text{ mL}.$
 Number of drops = 1200×15
 $= 18\ 000.$

(iv) METHOD 1

$$\text{Drip rate} = 18\ 000 \text{ drops} / 10 \text{ hours}$$

$$= 1800 \text{ drops} / \text{h}$$

$$= \frac{1800}{60} \text{ drops} / \text{min}$$

$$= 30 \text{ drops} / \text{min}.$$

METHOD 2

$$\text{Number of minutes in 10 hours} = 10 \times 60$$

$$= 600.$$

$$\text{Drip rate} = \frac{18\ 000}{600} \text{ drops} / \text{min}$$

$$= 30 \text{ drops} / \text{min}.$$

(b) Surface area of a sphere = $4\pi r^2.$
 Radius $r = \frac{1}{2} \times 45 \text{ cm} = 22.5 \text{ cm}.$
 Internal surface area = $\frac{1}{2} \times 4\pi r^2$

$$= \frac{1}{2} \times 4 \times \pi \times 22.5^2$$

$$= 3180.862\ 562 \dots \text{ cm}^2$$

$$\doteq 3181 \text{ cm}^2.$$

(c) Two applications of Simpson's rule

$$A \doteq \frac{h}{3} [d_f + 4d_m + d_l].$$

Left 'half': $A_1 \doteq \frac{3}{3} [4 + 4(5) + 5]$
 $= 29 \text{ m}^2.$

Right 'half': $A_2 \doteq \frac{3}{3} [5 + 4(4) + 0]$
 $= 21 \text{ m}^2.$

$$\therefore \text{Total area} \doteq 29 \text{ m}^2 + 21 \text{ m}^2$$

$$= 50 \text{ m}^2.$$

(d) (i) Volume of a cylinder $V = \pi r^2 h.$

$$r = \frac{1}{2} \times 10 = 5 \text{ cm}.$$

$$V = \pi \times 5^2 \times 7.8$$

$$= 612.610\ 5675 \dots$$

$$\doteq 613 \text{ cm}^3.$$

(ii) Engine capacity

$$= 8 \times 612.610\ 5675 \dots \text{ cm}^3$$

$$= 4900.884\ 54 \dots \text{ cm}^3$$

$$= 4900.884\ 54 \dots \text{ mL} \quad (1 \text{ mL} = 1 \text{ cm}^3)$$

$$= 4.900\ 884\ 54 \dots \text{ L}$$

$$< 5 \text{ L}.$$

\therefore Peta's engine meets the racing requirement of being under 5 litres.

Question 24

(a) $P = 24\ 000, r = \frac{0.0475}{12}$ (monthly),

$$n = 3 \times 12 = 36 \text{ months}.$$

$$A = P(1+r)^n$$

$$= 24\ 000 \left(1 + \frac{0.0475}{12} \right)^{36}$$

$$= 27\ 667.890\ 17 \dots$$

$$\doteq \$27\ 667.89.$$

(b) (i) Taxable income = $\$58\ 624 + 12 \times \900
 $= \$69\ 424.$

(ii) Tax payable
 $= \$17\ 730 + 0.48 \times (\$69\ 424 - \$66\ 000)$
 $= \$19\ 373.52.$

(iii) Annual tax on second job
 $= \$19\ 373.52 - \$14\ 410.80$ (from question)
 $= \$4962.72.$

METHOD 1

$$\text{Monthly tax} = \$4962.72 \div 12$$

$$= \$413.56.$$

$$\text{Monthly net income} = \$900 - \$413.56$$

$$= \$486.44.$$

METHOD 2

$$\begin{aligned} \text{Annual net income} &= 12 \times \$900 - \$4962.72 \\ &= \$5837.28. \end{aligned}$$

$$\begin{aligned} \text{Monthly net income} &= \$5837.28 \div 12 \\ &= \$486.44. \end{aligned}$$

(iv) Using future value of an annuity formula

$$A = M \left\{ \frac{(1+r)^n - 1}{r} \right\},$$

$$M = 486.44, r = \frac{0.04}{12} \text{ (monthly),}$$

$$n = 24 \text{ months.}$$

$$A = 486.44 \left\{ \frac{\left(1 + \frac{0.04}{12}\right)^{24} - 1}{\frac{0.04}{12}} \right\}$$

$$\begin{aligned} &= 12\,133.2183 \dots \\ &\doteq \$12\,133.21 \\ &> \$12\,000. \end{aligned}$$

\therefore Vicki will have enough in her account to pay for her holiday.

(c) Instalment = \$336, from the table.

$$\begin{aligned} \text{Total instalments} &= \$336 \times 26 \times 2 \\ &= \$17\,472. \end{aligned}$$

$$\begin{aligned} \therefore \text{Interest} &= \$17\,472 - \$15\,500 \\ &= \$1972. \end{aligned}$$

METHOD 1

Interest rate p.a.

$$= \frac{\$1972}{\$17\,472} \times 100\% \div 2 \text{ years}$$

$$= 6.361 \dots \%$$

$$\doteq 6.36\% \text{ (correct to 2 decimal places).}$$

METHOD 2

Using the simple interest formula $I = Prn$,

$$P = 15\,500, r = ?, n = 2, I = 1972.$$

$$\begin{aligned} 1972 &= 15\,500 \times r \times 2 \\ &= 31\,000r \end{aligned}$$

$$r = \frac{1972}{31\,000}$$

$$= 0.063\,612\,903$$

$$= 6.361\,290\,323 \dots$$

$$\doteq 6.36\% \text{ (correct to 2 dec. pl.).}$$

Question 25

(a) (i) (1) Mode = 2.

(2) The most common number of cars per household, or the number of cars per household that has the highest frequency.

(ii) The number of stickers/car
 $= 0 \times 2735 + 1 \times 12\,305 + 2 \times 13\,918$
 $+ 3 \times 3980 + 4 \times 233$
 $= 53\,013.$

(iii) Sector angle = $\frac{2735}{33\,171} \times 360^\circ$
 $= 29.682\,554\,04 \dots^\circ$
 $\doteq 30^\circ.$

(iv) Number of hours from 6 pm to 8:30 pm = 2.5. The parking fee = \$10 (from the step graph).

(b) (i) Possible answers:

- More secondary students travel by train/bus.
- More / most primary students travel by car/walking.
- Secondary students spread more evenly across the methods of travel.

(ii) Possible answers:

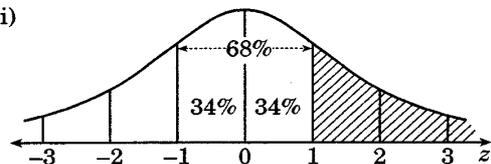
- Secondary students are more mature and can take public transport such as the bus or train.
- Secondary students often need to travel further to get to school and rely on public transport.

(iii) Primary students travelling by bus

$$\begin{aligned} &= 10\% \times 25\,000 \\ &= 2500. \end{aligned}$$

(c) (i) Hardev's score is one standard deviation above the mean.

(ii)



Shaded region is the required percentage.

METHOD 1

$$\text{Percentage} = \frac{100\% - 68\%}{2} = 16\%.$$

METHOD 2

$$\begin{aligned} \text{Percentage} &= 100\% - 34\% - 50\% \\ &= 16\%. \end{aligned}$$

\therefore 16% of people scored higher than Hardev.

Question 26

(a) (i) **METHOD 1**

$$a = 60\,000 \text{ (} a \text{ is the vertical intercept of the graph of } N = a - bt \text{).}$$

METHOD 2

When $t = 0$, $N = 60\,000$ (from graph).

$$N = a - bt$$

$$60\,000 = a - b(0)$$

$$60\,000 = a$$

$$a = 60\,000.$$

Explanation:

a was the number of people in the stadium immediately at the end of the game.

(ii) (1) **METHOD 1**

When $t = 30$, $N = 0$.

$$\begin{aligned} N &= a - bt \\ 0 &= 60\,000 - b(30) \\ &= 60\,000 - 30b \\ 30b &= 60\,000 \\ b &= \frac{60\,000}{30} \\ b &= 2000. \end{aligned}$$

METHOD 2

$-b$ is the gradient of the line.

$$\begin{aligned} -b &= -\frac{60\,000}{30} \left(\frac{\text{rise}}{\text{run}} \right) \\ -b &= -2000 \\ b &= 2000. \end{aligned}$$

(2) b is the rate at which people are leaving the stadium, in people / minute.

(iii) $N = a - bt$. $bt + N = a$
 $bt = a - N$
 $t = \frac{a - N}{b}$.

(iv) **METHOD 1**

When 10 000 people have left, there are $60\,000 - 10\,000 = 50\,000$ in the stadium.

$\therefore N = 50\,000$.

Using the formula from (iii):

$$\begin{aligned} t &= \frac{60\,000 - 50\,000}{2000} \\ &= \frac{10\,000}{2000} \\ &= 5 \text{ minutes.} \end{aligned}$$

METHOD 2

Using $N = a - bt$:

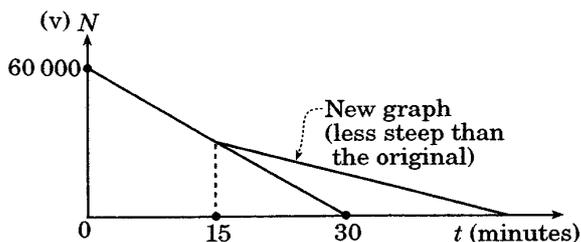
When $N = 50\,000$ (see Method 1),

$$\begin{aligned} 50\,000 &= 60\,000 - 2000t \\ 2000t + 50\,000 &= 60\,000 \\ 2000t &= 10\,000 \\ t &= \frac{10\,000}{2000} \\ &= 5 \text{ minutes.} \end{aligned}$$

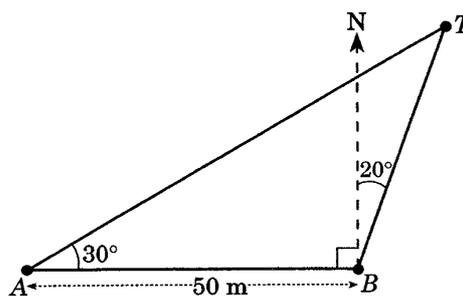
METHOD 3

2000 people / minute is the rate at which people are leaving the stadium, from (ii).

\therefore The time it takes 10 000 people to leave $= \frac{10\,000}{2000} = 5$ minutes.



(b)



(i) N is north.

$$\begin{aligned} \angle NBT &= 20^\circ \quad (T \text{ is } 020^\circ \text{ from } B) \\ \angle NBA &= 90^\circ \quad (B \text{ is due east of } A) \\ \therefore \angle ABT &= 20^\circ + 90^\circ \\ &= 110^\circ. \end{aligned}$$

(ii) $\angle T = 180^\circ - 30^\circ - 110^\circ$ (\angle sum of $\triangle TBA$)
 $= 40^\circ$.

By the sine rule,

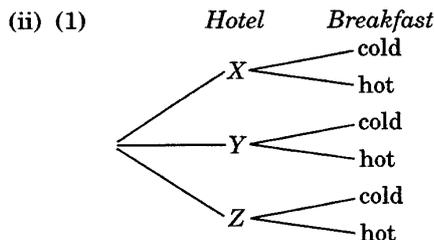
$$\begin{aligned} \frac{BT}{\sin 30^\circ} &= \frac{50}{\sin 40^\circ} \\ BT &= \frac{50 \sin 30^\circ}{\sin 40^\circ} \\ &= 38.893\,095\,67 \dots \\ &\doteq 39 \text{ m.} \end{aligned}$$

Question 27

(a) (i) $P(\text{staying at hotel } X) = 50\%$.

$$\begin{aligned} \therefore P(\text{staying at hotel } Y \text{ or } Z) &= 100\% - 50\% \\ &= 50\%. \end{aligned}$$

$$\begin{aligned} \therefore P(\text{staying at hotel } Z) &= \frac{1}{2} \times 50\% \text{ (equally likely)} \\ &= 25\% \left(\text{or } \frac{1}{4} \right). \end{aligned}$$



6 possible combinations:

- Hotel X, cold
- X, hot
- Y, cold
- Y, hot
- Z, cold
- Z, hot.

(2) These combinations are not all equally likely because the choice of hotels is not equally likely. (Karl is more likely to choose X over Y or Z.)

$$(3) P(Z, \text{ hot}) = 25\% \times \frac{1}{2} = 12.5\% \left(\text{or } \frac{1}{8} \right).$$

(b)

		2nd die					
		1	2	3	4	5	6
1st die	1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
	2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
	3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
	4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
	5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
	6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

There are 36 possible outcomes.

- (i) Number of outcomes that don't show 6 = 25 (as shaded on the above table).

$$\therefore P(\text{neither shows } 6) = \frac{25}{36}$$

- (ii) Number of outcomes that have both showing 6 = 1, (6, 6).

$$P(\text{both show } 6) = \frac{1}{36}$$

Number of outcomes that show only one 6 = 10 (row 6 and column 6, but not (6, 6) in the table above).

$$\therefore P(\text{only one } 6) = \frac{10}{36}$$

$$P(\text{neither shows } 6) = \frac{25}{36}, \text{ from (i).}$$

Financial expectation

$$= \$20\left(\frac{1}{36}\right) + \$2\left(\frac{10}{36}\right) + (-\$2)\left(\frac{25}{36}\right)$$

$$= -\$0.277\ 777\ 777\ \dots$$

$$\div -\$0.28 \text{ (correct to the nearest cent).}$$

(c) (i) $\tan 65^\circ = \frac{AB}{15}$

$$AB = 15 \tan 65^\circ$$

$$= 32.167\ 603\ 81\ \dots$$

$$\div 32.2 \text{ m.}$$

(ii) $CA = 2 \times 32.167\ 603\ 81\ \dots$
 $= 64.335\ 207\ 62\ \dots$

By Pythagoras' theorem on $\triangle CEA$:

$$CE^2 = CA^2 + AE^2$$

$$= (64.335\ 207\ 62\ \dots)^2 + 15^2$$

$$= 4364.018\ 939\ \dots$$

$$CE = \sqrt{4364.018\ 939\ \dots}$$

$$= 66.060\ 7216\ \dots$$

$$\div 66.1 \text{ m.}$$

Note: Do not round off partial answers such as 64.3352 ... and 4364.0189 ... until the end of the calculation.

Question 28

- (a) Let C = cost of pendant, L = length of pendant,
 $C \propto L^2$, $\therefore C = kL^2$.

When $L = 30$, $C = 130$: $130 = k(30^2) = 900k$

$$k = \frac{130}{900} = \frac{13}{90}$$

$$\therefore C = \frac{13}{90} L^2$$

When $L = 40$, $C = \frac{13}{90}(40^2)$
 $= 231.111\ 1111\ \dots$
 $\div 231 \text{ pesos.}$

- (b) (i) In $y = b(a^x)$, a is the growth constant and b is the initial population.

$$\therefore a = 1 + 1.57\%$$

$$= 1 + 0.0157$$

$$= 1.0157$$

$$b = 103\ 400\ 000$$

$$\therefore y = 103\ 400\ 000(1.0157)^x$$

- (ii) When $x = 2$, $y = 103\ 400\ 000(1.0157)^2$
 $= 106\ 672\ 247.1\ \dots$
 $\div 106\ 672\ 000.$

- (c) (i) (Celsius temp.) $\times 1.8 + 32$
 $=$ (Fahrenheit temp.).

$$A \times 1.8 + 32 = 77$$

$$1.8A + 32 = 77$$

$$1.8A = 45$$

$$\therefore A = \frac{45}{1.8} = 25.$$

- (ii) [(Celsius temp.) + 12] $\times 2$
 $=$ Fahrenheit temp.).

$$(C + 12) \times 2 = F$$

$$F = 2(C + 12) \text{ or } 2C + 24.$$

- (d) **Straight-line depreciation:**

METHOD 1

The camera depreciates \$800 in 4 years.
 \therefore It depreciates \$400 in 2 years (half the time).
 \therefore When $n = 2$, $S = \$800 - \$400 = \$400.$

METHOD 2

Using the formula $S = V_0 - Dn$, $V_0 = 800.$

When $n = 4$, $S = 0$: $0 = 800 - D(4)$
 $= 800 - 4D$

$$4D = 800$$

$$D = \frac{800}{4} = 200.$$

$$\therefore S = 800 - 200n.$$

When $n = 2$, $S = 800 - 200(2) = 400.$

Declining-balance depreciation:

Using the formula $S = V_0(1 - r)^n$, $V_0 = 800.$

When $n = 2$,

$$S = 400 \text{ (same as straight-line depreciation)}$$

$$400 = 800(1 - r)^2$$

$$\frac{400}{800} = (1 - r)^2$$

$$0.5 = (1 - r)^2$$

$$\sqrt{0.5} = 1 - r$$

$$r + \sqrt{0.5} = 1$$

$$r = 1 - \sqrt{0.5}$$

$$= 0.292\ 893\ 218\ \dots$$

$$R = 29.289\ 3218\ \dots\ \%$$

$$\div 29.3\%.$$