MANSW

2004 Higher School Certificate Solutions

General Mathematics

SECTION I

SUMMARY

1.	B	7.	D	13.	C	18.	C
2.	B	8.	B	14.	A	19.	B
3.	D	9.	C	15.	A	20.	A
4.	A	10.	C	16.	D	21.	D
5.	D	11.	B	17.	A	22.	A
6.	C	12.	C				

1. (B) $25\% = \frac{25}{100} = \frac{1}{4}$.

2. (B) Using the points (0, 10) and (4, 30):

$$\begin{aligned}\text{Gradient} &= \frac{\text{vertical rise}}{\text{horizontal run}}\\ &= \frac{30-10}{4-0}\\ &= \frac{20}{4}\\ &= 5.\end{aligned}$$

3. (D) $$\begin{aligned}K &= Ft^3\\ &= 5\times\left(0{\cdot}715\right)^3\\ &= 1{\cdot}827\ 629\ 375\ \ldots\\ &\doteqdot 1{\cdot}83 \text{ (3 significant figures)}.\end{aligned}$$

4. (A) Real estate agents receive a *commission* for selling property.

5. (D) $\tan 20° = \frac{\text{opposite}}{\text{adjacent}} = \frac{c}{a}$.

6. (C) $$\begin{aligned}\text{Range} &= \text{highest score} - \text{lowest score}\\ &= 12-1\\ &= 11.\end{aligned}$$

7. (D) 1, 3, 3, 3, 4, 5, 7, 7, 12
$\uparrow$

Median is the middle score for an odd set of scores. Median = 4.
Mode is the score with the highest frequency. Mode = 3.

8. (B) Sector angle for school $X = 100°$.
Number of prizes won by school X

$$\begin{aligned}&= \frac{100}{360}\times 116\\ &= 32{\cdot}2222\ \ldots\\ &\doteqdot 32.\end{aligned}$$

$\therefore$ School X won 32 prizes (number of prizes must be a whole number).

9. (C) $$\begin{aligned}\text{Area} &= \frac{1}{2}ab\sin C\\ &= \frac{1}{2}\times 30\times 20\times\sin 35°\\ &= 172{\cdot}072\ 930\ 9\ \ldots\\ &\doteqdot 172 \text{ m}^2 \left(\text{nearest m}^2\right).\end{aligned}$$

10. (C) Tax payable
$= \$8310 + 0{\cdot}4\times(\$47\,000 - \$45\,000)$
$= \$9110{\cdot}00$.

11. (B) $$\begin{aligned}d &= 6t^2\\ \frac{2400}{6} &= \frac{6t^2}{6}\\ 400 &= t^2\\ t &= \pm\sqrt{400}\\ t &= \pm 20.\end{aligned}$$

$\therefore\ t = 20$ is a possible value.

12. (C) Interquartile range
= upper quartile – lower quartile
$= 11 - 8$
$= 3$.

13. (C) Kath breathes 10×100 L = 1000 L.
Jim breathes 10×6 L = 60 L.
Difference = 1000 L – 60 L = 940 L.

14. (A) Financial expectation
$= \frac{1}{10}\times\$20 + \frac{1}{2}\times\$1 + \frac{2}{5}\times\left(-\$2\right)$
$= \$1{\cdot}70$.

15. (A) Shaded area
= area of ellipse – area of rectangle.

For the ellipse:
$a = \frac{1}{2}\times 60 = 30$
$b = \frac{1}{2}\times 40 = 20$

For the rectangle:
$L = 24$
$B = 10$

$$\begin{aligned}\therefore\ \text{Area} &= \pi ab - LB\\ &= \pi\times 30\times 20 - 24\times 10\\ &= 1644{\cdot}955\ 592\ \ldots\\ &\doteqdot 1645 \text{ mm}^2\left(\text{nearest mm}^2\right).\end{aligned}$$

16. (D) Both linear equations are of the form
$y = mx + b$ where m = gradient,
b = y-intercept.
$\therefore$ The graph of $y = 2x - 5$ has a gradient of 2 and a y-intercept of –5.
$\therefore$ The graph of $y = x + 6$ has a gradient of 1 and a y-intercept of 6.
(D) is the only diagram that shows 2 lines, both having positive gradient, one line with a positive y-intercept and the other line with a negative y-intercept.

17. (A) *METHOD 1*
110% of original price = \$880.
So 10% = \$880 ÷ 11
$\therefore$ GST refund = \$80.

METHOD 2
110% of original price = \$880.
1% = \$880 ÷ 110
1% = \$8.
So 10% = \$80
$\therefore$ GST refund = \$80.

18. (C)

	1	2	3	4	5	6
1	•	•	•	•	•	(1, 6)
2	•	•	•	•	•	(2, 6)
3	•	•	•	•	•	(3, 6)
4	•	•	•	•	•	(4, 6)
5	•	•	•	•	•	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	•

There are 36 possible outcomes when 2 dice are rolled.

$$\begin{aligned}P\left(\text{only one dice shows 6}\right) &= \frac{10}{36}\\ &= \frac{5}{18}.\end{aligned}$$

19. (B) $A = P(1+r)^n$ where A = final amount
$P = 250$
$r = 0{\cdot}05\%$
$= \frac{0{\cdot}05}{100}$ per day
$n = 30$ days.

$$\begin{aligned}A &= 250\times\left(1+\frac{0{\cdot}05}{100}\right)^{30}\\ &= 253{\cdot}777\ 314\ 8\ \ldots\end{aligned}$$

Final value = \$253·78 (to nearest cent).
$\therefore$ Interest charged = \$253·78 – \$250
= \$3·78.

20. (A) *METHOD 1*
Equivalent normal hours worked
$= 20 + 4\times 1{\cdot}5$
$= 26$.
So hourly rate = \$291·20 ÷ 26 = \$11·20.

METHOD 2
Let hourly rate be h.

$$\begin{aligned}20\times h + 4\times 1{\cdot}5\times h &= 291{\cdot}20\\ 20h + 6h &= 291{\cdot}20\\ 26h &= 291{\cdot}20\\ \therefore\quad h &= \frac{291{\cdot}20}{26}\\ &= 11{\cdot}2.\end{aligned}$$

$\therefore$ Hourly rate is \$11·20.

21. (D) Relationship is of the form $t = \frac{k}{n}$, where k is a constant.
A graph of this relationship would be a hyperbola. Note that a hyperbola is a curve that approaches each axis but has no point on either axis.

22. (A) One Australian dollar = 0·62 euros
×
ie. to convert Australian dollars to euros *multiply* by 0·62.
$\therefore$ Number of euros in 25 Australian dollars $= 25\times 0{\cdot}62$
= 15·5 euros.

One Vistabella dollar = 1·44 euros
÷
ie. to convert euros to Vistabella dollars *divide* by 1·44.
$\therefore$ Number of Vistabella dollars in 15·5 euros = 15·5 ÷ 1·44
= 10·763 888 89 …
= \$V10·76.

SECTION II

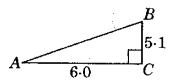
Question 23

(a) (i) Garden area
= area of ΔABC + area of ΔADC
$= \frac{1}{2}\times 10\times 5{\cdot}1 + \frac{1}{2}\times 10\times 6{\cdot}3$
$= 57 \text{ m}^2$.

(ii) $V = Ah$ where $A = 57 \text{ m}^2$, h = 5 cm
= 0·05 m
$\therefore$ Volume of straw $= 57\times 0{\cdot}05$
$= 2{\cdot}85 \text{ m}^3$.

(iii) Number of bags = 2·85 ÷ 0·25
= 11·4
$\doteqdot$ 12 (rounded up).
$\therefore$ Carmel needs 12 bags of straw.

(iv)



$AB^2 = 5{\cdot}1^2 + 6{\cdot}0^2$ (Pythagoras' theorem)
$= 62{\cdot}01$.

$AB = \sqrt{62 \cdot 01}$
$= 7 \cdot 874\,642\,849 \ldots$
$\doteqdot 8$ m.

$\therefore$ Length of the fence = 8 m (to the nearest metre).

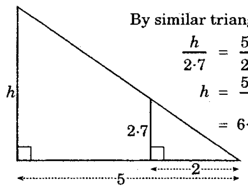
(b) (i) Cost of *Novirus* $= x + 115$.

(ii) The equation is

$$\begin{aligned} x + x + 115 &= 415 \\ 2x + 115 &= 415 \\ 2x &= 300 \\ \therefore \quad x &= 150. \end{aligned}$$

The cost of *Funmaths* is \$150·00.

(c)



By similar triangles

$$\frac{h}{2 \cdot 7} = \frac{5}{2}$$
$$h = \frac{5 \times 2 \cdot 7}{2} = 6 \cdot 75.$$

$\therefore$ The height of the tree is 6·75 metres.

Question 24

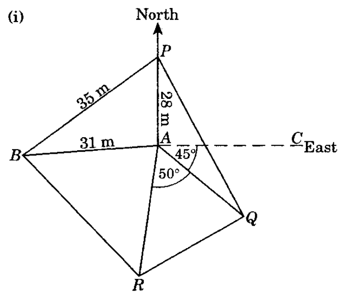
(a) (i) The mean 3 pm wind speed for September is 15 km/h.

(ii) The month with the lowest 3 pm wind speed is February.

(iii) The highest number of raindays occur in the three-month period January, February, March.

(iv) When the wind speed is high, as in July, August and September, the number of raindays is low. When the wind speed is low, as in February and March, the number of raindays is high.

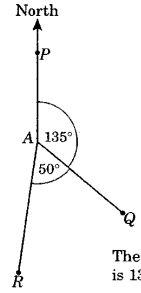
(b) (i)



Let C lie due east of A.

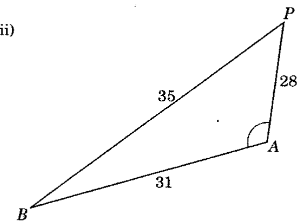
$$\begin{aligned} \angle PAQ &= \angle PAC + \angle CAQ \\ &= 90° + 45° \\ &= 135°. \end{aligned}$$

(ii)



The bearing of R from A is $135° + 50°$, ie. $185°$.

(iii)

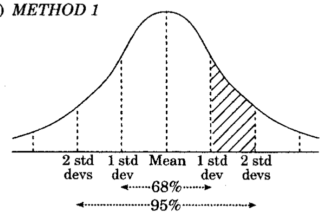


Using the cosine rule

$$\begin{aligned} \cos \angle PAB &= \frac{p^2 + b^2 - a^2}{2pb} \\ &= \frac{31^2 + 28^2 - 35^2}{2 \times 31 \times 28} \\ &= 0 \cdot 299\,539\,17 \ldots \\ \therefore \angle PAB &= 72 \cdot 570\,073\,25 \ldots \\ &\doteqdot 73° \text{ (correct to nearest degree).} \end{aligned}$$

(c) (i) Roberto's score is on the line between 180 and 190 (below the shaded area). It is a score between 1 and 2 standard deviations above the mean, so his z-score is a number between 1 and 2.

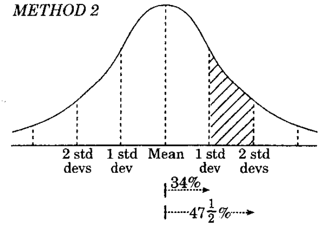
(ii) *METHOD 1*



For a normal distribution:
68% of scores lie within 1 standard deviation of the mean.
95% of scores lie within 2 standard deviations of the mean.

$\therefore$ shaded area $= \frac{1}{2} \times (95\% - 68\%)$
$= 13 \cdot 5\%$.

METHOD 2



For a normal distribution:
68% of scores lie within 1 standard deviation of the mean.
$\therefore$ 34% of scores lie within the mean and 1 standard deviation above the mean.
95% of scores lie within 2 standard deviations of the mean.
$\therefore$ $47\frac{1}{2}$% of scores lie within the mean and 2 standard deviations above the mean.

$\therefore$ shaded area $= 47\frac{1}{2}\% - 34\%$
$= 13\frac{1}{2}\%$.

Question 25

(a) (i) *METHOD 1*

2003 tax deduction = 40% of \$6500
= 0·4 × \$6500
= \$2600.

METHOD 2

$S = V_0(1 + r)^n$, $\quad V_0 = 6500$
$r = 40\% = 0 \cdot 40$
$n = 1$

The value of the computer after one year is

$S = 6500 \times (1 - 0 \cdot 40)^1$
$= \$3900.$

$\therefore$ The tax deduction = \$6500 − \$3900
= \$2600.

(ii) After 3 years, $S = 6500 \times (1 - 0 \cdot 40)^3$
$= \$1404.$

$\therefore$ The value of her computer at the start of the 2006 financial year is \$1404.

(b) (i) *METHOD 1*

The arrangements are: *CSV, CVS*
VSC, VCS
SVC, SCV

$\therefore$ There are 6 different ways of arranging the tubs in a row.

METHOD 2

There are 3 choices for 1st tub, 2 for 2nd tub, 1 for 3rd tub.
Number of choices $= 3 \times 2 \times 1$
$= 6.$

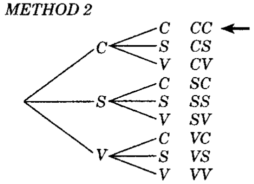
$\therefore$ There are 6 different ways of arranging the tubs in a row.

(ii) *METHOD 1*

P(choosing chocolate on one day) $= \frac{1}{3}$.

P(choosing chocolate on both days)
$= \frac{1}{3} \times \frac{1}{3}$
$= \frac{1}{9}$.

METHOD 2



$\therefore$ P(choosing chocolate on both days) $= \frac{1}{9}$.

(iii) *METHOD 1*

The two flavours in this case must be different. There are 3 choices for the first flavour, 2 choices for the second flavour.

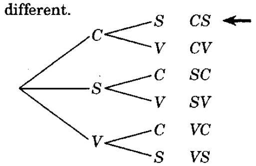
P(selecting chocolate as first flavour) $= \frac{1}{3}$.

P(selecting strawberry as second flavour) $= \frac{1}{2}$.

$\therefore$ P(selecting chocolate first then strawberry)
$= \frac{1}{3} \times \frac{1}{2}$
$= \frac{1}{6}$.

METHOD 2

The two flavours in this case must be different.



$\therefore$ P(chocolate first, then strawberry) $= \frac{1}{6}$.

(c) (i)

	Test indicated a lie	*Test did not indicate a lie*	*Total*
People who lied	40	10	50
People who did *not* lie	20	130	150

(ii) Number of people tested where lie-detector test was accurate = 40 + 130 = 170. (from table)

(iii) Percentage of people tested where the test was accurate $= \frac{170}{200} \times 100\%$ $= 85\%$.

Note: Denominator is 200 as required percentage is based on all the people tested.

(iv) *Note*: In this case only referring to the people who did *not* lie, so initial denominator will be 150.

P(test indicated a lie for a person who did *not* lie)
$= \frac{20}{150}$
$= \frac{2}{15}$.

Question 26

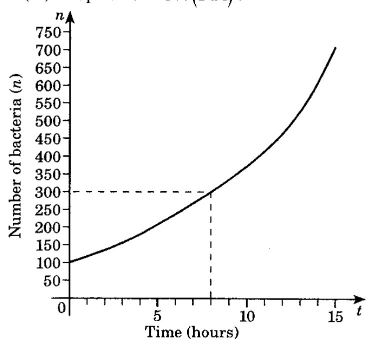
(a) (i) Percentage increase $= \frac{14}{100} \times 100\%$ $= 14\%$.

(ii) Given that $n = 100(1{\cdot}14)^t$.

When $t = 15$, $n = 100(1{\cdot}14)^{15}$
$= 713{\cdot}793\ 797\ 8\ldots$
$\doteqdot 714$.

$\therefore$ There are 714 bacteria after 15 hours.

(iii) Graph of $n = 100(1{\cdot}14)^t$.



(iv) From the graph, $n = 300$ when $t \doteqdot 8$ so the population is 300 after 8 hours.

An alternative method is to use 'guess and check' to solve $300 = 100(1{\cdot}14)^t$.

(b) (i) 131°E

Sorong — 1°S

Darwin — 12°S

Difference in latitude = 12° − 1° = 11°.

(ii) (1) *METHOD 1*

An angular distance of 1° on a great circle equals 60 nautical miles.

Distance = 11 × 60 nautical miles
= 11 × 60 × 1·852 km
= 1222·32 km
$\doteqdot$ 1200 km.

METHOD 2

The arc length formula can be used.

$\ell = \frac{\theta}{360} 2\pi r$ where $\theta = 11$, $r = 6400$

Distance $= \frac{11}{360} \times 2 \times \pi \times 6400$
$= 1228{\cdot}711\ 793\ldots$
$\doteqdot 1200$ km.

(2) *METHOD 1*

Assuming the tourists travel in a straight line, the distance is approximately 1200 km, or 1200 ÷ 1·852 $\doteqdot$ 647·95 nautical miles. At a speed of 15 nautical miles per hour (ie. 15 knots) this would take 647·95 ÷ 15 $\doteqdot$ 43·2 hours

$\left(\text{since time} = \frac{\text{distance}}{\text{speed}}\right)$.

So they would complete the trip in less than 48 hours.

METHOD 2

Distance = 11 × 60
= 660 nautical miles.

Using speed $= \frac{\text{distance}}{\text{time}}$, or the diagram D/ST,

time taken $= \frac{660}{15}$
= 44 hours.

So they would complete the trip in less than 48 hours.

METHOD 3

At 15 knots = 15 nautical miles per hour.

Distance travelled in 48 hours
= 15 × 48
= 720 nautical miles
= 720 × 1·852 km
= 1333·44 km.

So they would be able to travel the distance of 1200 km in less than 48 hours.

Question 27

(a) (i) 150 000 = 150 × 1000

From the table:

Monthly repayments = 7·75 × 150
= \$1162·50.

(ii) Total repayments = \$1162·50 × 20 × 12
= \$279 000.

Total interest repaid
= \$279 000 − \$150 000
= \$129 000.

(iii) *METHOD 1*

Interest paid if loan is paid after 15 years is: Interest = \$242 730 − \$150 000
= \$92 730.

$\therefore$ Savings in interest is:
Difference in interest
= \$129 000 − \$92 730
= \$36 270.

METHOD 2

The interest is the difference between the total payments.

So difference in interest
= \$279 000 − \$242 730
= \$36 270.

(b) (i) Hours worked:

Friday: 4 hours at \$18·00 per hour
Saturday: 6 hours time-and-a-half (6 × 1·5)
Sunday: 5 hours double time (5 × 2)

$\therefore$ Total pay $= (\$4 \times \$18) + (6 \times 1{\cdot}5 \times \$18) + (5 \times 2 \times \$18)$
$= \$414$.

(ii) *METHOD 1*

David's Saturday pay = 6 × 1·5 × \$18
= \$162.

Number of hours needed to work at the weekday rate (\$18 / h) = \$162 ÷ 18
= 9 hours.

METHOD 2

On a Saturday David works 6 hours time-and-a-half.

Equivalent weekday hours = 6 × 1·5
= 9 hours.

(c) The future value of \$200 per month at 6% pa compounded monthly over 4 years is found using the 'Future value of an annuity' formula.

In this case, $M = \$200$

$r = \frac{0{\cdot}06}{12} = 0{\cdot}005$ per month

$n = 12 \times 4 = 48$ months.

$A = M\left[\frac{(1+r)^n - 1}{r}\right]$

$= \$200\left[\frac{(1+0{\cdot}005)^{48} - 1}{0{\cdot}005}\right]$

$= \$10\ 819{\cdot}57$.

$\therefore$ Sanjeev will reach his goal of \$10 500 with \$319·57 in excess.

Question 28

(a) (i) In this case, $R = \frac{w}{h^2}$.

Substitute $w = 72$, $h = 1{\cdot}50$ (in metres).

$R = \frac{72}{(1{\cdot}5)^2}$
$= 32$.

Fred's health rating is 32.

(ii) Substitute $R = 25$, $h = 1{\cdot}60$ (in metres)

$25 = \frac{w}{(1{\cdot}6)^2}$

$w = 25 \times (1{\cdot}6)^2$
$= 64$ kg.

Difference in weights = 72 − 64
= 8 kg.

ie. Fred needs to lose 8 kg.

(b) (i) Relationship is of the form $C = kb^3$, where k is a constant.

(ii) Substitute $C = 50$, $b = 10$.

$50 = k \times 10^3$
$50 = 1000k$
$k = \frac{50}{1000}$
$\therefore\ k = 0{\cdot}05$.

(iii) *METHOD 1*

$C = 0{\cdot}05b^3$

Replace b with $2b$ in equation from (b)(i).

$C = k(2b)^3$
$= k \times 8b^3$
$= 8kb^3$.

$\therefore$ The cost is 8 times bigger, so Felicity is incorrect.

METHOD 2

$C = 0{\cdot}05b^3$.

Substitute $b = 10$: $C = 0{\cdot}05 \times 10^3 = 50$.

Substitute $b = 20$ (since the base is doubled):

$$C = 0{\cdot}05 \times (20)^3 = 400.$$

When the base is doubled the cost is 8 times more, so Felicity is incorrect.

(c) (i) Median height of middle section = 170.
Median weight of middle section = 55.
$\therefore$ Coordinates of C are (170, 55).

(ii) Jill needs to draw a line parallel to AB which is a third of the distance towards C.

(iii) (1) Weight in kg $= \frac{2}{3}(\text{height in cm}) - 50$.

Substitute weight = 75.

$$75 = \frac{2}{3}(\text{height in cm}) - 50$$

$$125 = \frac{2}{3}\text{ height in cm}$$

$$\text{height in cm} = \frac{125}{2} \times 3 = 187{\cdot}5.$$

$\therefore$ The height is predicted to be 187·5 cm.

(2) Suggested possible answers are:

- the sample used is small (only 9 people)

OR

- tall people are not necessarily heavier

OR

- the model cannot be used outside the range of the sample

OR

- the accuracy with which the graph is drawn.

END OF GENERAL MATHEMATICS SOLUTIONS