Yet another way to prove the irrationality of e

This follows the method suggested in the Independent Trial for Mathematics Extension 2 for 2007.

Proposition. e is irrational.

Proof. If $e \in \mathbb{Q}$, $\exists m, n \in \mathbb{Z} : e = \frac{m}{n}$ and if $a := n!(e - \sum_{r=0}^{n} \frac{1}{r!})$, since $e = \sum_{r=0}^{\infty} \frac{1}{r!}$ $0 < a = n!(\sum_{r=0}^{\infty} \frac{1}{r!} - \sum_{r=0}^{n} \frac{1}{r!}) = \sum_{r=n+1}^{\infty} \frac{n!}{r!} = \sum_{r=1}^{\infty} \prod_{j=1}^{r} \frac{1}{n+j} < \sum_{r=1}^{\infty} \frac{1}{(n+1)^r} = \frac{1/(n+1)}{1-(1/(n+1))} = \frac{1}{n} \leq 1$ if $n \geq 1$. Hence $a \notin \mathbb{Z}$. But $a = n!(\frac{m}{n} - \sum_{r=0}^{n} \frac{1}{r!}) = (n-1)!m - \sum_{r=0}^{n} \frac{n!}{r!} \in \mathbb{Z}$ - contradiction. $\therefore e \notin \mathbb{Q}$. $\square$