

Mathematics

HSC Mathematics Trial Examination

Term 3 2011

Name: _____

General Instructions

- Time allowed – **3 hours + 5 minutes reading time**
- Write using blue or black pen; diagrams may be in pencil.
- Answer each question in a **SEPARATE** writing booklet.
- Show all necessary working.
- Board-approved calculators may be used.
- **Total- 120 marks**
- Attempt questions 1-10.
- All questions are of equal value.

Total Marks –120

Attempt Questions 1-10

All questions are of equal value

Answer the questions on the writing booklets provided. Start each question on a new page.

Question 1 (12 marks)

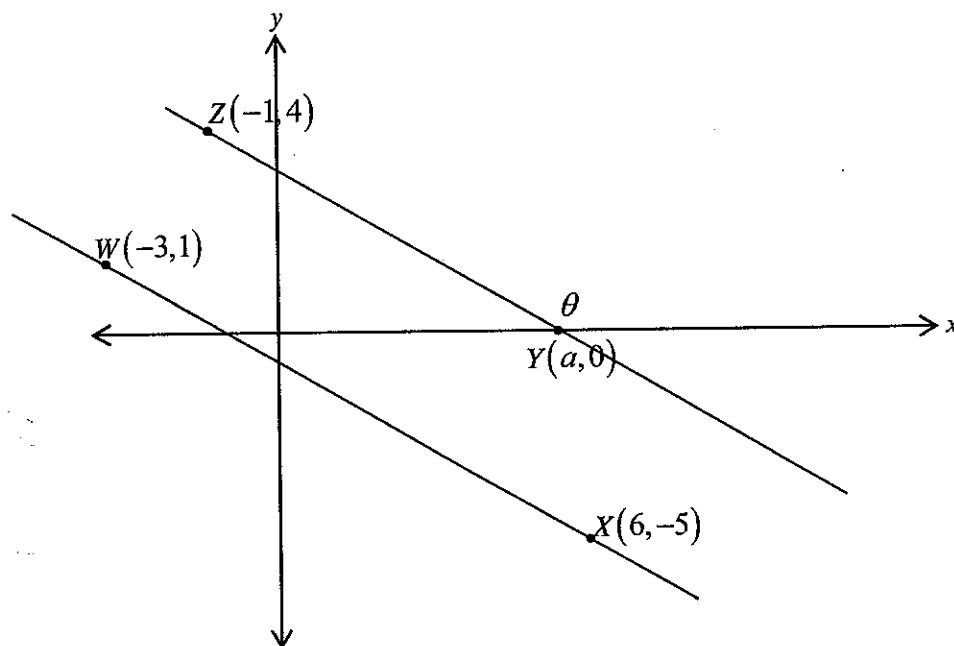
Marks

- (a) Evaluate $5e^{2.302}$ correct to 3 significant figures. 2
- (b) Factorise fully $3x^2 + 5x - 2$. 2
- (c) Express $\frac{5}{\sqrt{3}+2}$ in the form $a\sqrt{3}+b$. 2
- (d) Solve $|2x-1| < 3$. 2
- (e) If $f(x) = \sin 3x$ find the exact value of $f'\left(\frac{\pi}{18}\right)$. 2
- (f) Express $\frac{3}{a+1} - \frac{a-4}{a}$ as a single fraction in its simplest form. 2

Question 2 (12 marks)

Marks

(a)



The diagram shows the points $W(-3,1)$, $X(6,-5)$ and $Z(-1,4)$ in the Cartesian plane. The point $Y(a,0)$ is the point where the line ZY intersects with the x -axis.

- (i) Show that the gradient of WX is $-\frac{2}{3}$. 1
- (ii) Show that the equation of WX is $2x+3y+3=0$. 1
- (iii) If $WX \parallel ZY$ find the value of a , the x -coordinate of the point Y . 1
- (iv) Show the distance WZ is $\sqrt{13}$ 1
- (v) Find the perpendicular distance from Z to the line WX . 2
- (vi) Find the size of angle θ correct to the nearest minute. 2
- (vii) Find the equation of the circle centre W which has the line ZY as a tangent. 2

- (b) Many concave up parabolas have roots of $x=-4$ and $x=3$. Give the equation of one such parabola in general form. 2

Question 3 (12 marks)

Marks

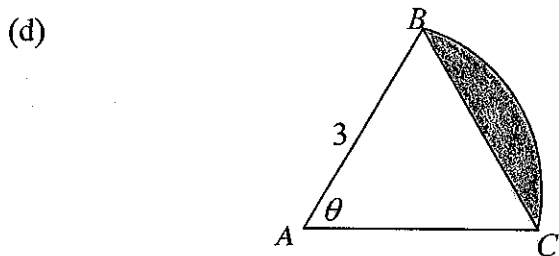
(a) Consider the function $f(x) = \begin{cases} 2x-1 & \text{if } x < 2 \\ x^2 & \text{if } x \geq 2 \end{cases}$

i) Evaluate $f(2) - f(-2)$ **1**

ii) Sketch the graph of $f(x)$, indicating all important points. **2**

(b) State the domain and range of $y = \frac{1}{\frac{1}{2}x - 3}$ **2**

(c) Find the value(s) of k for which the quadratic equation $x^2 - (k+2)x - (3k+11) = 0$ has real roots. **3**



In this diagram $\triangle ABC$ is an equilateral triangle and $AB = 3$.

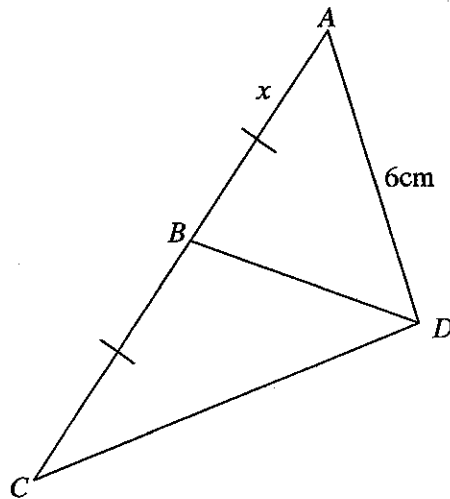
(i) Find the length of arc BC . **1**

(ii) Find the exact area of the minor segment subtended by chord BC **3**

Question 4 (12 marks)

Marks

- (a) Find the equation of the normal to $y = 2x^2 + 3x - 5$ at the point where $x = 1$.
Give your answer in general form. 3
- (b) In the diagram below, $AB = BC = x$, $AD = 6\text{cm}$ and $\angle ADB = \angle ACD$.



- (i) Prove $\triangle ABD \cong \triangle ADC$. 2
- (ii) Find the exact value of x . 2
- (c) Find $\int \frac{x}{x^2 - 1} dx$ 2
- (d) Solve for x : $\log x + \log(x - 1) = \log 4x$ 3

Question 5 (12 marks)**Marks**

- (a) The gradient function of a curve is $\frac{dy}{dx} = 3 - 2x^2$. Find the equation of the curve if it passes through the point $(1, -2)$. 2
- (b) Maureen is raising money for charity by jumping on a Pogo stick. Her challenge is jump between two points, A and B, 20 times. On her first attempt she takes 45 jumps. On her second attempt she takes 48 jumps. On her third attempt she takes 51 jumps. She continues this pattern for all 20 attempts.
- (i) How many jumps did she make on her 20th attempt? 2
- (ii) How many jumps did she make altogether? 1
- (c) Differentiate with respect to x
- (i) $3xe^{x^2}$ 2
- (ii) $\frac{\ln x}{x^2}$ 2
- (d) Show that $\frac{\tan \theta}{\sec \theta - 1} - \frac{\tan \theta}{\sec \theta + 1} = 2 \cot \theta$ 3

Question 6 (12 marks)

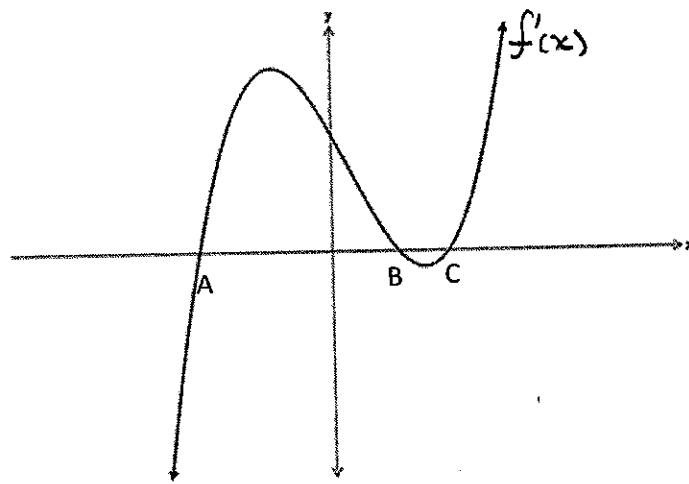
Marks

(a) Evaluate $\int_0^{\pi} \left(\sin \frac{x}{2} + \sqrt{x} \right) dx$. Leave answer in exact form.

2

(b) The graph of $f'(x)$ is shown below. Sketch a possible graph of $f(x)$.

2



(c) Consider the curve $y = x^3 - 12x^2 + 36x$.

(i) Find the x and y intercepts.

2

(ii) Find any stationary points and determine their nature.

3

(iii) Find the point of inflection.

1

(iii) Hence, sketch for $-1 \leq x \leq 7$, showing all features from parts (i) and (ii).

2

Question 7 (12 marks)

Marks

(a) Given $\sum_{n=0}^{\infty} \frac{9}{x^{n+1}} = 18$, find:

(i) the first three terms of the series $\sum_{n=0}^{\infty} \frac{9}{x^{n+1}}$ 1

(ii) the value of x . 2

(b) (i) Graph the region bounded by the curve $y = \frac{4}{x}$, the x -axis and the lines $x=1$ and $x=2$. 1

(ii) Find the exact area of the region in part (i). 2

(ii) Use the table below and Simpson's Rule to find an approximation for the area bounded by the curve $y = \frac{4}{x}$, the x -axis and the lines $x=1$ and $x=2$. 2

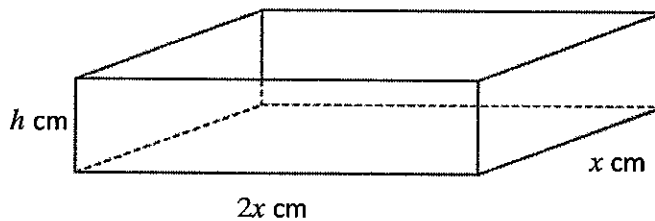
x	1	$1\frac{1}{4}$	$1\frac{1}{2}$	$1\frac{3}{4}$	2
y	4	$\frac{16}{5}$	$\frac{8}{3}$	$\frac{16}{7}$	2

(c) Solve for x : $\sqrt{2} \sin x = \tan x$ for $0 \leq x \leq 2\pi$. 4

Question 8 (12 marks)

Marks

- (a) Joe is building a small *open topped* toy box. The box is twice as long as it is wide. The box has a total external surface of 3750cm^2 .
Note: the box does not have a lid.



- (i) Show that the height h of the toy box is given by $h = \frac{625}{x} - \frac{x}{3}$. 1
- (ii) Find the dimensions of the box which give a maximum volume. 3
- (b) Find the exact volume generated when the area bounded by the curve $y = \sec \frac{x}{2}$, the x and y axes and the line $x = \frac{\pi}{2}$ is rotated about the x axis. 3

- (c) A radioactive isotope is decaying at a rate proportional to its mass according to the formula

$$M = Ae^{-kt}$$

Time t is in hours. Initially the isotope has a mass of 125 g.
After 5 hours it has decayed to a mass of 118 g.

- (i) Show that $k = 0.0115$ correct to 3 significant figures 2
- (ii) Find, to the nearest gram, the mass of the isotope after 1 day (24 hours). 1
- (iii) Find the time taken for the isotope to decay to a mass of 20 g.
Give your answer to the nearest hour. 2

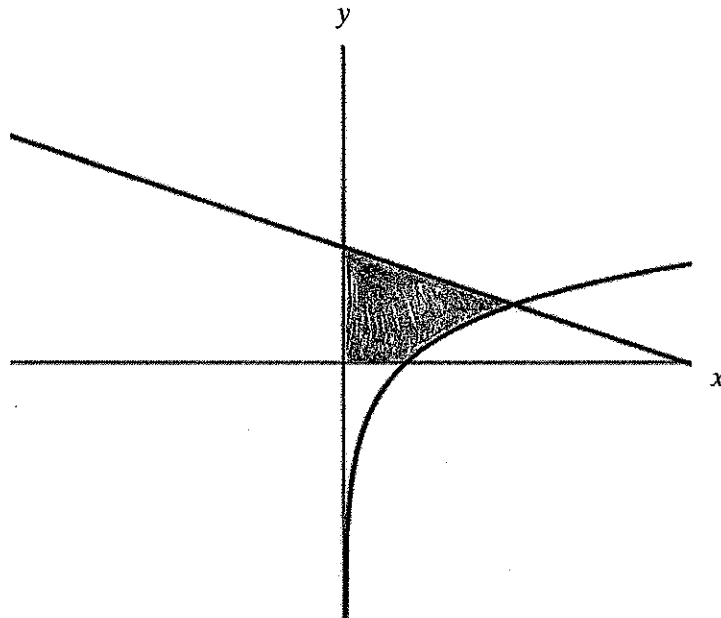
Question 9 (12 marks)

Marks

- (a) A rain water tank attached to a house holds 5600 litres of water when full. The tank is initially empty. During a recent thunderstorm the tank took 2 hours to fill. The volume V litres of water flowing off the roof into the tank t minutes after the storm commences is given by $V = \frac{7t^2}{2700}(270-t)$

- (i) Find the volume of water in the tank after 25 minutes. 1
- (ii) Find the rate at which the tank is filling after one hour. 2
- (iii) Find the time when the tank is filling at the fastest rate. 2
- (iv) Find the amount of water which overflowed the tank if it rained for 3 hours. 2

- (b) The shaded region below represents the area bounded by the x and y axes and the curves $y = 2 - \frac{x}{e}$ and $y = \ln x$.



- (i) Show by substitution that the curves $y = 2 - \frac{x}{e}$ and $y = \ln x$ intersect at the point $(e, 1)$. 1
- (ii) Hence, find the exact area of the shaded region. 4

Question 10 (12 marks)

Marks

- (a) Bob and Beryl take out a home loan of \$460 000 over 30 years at 9% p.a. compounded monthly. M represents the monthly repayments and n represents the number of payments made.

- (i) Show that the amount owing after 3 months is

$$A_3 = 460000(1.0075)^3 - M(1 + 1.0075 + 1.0075^2). \quad 2$$

- (ii) Give an expression for A_n . 1

- (iii) Find the amount of M , the monthly repayment. 2

- (iv) How much interest did Bob and Beryl pay after 30 years? 1

- (b) Two particles A and B start moving on the x axis at time $t = 0$. Particle A starts from a point 7 metres to the right of the origin with a velocity of 12 m/s and has an acceleration of -8 m/s^2 . The position of particle B is given by $x = \frac{25}{t+1} + 4t$.

- (i) Show that the position of particle A is given by $x = 7 + 12t - 4t^2$. 2

- (ii) At what time and where is particle A at rest? 2

- (iii) Where on the x -axis does particle B start and in what direction is it moving? 2

End of Paper

