EXT SOLS

Q1 (a) $2(2x-1)(4x^2+2x+1)$ (2)

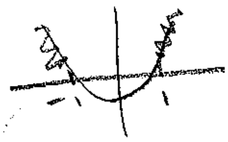
(b) $\dfrac{2(6)-3}{3}$, $\dfrac{-4(2)+2(1)}{3}$ (2)

$P(3,-2)$

(c) $\dfrac{2}{x+1} < 1$

$2x+2 < x^2+2x+1$ (2)

$0 < x^2-1$



$x < -1$ or $x > 1$

(d) $f'(x) = 27x^2$ and $x^2 > 0$ (2)

so $27x^2$ is always > 0

$\therefore$ pos. gradient

always increasing

(e) (i) $u = x$ $\quad v = \cos^2 3x$

$u' = 1$ $\quad v' = -6\cos 3x \sin 3x$

$\therefore \dfrac{d}{dx}(x\cos^2 3x) = \cos^2 3x - 6x\cos 3x \sin 3x$ (2)

(ii) $u = x^2$ $\quad v = \tan^{-1} x$

$u' = 2x$ $\quad v' = \dfrac{1}{1+x^2}$

$2x\tan^{-1}x + \dfrac{x^2}{1+x^2}$ (2)

Q2

(a) $P(2) = 2^3 - 2(2)^2 + k(2) + k^2$

$0 = 8 - 8 + 2k + k^2$

$0 = k(2+k)$

$k = 0, -2.$

2

(b) $y = 3x + 1 \quad m_1 = 3$

$y = -x + 5 = 0 \quad m_2 = -1$

$\tan\theta = \left| \dfrac{3-(-1)}{1 \times 3(-1)} \right| = 2$

$\therefore \theta \doteqdot 63^\circ$

2

(c) $\displaystyle\int_0^1 \frac{dx}{\sqrt{4-x^2}} = \left[\sin^{-1} \frac{x}{2} \right]_0^1$

$= \sin^{-1}\left(\tfrac{1}{2}\right) - \sin^{-1}(0)$

$= \dfrac{\pi}{6} - 0$

$= \dfrac{\pi}{6}$

2

(d) $\left(1 - \dfrac{1-t^2}{1+t^2}\right) \div \dfrac{2t}{1+t^2}$ 1.

$\dfrac{1+t^2-1+t^2}{\cancel{1+t^2}} \times \dfrac{\cancel{1+t^2}}{2t}$ 1

$\dfrac{2t^2}{2t} = t$

but $t = \tan\dfrac{x}{2}$ 1

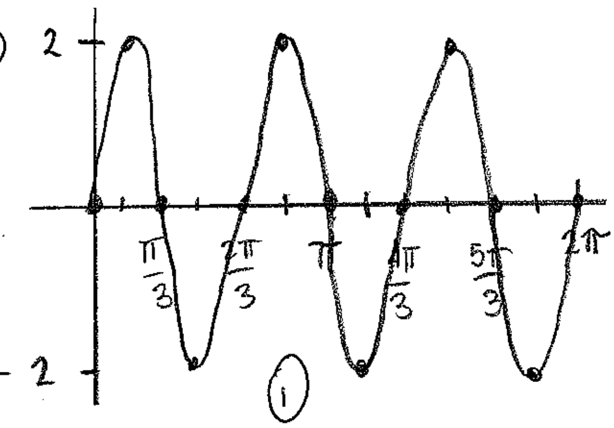
Total 3

(e) $y = 2\sin 3x$

$\dfrac{2\pi}{3} = \text{per}$

1

1



1

Q3

(a) $\alpha+\beta+\gamma = \frac{-b}{a}$

$\alpha\beta+\alpha\gamma+\beta\gamma = \frac{c}{a}$

$\alpha\beta\gamma = \frac{-d}{a}$

$\frac{\beta\gamma+\alpha\gamma+\alpha\beta}{\alpha\beta\gamma} = \frac{\frac{c}{a}}{\frac{-d}{a}}$

$\frac{c}{a} \times \frac{a}{-d}$

$-\frac{29}{\cancel{5}} \times \frac{\cancel{5}}{-6} = \frac{29}{6}$

2

(b) Show true for $n=1$

$2^1 \geq 1+1$

$2 \geq 2$ ✓

$\therefore$ true for $n=1$

assume true for $n=k$

$2^k \geq 1+k$

Prove true for $n=k+1$

LHS $= 2^{k+1}$ RHS $= k+1+1$

$= k+2$

$2^k \cdot 2$

but $2^k \geq 1+k$ if $k \geq 1$

$\therefore 2 \times 2^k \geq 2(1+k)$ which is greater than $k+2$

$= 2+2k$

4

c) $u = x^2+1$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$\therefore \frac{1}{2}\int_2^{50} \frac{du}{u^2} = -\frac{1}{2}\left[\frac{1}{u}\right]_2^{50}$

$= -\frac{1}{2}\left[\frac{1}{50}-\frac{1}{2}\right]$

$= 6/25 = 0.24$

4

d) $S_\infty = \frac{a}{1-r}$

$= \frac{\sin^2 x}{1-\sin^2 x}$

$= \frac{\sin^2 x}{\cos^2 x}$

$= \tan^2 x$

2

4a) $\frac{AC}{h} = \tan 60^\circ$

$AC = h \tan 60$

$AC = h\sqrt{3}$

$BC = h \tan 45^\circ$

$BC = h$

$900 = 3h^2 + h^2 - 2\sqrt{3}h^2 \cos 120^\circ$

$900 = h^2(3 + 1 - 2\sqrt{3} \times -1/2)$

$900 = h^2(5.732\ldots)$

$157.01\ldots = h^2$

$h \doteq 12.53$

$\sqrt{3}\cos x - \sin x = r(\cos x \cos\alpha - \sin\alpha \sin x)$

$r = \sqrt{\sqrt{3}^2 + 1^2} = \sqrt{4} = 2$

$2\cos\alpha = \sqrt{3}$ $\quad$ $2\sin\alpha = 1$

$\cos\alpha = \frac{\sqrt{3}}{2}$ $\quad$ $\sin\alpha = 1/2$

$\tan\alpha = \frac{1}{\sqrt{3}}$

$\alpha = 30^\circ$

$2\cos(x + 30^\circ) = 1$

$\cos(x + 30^\circ) = 1/2$

$\cos(x + 30^\circ) = \cos(60^\circ)$

$x + 30 = 360n \pm 60^\circ$

$x = 360n + 30^\circ$ or $360^\circ n - 90^\circ$

4b) $x = 2t$

$y = 2t^2$

(i)

$$\frac{x}{2} = t$$

$$\therefore \quad y = 2\left(\frac{x}{2}\right)^2$$

$$y = 2\left(\frac{x^2}{4}\right)$$

$$y = \frac{x^2}{2}$$

$$2y = x^2. \qquad ①$$

(ii)

$$y' = \frac{1}{2} \times 2x$$

$$= x$$

@ $x = 2 \quad m_t = 2$

@ $x = 2 \quad y = \cancel{8}\, 2$

$\therefore$ equation of tangent is $y - 2 = 2(x - 2)$

$$y - 2 = 2x - 4$$

$$y = 2x - 2 \qquad ②$$

(c)

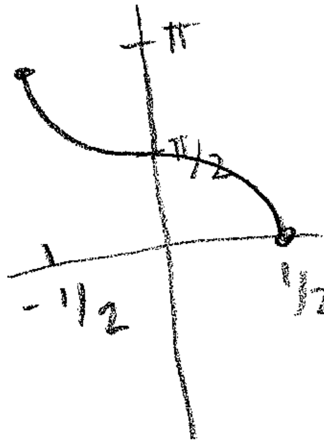
$$\int \sin^2 2x \, dx$$

$$= \frac{1}{2}\int (1 - \cos 4x)\, dx$$

$$= \frac{1}{2}x - \frac{1}{2} \times \frac{1}{2}\sin 4x + c$$

$$= \frac{1}{2}x - \frac{1}{4}\sin 4x + c \qquad ②$$

Q4) d) (ii)



①

(i) D : $-\frac{1}{2} \le x \le \frac{1}{2}$

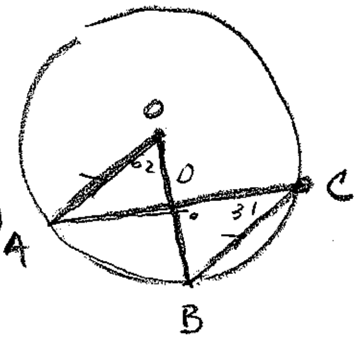
R : $0 \le y \le \pi$. ②

Q5) a) (i) $\angle AOB = 62°$ angle at the centre is twice the angle on circumference standing on the same arc.

②

(ii) $\angle CBD = 62$ (alt angles bet parallel lines)

and ∴ $\angle CDB = 87°$ (angle sum of Δ)



(b) (i) $N = 80 + Ae^{0.1t}$

$\frac{dN}{dt} = 0.1Ae^{0.1t}$ ①

$= 0.1(N-80)$

(ii) $N = 100$ when $t = 0$ ∴ $A = 20$

$200 = 80 + 20e^{0.1t}$

$e^{0.1t} = 6$

$0.1t = \ln 6$

$t = 10\ln 6$

∴ $t = 17.92$

So in 2017 the pop'n reaches 200.

Q6)

a i) $v = \frac{1}{x+1}$

$\frac{d}{dx}\left(\frac{1}{2(x+1)^2}\right) = a$

$= \frac{-1}{(x+1)^3}$.

(ii) $\frac{dx}{dt} = \frac{1}{x+1}$

$\therefore \frac{dt}{dx} = x+1$

$t = \int (x+1)\,dx$

$= x^2 + x + c$

when $t=0$ $x=0$

$\therefore c = 0$

$t = x^2 + x$

$\cancel{t = x^2}$

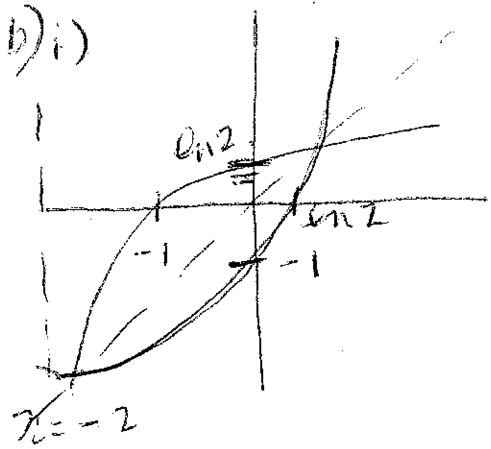
$t + \frac{1}{4} = x^2 + x + \frac{1}{4}$

$t + \frac{1}{4} = (x+\frac{1}{2})^2$

$\pm\sqrt{t+\frac{1}{4}} = x + \frac{1}{2}$

$\therefore -\frac{1}{2} \pm \sqrt{t + 1/4} = x$

6 b) i)



(2)

(ii) intersect where $y = x$

$\ln(x+2) = x$

$x+2 = e^x$

$e^x - x - 2 = 0$ ✓ (2)

(iii) $g(x) = e^x - x - 2$

$g(1) = e - 3 < 0$ and

$g(2) = e^2 - 4 > 0$

$\therefore 1 < \alpha < 2$ where α is a root.

(iv) $g'(x) = e^x - 1$

$1.2 - \frac{e^{1.2} - 3.2}{e^{1.2} - 1} \doteqdot 1.1$

(2)

7a)

(i) $x = 1 + 3\cos\frac{t}{2}$, V m/sec a m/sec

$$v = \frac{dx}{dt} = -\frac{3}{2}\sin\frac{t}{2}$$

$$a = \frac{d}{dt}\left(\frac{dx}{dt}\right) = -\frac{3}{4}\cos\frac{t}{2}$$

but $x - 1 = 3\cos\frac{t}{2}$

2

$$\therefore\ a = -\frac{1}{4}(x-1)\ ✓$$

(ii) $v = 0$ when $-\frac{3}{2}\sin\frac{t}{2} = 0$

at rest

$$\sin\frac{t}{2} = 0$$

$$t = 0,\quad t = 2\pi,\quad t = 4\pi \ldots$$

$\therefore$ when $t = 0$ $x = 1 + 3 = 4$

$t = 2\pi$ $x = 1 - 3 = -2$

$\therefore$ from 0 and back is 12 m.

and it takes 4π sec

period $\frac{2\pi}{1/2} = 4\pi$

b.

(i) $\dot{y} = -10t + V\sin\alpha$ and $\dot{y} = 0$ when $t = 3$

$0 = -30 + V\sin\alpha$ (1)

$\therefore V\sin\alpha = 30$.

(ii) $y = -80 = -5t^2 + 30t$

$\therefore t^2 - 6t - 16 = 0 \quad (t \geq 0)$ (2)

$(t-8)(t+2) = 0 \quad \therefore t = 8$.

(iii) $x = 320$ when $t = 8$ (1)

$320 = 8V\cos\alpha$

$\therefore V\cos\alpha = 40$

(iv) $V^2(\sin^2\alpha + \cos^2\alpha) = 30^2 + 40^2$ (2)

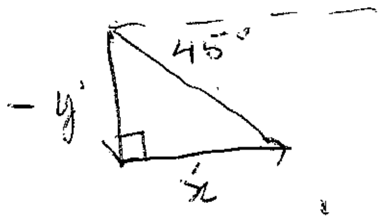
$V^2 = 2500$

$\therefore V = 50$

$$\frac{V\sin\alpha}{V\cos\alpha} = \frac{30}{40} \quad \therefore \tan\alpha = \frac{3}{4}$$

and $\alpha = 36°52'$

(v) (2)



$\dot{y} = -\dot{x}$

$-10t + 30 = -40$

$\therefore 10t = 70$

$t = 7$

hence after 7 seconds