QUESTION 1.

(a) (i)

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1-x}}\,dx = \int \sqrt{1+x}\,dx$$

$$= \frac{2}{3}(1+x)^{\frac{3}{2}} + c$$

(ii)

$$\int x\cos x\,dx = x\sin x - \int \sin x\,dx$$

$$= x\sin x + \cos x + c$$

(b)

$$\frac{8-2x}{(1+x)(4+x^2)} \equiv \frac{a}{1+x} + \frac{bx+c}{4+x^2}$$

$$8-2x \equiv a(4+x^2) + (bx+c)(1+x)$$

sub. $x=-1$: $10 = 5a \Rightarrow a = 2$

equate coeffs of x^2: $0 = a+b \Rightarrow b = -2$

sub. $x = 0$: $8 = 4a + c \Rightarrow c = 0$

$$\int_0^4 \frac{8-2x}{(1+x)(4+x^2)}\,dx = \int_0^4 \left(\frac{2}{(1+x)} + \frac{-2x}{4+x^2}\right)dx$$

$$= \left[2\ln(1+x) - \ln(4+x^2)\right]_0^4$$

$$= 2(\ln 5 - \ln 1) - (\ln 20 - \ln 4)$$

$$= \ln 5$$

(c)

$$t = \tan\frac{x}{2}$$

$$dt = \frac{1}{2}\sec^2\frac{x}{2}\,dx$$

$$= \frac{1}{2}(1+t^2)\,dx$$

$$dx = \frac{2}{1+t^2}\,dt$$

$$x = 0 \Rightarrow t = 0$$

$$x = \frac{\pi}{2} \Rightarrow t = 1$$

$$5 + 3\cos x$$

$$= 5 + \frac{3(1-t^2)}{1+t^2}$$

$$= \frac{8+2t^2}{1+t^2}$$

$$\int_0^{\frac{\pi}{2}} \frac{2}{5+3\cos x}\,dx = \int_0^1 \frac{2(1+t^2)}{2(4+t^2)} \cdot \frac{2}{(1+t^2)}\,dt$$

$$= \int_0^1 \frac{2}{(4+t^2)}\,dt$$

$$= \left[\tan^{-1}\frac{t}{2}\right]_0^1$$

$$= \tan^{-1}\frac{1}{2}$$

$$\approx 0\cdot 46 \quad (2\ sig.\ figs.)$$

(d) (i)

$$u = \frac{\pi}{4} - x$$

$$du = -dx$$

$$x = 0 \Rightarrow u = \frac{\pi}{4}$$

$$x = \frac{\pi}{4} \Rightarrow u = 0$$

$$\int_0^{\frac{\pi}{4}} \ln(1+\tan x)\,dx = \int_{\frac{\pi}{4}}^0 \ln\left\{1 + \tan\left(\frac{\pi}{4} - u\right)\right\}.-du$$

$$= \int_0^{\frac{\pi}{4}} \ln\left\{1 + \frac{1-\tan u}{1+\tan u}\right\}du$$

$$= \int_0^{\frac{\pi}{4}} \ln\left(\frac{2}{1+\tan u}\right)du$$

$$= \int_0^{\frac{\pi}{4}} \ln\left(\frac{2}{1+\tan x}\right)dx$$

(ii)

$$\int_0^{\frac{\pi}{4}} \ln(1+\tan x)\,dx = \int_0^{\frac{\pi}{4}} \{\ln 2 - \ln(1+\tan x)\}\,dx$$

$$2\int_0^{\frac{\pi}{4}} \ln(1+\tan x)\,dx = \int_0^{\frac{\pi}{4}} \ln 2\,dx$$

$$= \frac{\pi}{4}\ln 2$$

$$\therefore \int_0^{\frac{\pi}{4}} \ln(1+\tan x)\,dx = \frac{\pi}{8}\ln 2$$

QUESTION 2.

(a) Let $z = a + ib$, $(a, b \text{ real})$.

Then $z^2 + 2\bar{z} + 6 = 0$

$a^2 + 2iab - b^2 + 2a - 2ib + 6 = 0$

$(a^2 + 2a - b^2 + 6) + 2ib(a-1) = 0$

$\{(a+1)^2 + 5 - b^2\} + 2ib(a-1) = 0$

Equating real and imaginary parts:

$(a+1)^2 + 5 - b^2 = 0 \quad [1]$

$b(a-1) = 0 \quad [2]$

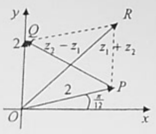
From $[2]$, $b = 0$ or $a = 1$

But from [1], there is no real a for which $b = 0$.

$\therefore a = 1$, and from $[1]$ $b = \pm 3$.

Hence $z = 1 \pm 3i$

(b) (i)



(ii) Complete the parallelogram $OPRQ$. Then the diagonals, $\overline{OR}$, $\overline{PQ}$ represent $z_1 + z_2$, $z_2 - z_1$ respectively.

Now $|z_1| = |z_2| = 2. \therefore OPRQ$ is a rhombus.

$\therefore \angle POR = \frac{1}{2}\left(\frac{\pi}{2} - \frac{\pi}{12}\right) = \frac{5\pi}{24}$

$\therefore \arg(z_1 + z_2) = \frac{\pi}{12} + \frac{5\pi}{24} = \frac{7\pi}{24}$

Also $OR \perp PQ$

$\therefore \arg(z_2 - z_1) = \frac{\pi}{2} + \arg(z_1 + z_2) = \frac{19\pi}{24}$

(c) (i) $z = 4\left(\frac{\sqrt{3}}{2} + \left(-\frac{1}{2}\right)i\right)$

$z = 4\left(\cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right)\right)$

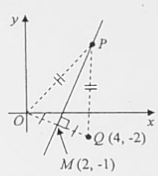
(ii) $z^{\frac{1}{2}} = \pm 2\left(\cos\left(-\frac{\pi}{12}\right) + i\sin\left(-\frac{\pi}{12}\right)\right)$.

The two values of $z^{\frac{1}{2}}$ are

$2\left(\cos\left(-\frac{\pi}{12}\right) + i\sin\left(-\frac{\pi}{12}\right)\right)$,

$2\left(\cos\left(\frac{11\pi}{12}\right) + i\sin\left(\frac{11\pi}{12}\right)\right)$

(d) (i)



$Q(4, -2)$ represents $(4 - 2i)$ in the Argand diagram. Then $|z| = |z - (4 - 2i)| \Rightarrow PO = PQ$.

Hence the locus of P is the perpendicular bisector of OQ.

But gradient OQ is -½ and midpoint of OQ is $M(2, -1)$.

Locus of P has equation $y + 1 = 2(x - 2)$

$2x - y - 5 = 0$

(ii) $\min|z| = OM = \sqrt{5}$

QUESTION 3.

(a) (i)

$$y = xe^{\frac{-1}{2}x^2}$$

$$\frac{dy}{dx} = 1.e^{\frac{-1}{2}x^2} + xe^{\frac{-1}{2}x^2}.(-x)$$

$$= (1-x^2)e^{\frac{-1}{2}x^2}$$

$$\frac{dy}{dx} = 0 \Rightarrow x = \pm 1$$

Stationary points are $A\left(-1, -e^{\frac{-1}{2}}\right), B\left(1, e^{\frac{-1}{2}}\right)$

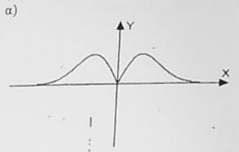
(ii) $x = 0 \Rightarrow \frac{dy}{dx} = 1$

Tangent at O has gradient 1.

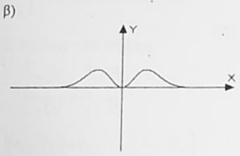
For $f(x) = kx$ to have 3 real roots, the line $y = kx$ must cut the curve in three points. Hence $0 < k < 1$.

(iii)

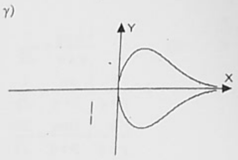
α)



β)



γ)



(b) (i)

$$\frac{dy}{dx}\left\{\tan^{-1}e^x + \tan^{-1}e^{-x}\right\} = \frac{e^x}{1+e^{2x}} + \frac{-e^{-x}}{1+e^{-2x}}$$

$$= \frac{e^x}{1+e^{2x}} - \frac{e^{-x}.e^{2x}}{e^{2x}+e^{-2x}.e^{2x}}$$

$$= \frac{e^x}{1+e^{2x}} - \frac{e^x}{e^{2x}+1}$$

$$= 0$$

(ii) Since the function is continuous, $\tan^{-1}e^x + \tan^{-1}e^{-x} = c$, for some constant c.

But $\tan^{-1}e^0 + \tan^{-1}e^{-0} = \frac{\pi}{4} + \frac{\pi}{4}$

$$\therefore \tan^{-1}e^x + \tan^{-1}e^{-x} = \frac{\pi}{2}$$

(iii)

$$f(x) = \tan^{-1}e^x - \frac{\pi}{4}$$

$$f(-x) = \tan^{-1}e^{-x} - \frac{\pi}{4}$$

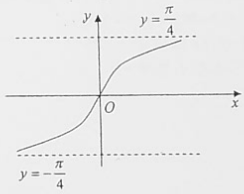
$$= \left(\frac{\pi}{2} - \tan^{-1}e^x\right) - \frac{\pi}{4}$$

$$= -\left(\tan^{-1}e^{-x} - \frac{\pi}{4}\right)$$

$$= -f(x)$$

Hence $f(x)$ is an odd function.

(iv) $y = \tan^{-1}e^x - \frac{\pi}{4}$

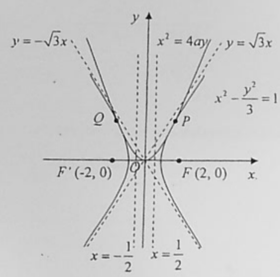


QUESTION 4.

(a) (i)

$3 = 1(e^2 - 1)$

$e = 2$



(ii)

At P, Q $x^2 = 4ay$ and $x^2 - \frac{y^2}{3} = 1$

By symmetry, the y coordinates of P and Q are equal.
This quadratic equation in y must have equal roots.

$$4ay - \frac{y^2}{3} = 1$$
$$12ay - y^2 = 3$$
$$y^2 - 12ay + 3 = 0$$

$\therefore \Delta = (-12a)^2 - 12 = 0 \quad \therefore a = \frac{1}{\sqrt{12}} = \frac{\sqrt{3}}{6}$

Then the equation becomes $\left(y - \sqrt{3}\right)^2 = 0 \quad \therefore y = \sqrt{3}$

Hence $P\left(\sqrt{2}, \sqrt{3}\right)$ and $Q\left(-\sqrt{2}, \sqrt{3}\right)$.

(b) (i)

$x = ct \qquad y = \frac{c}{t}$

$\frac{dx}{dt} = c \qquad \frac{dy}{dt} = -\frac{c}{t^2}$

$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = -\frac{1}{t^2}$ Tangent at T has gradient $-\frac{1}{t^2}$ and equation

$$y - \frac{c}{t} = -\frac{1}{t^2}(x - ct)$$
$$t^2 y - ct = -x + ct$$
$$\therefore x + t^2 y = 2ct$$

(ii) At R, $x + p^2 y = 2cp$ [1]

$x + q^2 y = 2cq$ [2]

$[1] - [2] \Rightarrow (p^2 - q^2)y = 2c(p - q)$

$(p - q)(p + q)y = 2c(p - q)$

$y = \frac{2c}{p + q}$

$q^2 \times [1] - p^2 \times [2] \Rightarrow$

$(q^2 - p^2)x = 2cpq(q - p)$

$(q - p)(q + p)x = 2cpq(q - p)$

$x = \frac{2cpq}{p + q}$

Hence R has coordinates $\left(\frac{2cpq}{p+q}, \frac{2c}{p+q}\right)$.

(iii)

gradient PS = gradient QS

$$\frac{\sqrt{2} - \frac{1}{p}}{\sqrt{2} - p} = \frac{\sqrt{2} - \frac{1}{q}}{\sqrt{2} - q}$$
$$\left(\sqrt{2} - \frac{1}{p}\right)\left(\sqrt{2} - q\right) = \left(\sqrt{2} - p\right)\left(\sqrt{2} - \frac{1}{q}\right)$$
$$2 - \frac{1}{p}\sqrt{2} - q\sqrt{2} + \frac{q}{p} = 2 - p\sqrt{2} - \frac{1}{q}\sqrt{2} + \frac{p}{q}$$
$$\sqrt{2}\left(\frac{1}{q} - \frac{1}{p}\right) + \sqrt{2}(p - q) = \frac{p}{q} - \frac{q}{p}$$
$$(p - q)\sqrt{2}(1 + pq) = (p - q)(p + q)$$
$$\sqrt{2}(1 + pq) = p + q \quad \text{Q.E.D.}$$

iv)

$$\frac{1 + pq}{p + q} = \frac{1}{\sqrt{2}}$$
$$\frac{1}{p + q} + \frac{pq}{p + q} = \frac{1}{\sqrt{2}}$$
$$\frac{2c}{p + q} + \frac{2cpq}{p + q} = \frac{2c}{\sqrt{2}}$$

Hence the coordinates (x, y) of R satisfy $y + x = c\sqrt{2}$ (which is therefore the equation of the locus of R).

QUESTION 5.

(a) (i) The x coordinates of P and Q satisfy

$$x(k-x)=\frac{k^2}{x}$$
$$kx^2-x^3=k^2$$
$$kx^2-x^3-k^2=0$$

Since the curves touch at P, the x coordinate of P is a repeated real root of this equation. Since the curves intersect at a second point Q the x coordinate of Q is another real root of the equation. Hence the equation has 3 real roots of the form α, α, β where $\alpha \neq \beta$.

(ii) α, α, β are roots of $x^3-kx^2+k^2=0$.

Coefficient of x is zero, hence

$$\alpha^2+2\alpha\beta=0$$
$$\alpha(\alpha+2\beta)=0$$
$$\because \alpha \neq 0 \Rightarrow \alpha=-2\beta$$

Also $2\alpha+\beta=k$ and $\alpha^2\beta=-k^2$

$\therefore k=-3\beta$ and $k^2=-4\beta^3$

Hence $9\beta^2=-4\beta^3 \quad \because \beta \neq 0 \Rightarrow \beta=-\dfrac{9}{4}$

Then $\alpha=\dfrac{9}{2}$ and $k=\dfrac{27}{4}$

(b) (i)

$$1-(\cos n\theta+i\sin n\theta)=(1-\cos n\theta)-i\sin n\theta$$
$$=2\sin^2\frac{n\theta}{2}-i\left(2\sin\frac{n\theta}{2}\cos\frac{n\theta}{2}\right)$$
$$=-2i\sin\frac{n\theta}{2}\left(\cos\frac{n\theta}{2}+i\sin\frac{n\theta}{2}\right)$$

(ii) $z+z^2+z^3+\ldots+z^n=\dfrac{z(1-z^n)}{1-z}$ for $z \neq 1$ (sum of a GP with common ratio z)

(iii) $z^n=\cos n\theta+i\sin n\theta$. Hence

$\text{Re}(z+z^2+z^3+\ldots.+z^n)=\cos\theta+\cos 2\theta+\cos 3\theta+\ldots+\cos n\theta$

Also
$$\frac{z(1-z^n)}{1-z}=\frac{(\cos\theta+i\sin\theta)\left\{-2i\sin\frac{n\theta}{2}\left(\cos\frac{n\theta}{2}+i\sin\frac{n\theta}{2}\right)\right\}}{-2i\sin\frac{\theta}{2}\left(\cos\frac{\theta}{2}+i\sin\frac{\theta}{2}\right)}$$
$$=\frac{\left(\cos\frac{\theta}{2}+i\sin\frac{\theta}{2}\right)\left\{\sin\frac{n\theta}{2}\left(\cos\frac{n\theta}{2}+i\sin\frac{n\theta}{2}\right)\right\}}{\sin\frac{\theta}{2}}$$

$$\therefore \text{Re}\left\{\frac{z(1-z^n)}{1-z}\right\}=\frac{\sin\frac{n\theta}{2}\left(\cos\frac{\theta}{2}\cos\frac{n\theta}{2}-\sin\frac{\theta}{2}\sin\frac{n\theta}{2}\right)}{\sin\frac{\theta}{2}}=\frac{\sin\frac{n\theta}{2}\cos\frac{(n+1)\theta}{2}}{\sin\frac{\theta}{2}}$$

Hence $\cos\theta+\cos 2\theta+\cos 3\theta+\ldots+\cos n\theta=\dfrac{\sin\frac{n\theta}{2}\cos\frac{(n+1)\theta}{2}}{\sin\frac{\theta}{2}}$ for $\sin\dfrac{\theta}{2}\neq 0 \quad (z \neq 1)$

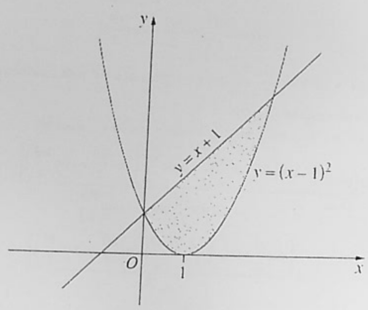
(iv) $\sin\frac{\theta}{2}=0$ for $\theta=0, 2\pi$, and these are not solutions of the equation.

For $\sin\frac{\theta}{2}\neq 0$, $\cos\theta+\cos 2\theta+\cos 3\theta=0 \Rightarrow \sin\frac{3\theta}{2}\cos\frac{4\theta}{2}=0$

$$\therefore \sin\tfrac{3\theta}{2}=0 \text{ or } \cos 2\theta=0,\ 0<\theta<2\pi$$
$$\tfrac{3\theta}{2}=\pi, 2\pi \text{ or } 2\theta=\tfrac{\pi}{2}, \tfrac{3\pi}{2}, \tfrac{5\pi}{2}, \tfrac{7\pi}{2}$$
$$\theta=\tfrac{2\pi}{3}, \tfrac{4\pi}{3}, \tfrac{\pi}{4}, \tfrac{3\pi}{4}, \tfrac{5\pi}{4}, \tfrac{7\pi}{4}$$

QUESTION 6.

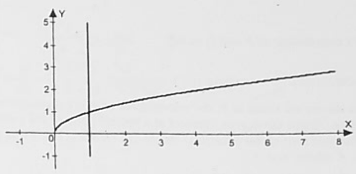
(a) (i)



(ii)

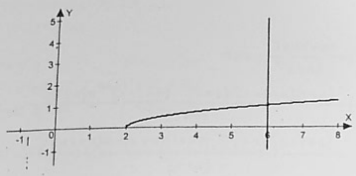
$$V = \int_0^3 2\pi x\left[(x+1)-(x-1)^2\right]dx$$
$$= 2\pi\int_0^3\left(x^2+x-x^3+2x^2-x\right)dx$$
$$= 2\pi\left[\frac{-x^4}{4}+x^3\right]_0^3$$
$$= 2\pi\left(27-\frac{81}{4}\right)$$
$$= \frac{27\pi}{2} \text{ units}^3$$

(b)



$$V = 2\pi\int_0^1 x^{3/2}\,dx$$
$$= 2\pi\left[\frac{2}{5}x^{5/2}\right]_0^1$$
$$= 2\pi\left[\frac{2}{5}-0\right]$$
$$= \frac{4\pi}{5} \text{ units}^3$$

(c)



$y=\frac{1}{2}\sqrt{x-2}$, $y=1$ when $x=6$

$2y=\sqrt{x-2}$

$x-2=4y^2$

$x=4y^2+2$

$$V=\int_0^1 \pi\left(6-\left(4y^2+2\right)\right)^2 dy$$

$$=\int_0^1 \pi\left(4-4y^2\right)^2 dy$$

$$=\pi\int_0^1\left(16-32y^2+16y^4\right)dy$$

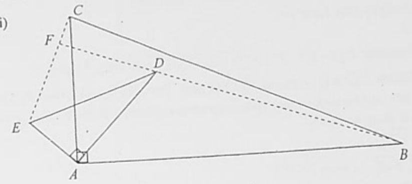
$$=\pi\left[16y-\frac{32y^3}{3}+\frac{16y^5}{5}\right]_0^1$$

$$=\pi\left[\left(16-\frac{32}{3}+\frac{16}{5}\right)-0\right]$$

$$=\frac{128\pi}{15}\text{ units}^3$$

QUESTION 7.

(a) (i)



(ii) $\Delta ABC \parallel\!\mid \Delta ADE$

$\therefore \frac{AC}{AE}=\frac{AB}{AD}$ (corresponding sides in similar Δs)

In $\Delta BDA, \Delta CEA$

$\frac{AB}{AC}=\frac{AD}{AE}$

$\angle BAD=\angle CAE$ (both complementary $\angle$s with $\angle CAD$)

$\therefore \Delta BDA \parallel\!\mid \Delta CEA$ (one pair of equal $\angle$s and included sides in proportion)

(iii) $\angle ADB=\angle AEC$ (corresponding $\angle$s of similar Δs are equal)

$\therefore ADFE$ is a cyclic quadrilateral (one exterior $\angle$ equal to interior opposite $\angle$)

(iv) $\therefore \angle DFE=180°-\angle DAE$ (opposite $\angle$s of cyclic quadrilateral sum to 180°)

$\therefore \angle DFE=90°$ and hence $BF\perp CE$

(b) (i) Let $S(n)$, $n = 1, 2, 3, \dots\dots$ be the sequence of statements $T_n < 3$.

Step 1. Consider $S(1)$: $T_1 = 1 < 3$. Hence $S(1)$ is true.

Step 2. Assume true for $n = k$.
i.e. $T_k < 3$.

Step 3. Consider $S(k+1)$: $T_{k+1} = \sqrt{3+2T_k} < \sqrt{3+6} = 3$ if $S(k)$ is true.

Step 4. Hence $S(k+1)$ is true if $S(k)$ is true. But $S(1)$ is true, hence $S(2)$ is true, and then $S(3)$ is true and so on. Hence by Mathematical Induction, $S(n)$ is true for all positive integers $n \ge 1$.

(ii)
$$\begin{aligned} T_{n+1} &= \sqrt{3+2T_n} \\ &> \sqrt{T_n + 2T_n} \quad (\text{since } 3 > T_n) \\ &= \sqrt{3T_n} \\ &> \sqrt{T_n . T_n} \quad (\text{since } 3 > T_n) \\ &= T_n \end{aligned}$$
$$\therefore T_{n+1} > T_n$$

(iii) $(T_{n+2} - T_{n+1})(T_{n+2} + T_{n+1}) = T_{n+2}{}^2 - T_{n+1}{}^2$.

$$\begin{aligned} T_{n+2} - T_{n+1} &= \frac{T_{n+2}{}^2 - T_{n+1}{}^2}{T_{n+2} + T_{n+1}} \\ &= \frac{(3+2T_{n+1}) - (3+2T_n)}{T_{n+2} + T_{n+1}} \\ &= \frac{2(T_{n+1} - T_n)}{T_{n+2} + T_{n+1}} \end{aligned}$$

Clearly $T_{n+2} > T_{n+1} > T_n > \dots > T_1 = 1$

Hence for $n = 1, 2, 3, \dots$, $T_{n+2} > 1, T_{n+1} > 1$ and

$T_{n+2} + T_{n+1} > 2$, giving $1 > \dfrac{2}{T_{n+2} + T_{n+1}}$.

$\therefore T_{n+2} - T_{n+1} < T_{n+1} - T_n$ for $n = 1, 2, 3, \dots$

QUESTION 8.

(a) (i)

$$f(x) = \frac{(n+1+x)^{n+1}}{(n+x)^n}, \; x \ge 0$$

$$\begin{aligned} f'(x) &= \frac{(n+1)(n+1+x)^n (n+x)^n - (n+1+x)^{n+1} n(n+x)^{n-1}}{(n+x)^{2n}} \\ &= \frac{(n+1+x)^n (n+x)^{n-1} \{(n+1)(n+x) - n(n+1+x)\}}{(n+x)^{2n}} \end{aligned}$$

$$f'(x) = \frac{x(n+1+x)^n}{(n+x)^{n+1}} > 0 \quad \text{for } x > 0$$

Hence $f(x)$ is increasing for $x > 0$.

(ii) $x > 0 \Rightarrow f(x) > f(0) \Rightarrow \dfrac{(n+1+x)^{n+1}}{(n+x)^n} > \dfrac{(n+1)^{n+1}}{n^n}$

$\therefore$ for $x > 0$, $\dfrac{(n+1+x)^{n+1}}{(n+1)^{n+1}} > \dfrac{(n+x)^n}{n^n}$

$$\left(1 + \frac{x}{n+1}\right)^{n+1} > \left(1 + \frac{x}{n}\right)^n$$

(iii) Substituting $x = 1$, $\left(1 + \dfrac{1}{n+1}\right)^{n+1} > \left(1 + \dfrac{1}{n}\right)^n$

$$\left(\frac{n+2}{n+1}\right)^{n+1} > \left(\frac{n+1}{n}\right)^n$$

$$(n+2)^{n+1} n^n > (n+1)^{2n+1}$$

(b) (i) The remaining 5 letters can be arranged in the remaining 5 envelopes in $5! = 120$ ways. Hence number of arrangements with A in envelope 2 is 120.

(ii) Choose an envelope for A in 4 ways, then choose an envelope for B in 4 ways, then arrange the remaining 4 letters in the remaining 4 envelopes in 4! ways. Hence the number of such arrangements is $4 \times 4 \times 4! = 384$.

(iii) The event {A is not in envelope 1 and B is not in envelope 2} is the union of the two events {A is in envelope 2}, {A is in neither envelope 1 nor envelope 2 and B is not in envelope 2}.
Hence the number of suitable arrangements is $120 + 384 = 504$.

(c) (i) $a^2+b^2-2ab=(a-b)^2\ge 0$

$\therefore\ a^2+b^2\ge 2ab$

(ii)

$a^2+b^2\ge 2ab$	$b^2+c^2\ge 2bc$
$a^2+c^2\ge 2ac$	$b^2+d^2\ge 2bd$
$a^2+d^2\ge 2ad$	$c^2+d^2\ge 2cd$

$$3(a^2+b^2+c^2+d^2)\ge 2(ab+ac+ad+bc+bd+cd)$$

(iii) $a^2+b^2+c^2+d^2=(a+b+c+d)^2-2(ab+ac+ad+bc+bd+cd)$

But $a+b+c+d=1$. Hence, using ii),

$$3(a^2+b^2+c^2+d^2)=3-6(ab+ac+ad+bc+bd+cd)$$

$$2(ab+ac+ad+bc+bd+cd)\le 3-6(ab+ac+ad+bc+bd+cd)$$

$$8(ab+ac+ad+bc+bd+cd)\le 3$$

$$(ab+ac+ad+bc+bd+cd)\le \tfrac{3}{8}$$