Student Number: …………………………

Barker College

2011
YEAR 12
TRIAL HSC
EXAMINATION

MATHEMATICS

Staff Involved:

THURSDAY 4TH AUGUST

- KJL
- GPF
- RMH
- BJB*
- JGD*
- TZR
- DZP
- GIC
- AJD

110 copies

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen
- Write your Barker Student Number on all pages of your answers
- Board-approved calculators may be used
- A Table of Standard Integrals is provided at the back of this paper which may be detached for your use
- ALL necessary working MUST be shown in every question
- Marks may be deducted for careless or badly arranged working

Total marks - 120

- Attempt Questions 1 - 10
- All questions are of equal value
- BEGIN your answer to EACH QUESTION on a NEW PIECE of the separate lined paper
- Write only on ONE SIDE of the separate lined paper

-**BLANK PAGE -**

Total marks - 120
Attempt Questions 1 - 10
All questions are of equal value

Answer each question on a separate A4 lined sheet of paper.

Marks

Question 1 (12 marks) **[START A NEW PAGE]**

(a) Evaluate, to 3 significant figures, $\dfrac{\sqrt{9^2+144}}{14-3}$ **2**

(b) Simplify fully $\dfrac{3}{x+3} - \dfrac{1}{x-3}$ **2**

(c) If $\dfrac{14}{3+\sqrt{2}} = a+b\sqrt{2}$, find the values of a and b **2**

(d) Solve $|4x-1| = 3$ **2**

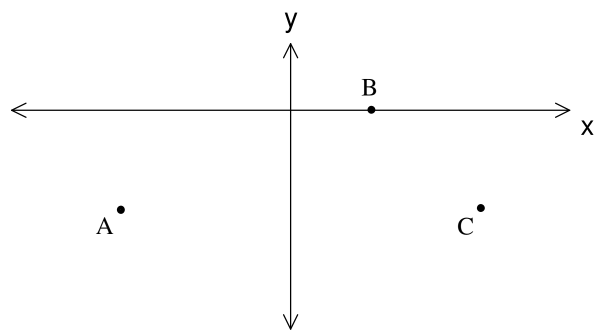
(e) Factorise fully: $2x^3 - 54y^3$ **2**

(f) Given $\log_a 3 = 0.6$ and $\log_a 2 = 0.4$, find $\log_a 18$ **2**

End of Question 1

Marks

Question 2 (12 marks) **[START A NEW PAGE]**



The coordinates of the points A, B and C are $(-3, -2)$, $(1, 0)$ and $(5, -2)$ respectively

(i) Calculate the length of the interval AB **1**

(ii) Find the gradient of the line AB **1**

(iii) Show that the equation of line l, drawn through C parallel to AB is $x - 2y - 9 = 0$ **1**

(iv) Find the coordinates of D, the point where l intersects the x-axis **1**

(v) What is the size of the acute angle (to the nearest degree) made by the line AB with the positive direction of the x-axis? **1**

(vi) Hence, determine the size of $\angle ABD$ **1**

(vii) Find the perpendicular distance of the point A from the line l **2**

(viii) Find the area of quadrilateral ABDC **2**

(ix) Sketch the line l and shade the area satisfied by the following simultaneously **2**
$x \geq 0$, $y \leq 0$, $x - 2y - 9 \geq 0$

End of Question 2

Marks

Question 3 (12 marks) **[START A NEW PAGE]**

(a) Differentiate with respect to x:

(i) $3\tan x$ **1**

(ii) $(5-2x)^7$ **2**

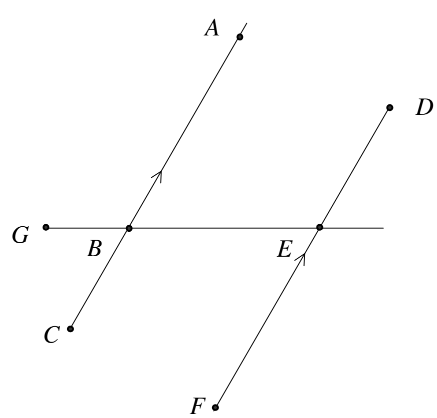
(b) Find:

(i) $\int_0^1 3\sqrt{x}\ dx$ **2**

(ii) $\int \frac{8x+10}{2x^2+5x}\ dx$ **2**

(c) A curve $y = f(x)$ has the following properties in the interval $a \le x \le b$: **2**
$f(x)>0,\ f'(x)>0,\ f''(x)<0$
Sketch a curve satisfying these conditions.

(d)



In the diagram, $AB = AE,\ AC \parallel DF$, $\angle ABG = 146^\circ$ and $\angle AED = x^\circ$

(i) Copy this diagram into your writing booklet and place all the information onto the diagram. **1**

(ii) Find the value of x, giving complete reasons. **2**

End of Question 3

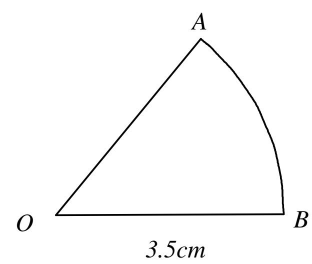
Marks

Question 4 (12 marks) **[START A NEW PAGE]**

(a) In ΔRST, $\angle RTS = 150^\circ$, $ST = 3cm$ and $RT = 5cm$. **2**
Find the length of RS correct to one decimal place.

(b) A sector AOB of a circle has a radius of $3.5cm$.
Its perimeter is $9.5cm$.

NOT TO SCALE



(i) Find the length of the arc AB **1**

(ii) Find the size of $\angle AOB$ in radians **2**

(iii) Find the area of the sector AOB **2**

(c) Is $f(x) = \dfrac{3^x + 3^{-x}}{2x^2}$ an odd function or an even function? **2**

Give reasons for your answer.

(d) Solve $2^{2x} - 9(2^x) + 8 = 0$ **3**

End of Question 4

Marks

Question 5 (12 marks) **[START A NEW PAGE]**

(a) Consider the function $f(x) = x^3 + 6x^2 + 9x + 4$ in the domain $-4 \le x \le 1$

(i) Find the coordinates of any stationary points and determine their nature. **3**

(ii) Determine the coordinates of its point(s) of inflexion. **2**

(iii) Draw a sketch of the curve $y = f(x)$ in the domain $-4 \le x \le 1$ clearly showing all its essential features. **2**

(iv) What is the maximum value of the function $y = f(x)$ in the domain $-4 \le x \le 1$? **1**

(b) Find the equation of the tangent to $y = \ln(3x + 1)$ at the point (2, 5) **2**

(c) Solve $\log_7 x^2 = 3$ **2**

End of Question 5

Marks

Question 6 (12 marks) **[START A NEW PAGE]**

(a) If $\sin\theta = -\dfrac{8}{17}$ and $\tan\theta > 0$, find the exact value of $\cos\theta$ **2**

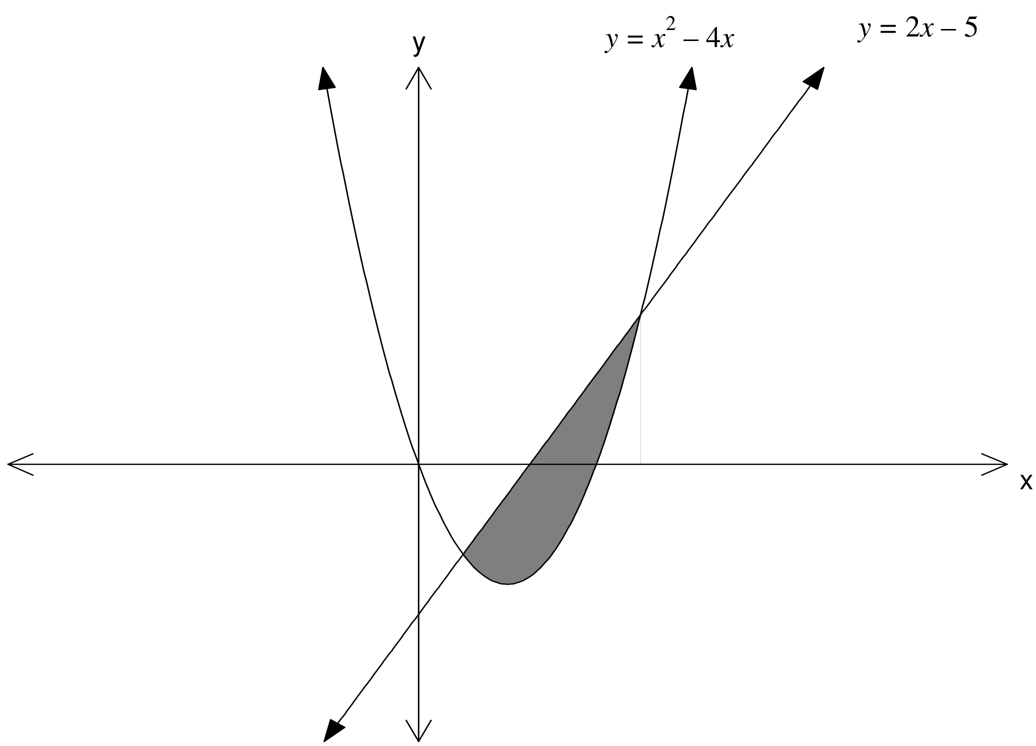
(b) The first four terms of a sequence are 3, 6, 9, 12

(i) Show that 102 is a term of this sequence **2**

(ii) Hence, or otherwise, find the sum of the terms of this sequence between 100 and 200 **3**

(c) (i) Show that $y = x^2 - 4x$ and $y = 2x - 5$ intersect when $x = 1$ and $x = 5$ **2**

(ii) Hence, find the shaded area below **3**



End of Question 6

Marks

Question 7 (12 marks) **[START A NEW PAGE]**

(a) Evaluate $\lim_{x \to 3} \frac{x^2 - x - 6}{x - 3}$ **2**

(b) The curve $y = ax^3 + bx$ passes through the point $(1, 7)$. The tangent at this point is parallel to the line $y = 2x - 6$. Find the values of a and b. **4**

(c) Find the equation of the locus of $P(x, y)$, if P is always equidistant from $A(3, 1)$ and $B(1, 3)$. Give a geometric description of this locus. **3**

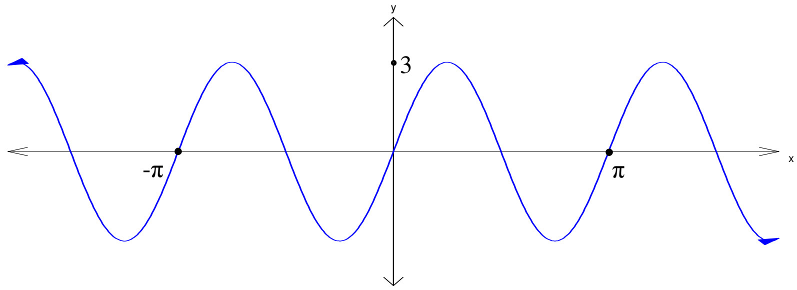
(d) A retirement fund pays 8% per annum compound interest on the money invested in it. What investment must a worker make at the beginning of each year if he wishes to retire with a lump sum of \$200 000 after 25 years (with his last investment at the beginning of the 25^{th} year)? **3**

End of Question 7

Marks

Question 8 (12 marks) **[START A NEW PAGE]**

(a)



Not to scale

For the above graph, write down:

(i) the period of the function **1**

(ii) the amplitude of the function **1**

(iii) a possible equation of the function **1**

(b) Given that $\frac{d}{dx}(xe^x) = xe^x + e^x$

evaluate $\int_0^2 \frac{xe^x}{2}\, dx$ **3**

(c) Use the trapezoidal rule with 5 function values to find an approximation to

$$\int_0^2 \frac{1}{x+1}\, dx$$ **3**

(d) Show that $\frac{\cos\theta}{1-\sin\theta} - \frac{\cos\theta}{1+\sin\theta} = 2\tan\theta$ **3**

End of Question 8

Marks

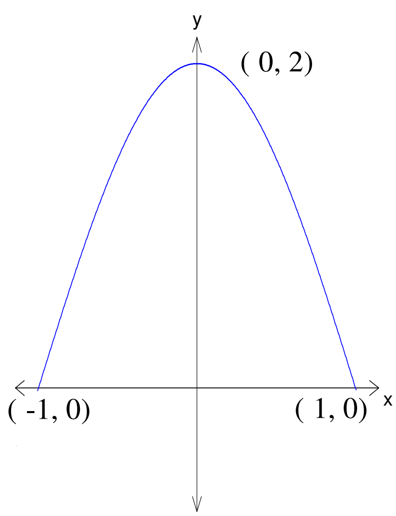
Question 9 (12 marks) **[START A NEW PAGE]**

(a) If p, q and 32 are the first three terms of a geometric sequence and q, 4, p are the first three terms of another geometric sequence, find p and q. **4**

(b) (i) Sketch the curve $y = \log_e x$ **1**

(ii) The curve $y = \log_e x$, between $x = 1$ and $x = e$, is rotated 360° about the y-axis. Find the exact value of the volume of the solid formed. **4**

(c) An ornamental arch window 2 metres wide at the base and 2 metres high is to be made in the shape of a cosine curve. Find the area of the window in terms of π , if $y = 2\cos\left(\frac{\pi}{2}x\right)$. **3**



End of Question 9

Marks

Question 10 (12 marks) **[START A NEW PAGE]**

(a) During the normal operation of a petrol driven engine, the volume V litres of petrol left in the tank reduces at a rate $\frac{dV}{dt} = -3e^{0.4t}$ where t is measured in minutes since the engine was switched on and the 100 litre tank was full.

(i) At what rate is the petrol used, initially? **1**

(ii) Use integration to show that volume remaining can be expressed as

$$V = \frac{-30}{4}e^{0.4t} + 107.5$$ **2**

(iii) How long can the machine operate until the tank is only half full?
Give your answer correct to the nearest minute. **2**

(b) (i) Find the value of x for which the function

$$y = \frac{x^2 - x + 2}{x^2 - x + 1} \text{ is equal to } \frac{7}{3}.$$ **2**

(ii) Show that the function $\frac{x^2 - x + 2}{x^2 - x + 1}$ can never exceed $\frac{7}{3}$ **3**

(iii) Hence, the range of this function must be $a < y \leq \frac{7}{3}$
Find the value of a. **2**

End of Question 10

End of Paper

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

2 Unit Mathematics Trial Paper 2011 - Solutions

Question 1

a. $\frac{15}{11}$

$= 1.3636\ldots$

$= 1.36$ (3 sig. fig)

b. $= \frac{3(x-3) - (x+3)}{(x+3)(x-3)}$

$= \frac{3x-9-x-3}{x^2-9}$

$= \frac{2x-12}{x^2-9}$

c. $= \frac{14}{3+\sqrt{2}} \times \frac{3-\sqrt{2}}{3-\sqrt{2}}$

$= \frac{14(3-\sqrt{2})}{9-2}$

$= 2(3-\sqrt{2})$

$\therefore 6-2\sqrt{2} = a+b\sqrt{2}$

$\therefore a=6, \quad b=-2$

d. $|4x-1| = 3$

$4x-1=3$ $4x-1=-3$

$4x=4$ $4x=-2$

$x=1$ $x=-\frac{1}{2}$

$\therefore x = -\frac{1}{2}, 1$

e. $2x^3-54y^3 = 2(x^3-27y^3)$

$= 2(x-3y)(x^2+3xy+9y^2)$

f. $\log_a 18 = \log_a (3^2 \times 2)$

$= 2\log_a 3 + \log_a 2$

$= 2 \times 0.6 + 0.4$

$= 1.6$

Question 2

i. $A(-3,-2)$ $B(1,0)$

$d = \sqrt{(1+3)^2 + (0+2)^2}$

$d = \sqrt{16+4}$

$d = \sqrt{20}$

$d = 2\sqrt{5}$

ii. $m_{AB} = \frac{0+2}{1+3}$

$= \frac{1}{2}$

iii. $m = \frac{1}{2}$ $C(5,-2)$

$y+2 = \frac{1}{2}(x-5)$

$2y+4 = x-5$

$\therefore x-2y-9=0$

iv. x-intercepts occur when $y=0$

$x-2(0)-9=0$

$x=9$

$\therefore D(9,0)$

v. $m = \tan\theta$

$\therefore \tan\theta = \frac{1}{2}$

$\theta = 26°33'54''$

$\therefore \theta = 27°$ (to nearest degree)

vi. $\angle ABD = 180° - 27°$

$= 153°$

vii. $d = \frac{|ax+by+c|}{\sqrt{a^2+b^2}}$

$d = \frac{|1(-3) + (-2)(-2) + (-9)|}{\sqrt{(1)^2+(-2)^2}}$

$d = \frac{|-3+4-9|}{\sqrt{1+4}}$

$d = \frac{|-8|}{\sqrt{5}}$

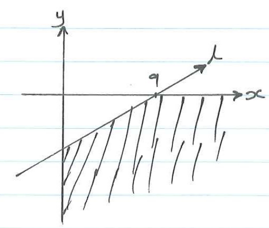
$\therefore d = \frac{8}{\sqrt{5}}$

viii Area $= \frac{1}{2}$ dist $\times$ AB

$= \frac{8}{\sqrt{5}} \times 2\sqrt{5}$

$= 16$ units^2

ix



$x \geq 0, \; y \leq 0, \; x-2y-9 \geq 0$

Question 3

a. i. $\frac{d}{dx}(3\tan x) = 3\sec^2 x$

ii $\frac{d}{dx}(5-2x)^7 = 7(5-2x)^6 \times -2$

$= -14(5-2x)^6$

b. i. $\int_0^1 3\sqrt{x}\,dx = \left[\frac{3x^{3/2}}{3/2}\right]_0^1$

$= \left[2x^{3/2}\right]_0^1$

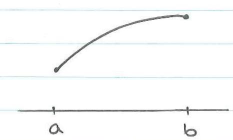
$= 2-0$

$= 2$

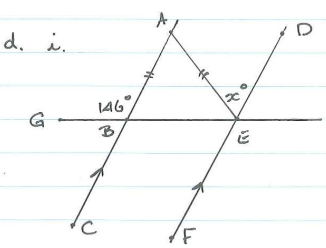
ii. $\int \frac{8x+10}{2x^2+5x}\,dx$

$= 2\int \frac{4x+5}{2x^2+5x}\,dx = 2\ln(2x^2+5x)+C$

c.



d. i.



ii. $\angle ABE = 34°$ (angle sum st. line)

$\angle ABE = \angle AEB$ (base ∠'s of isos △)

$\angle ABE + (\angle AEB + \angle DEA) = 180°$

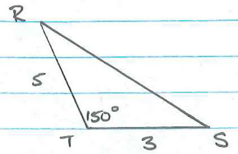
(co-int ∠'s, AC∥DF)

$\therefore 34° + 34° + x° = 180°$

$x = 112°$

Question 4

a.



$RS^2 = 5^2 + 3^2 - 2 \times 5 \times 3 \times \cos 150°$

$RS^2 = 59.98076211...$

$RS = 7.7$ (1 d.p)

b. i. $AB = 9.5 - 2 \times 3.5$

$= 2.5$

ii. $l = r\theta$

$\theta = \frac{l}{r}$

$\theta = \frac{2.5}{3.5}$

$\theta = \frac{5}{7}$ radians

iii. Area $= \frac{1}{2} r^2 \theta$

$= \frac{1}{2} \times 3.5^2 \times \frac{5}{7}$

$= \frac{35}{8}$ cm^2

c. $f(a) = \frac{3^a + 3^{-a}}{2a^2}$

$f(-a) = \frac{3^{-a} + 3^a}{2(-a)^2}$

$= \frac{3^a + 3^{-a}}{2a^2}$

$f(a) = f(-a)$

∴ even function

d. $2^{2x} - 9(2^x) + 8 = 0$

let $u = 2^x$

$u^2 - 9u + 8 = 0$

$(u-8)(u-1) = 0$

∴ $u = 8, 1$

∴ $2^x = 8$ $\qquad 2^x = 1$

$2^x = 2^3$ $\qquad 2^x = 2^0$

$x = 3$ $\qquad x = 0$

∴ $x = 0, 3$

Question 5

a. $f(x) = x^3 + 6x^2 + 9x + 4$

$f'(x) = 3x^2 + 12x + 9$

$f''(x) = 6x + 12$

i. $f'(x) = 0$

$3x^2 + 12x + 9 = 0$

$3(x+3)(x+1) = 0$

∴ $x = -1, -3$

when $x = -1$, $f(-1) = 0$

$f''(-1) = 6 \times (-1) + 12$

$= 6$

i.e. $f''(-1) > 0$ ∪

∴ min turning point $(-1, 0)$

when $x = -3$, $f(-3) = 4$

$f''(-3) = 6 \times (-3) + 12$

$= -6$

i.e. $f''(-3) < 0$ ∩

∴ max turning point $(-3, 4)$

ii. $f''(x) = 0$

$6x + 12 = 0$

$6x = -12$

$x = -2$

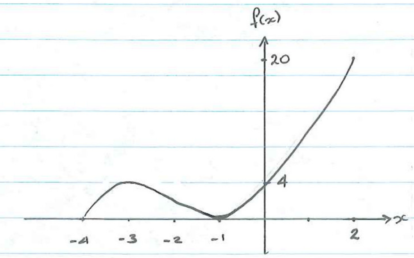
Test

x	-3	-2	-1
$f''(x)$	-6	0	6
concavity	∩	·	∪

$f(-2) = 2$

∴ pt of inflexion at $(-2, 2)$

iii.



iv. max value = 20

b. $y = \ln(3x+1)$ at $(2, 5)$

$\frac{dy}{dx} = \frac{3}{3x+1}$

when $x = 2$

$\frac{dy}{dx} = \frac{3}{3 \times 2 + 1}$

$= \frac{3}{7}$

$y - 5 = \frac{3}{7}(x - 2)$

$7y - 35 = 3x - 6$

∴ $3x - 7y + 29 = 0$

c. $\log_7 x^2 = 3$

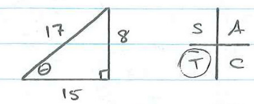
$x^2 = 7^3$

$x^2 = 343$

$x = \pm\sqrt{343}$

Question 6

a.



∴ $\cos\theta = -\frac{15}{17}$

b. 3, 6, 9, 12

i. $T_n = a + (n-1)d$

$102 = 3 + (n-1)3$

$102 = 3n$

$n = 34$

ii. $T_{34} = 102$

$T_{66} = 198$

$S_n = \frac{n}{2}(a + l)$

$= \frac{33}{2}(102 + 198)$

$= 4950$

c. i. $y = x^2 - 4x$

$y = 2x - 5$

∴ $x^2 - 4x = 2x - 5$

$x^2 - 6x + 5 = 0$

$(x-5)(x-1) = 0$

∴ $x = 5, 1$

∴ $(1, -3)$, $(5, 5)$

ii $A = \int_1^5 (2x-5) - (x^2-4x)\,dx$

$= \int_1^5 6x - 5 - x^2\,dx$

$= \left[3x^2 - 5x - \frac{x^3}{3}\right]_1^5$

$= \left(\frac{25}{3} + \frac{7}{3}\right)$

$= 10\frac{2}{3}$ units^2

Question 7

a. $= \lim_{x \to 3} \frac{(x-3)(x+2)}{(x-3)}$

$= \lim_{x \to 3} (x+2)$

$= 5$

b. $y = ax^3 + bx \quad (1,7)$

$7 = a(1)^3 + b(1)$

$7 = a + b$

$y' = 3ax^2 + b$

$y' = 2$ when $x = 1$

$2 = 3a(1)^2 + b$

$2 = 3a + b$

$-\begin{vmatrix} a+b=7 \\ 3a+b=2 \end{vmatrix}$

$-2a = 5$

$a = -\frac{5}{2}$

$\therefore -\frac{5}{2} + b = 7$

$b = \frac{19}{2}$

$\therefore a = -\frac{5}{2},\ b = \frac{19}{2}$

c. $PA = PB$

$\sqrt{(x-3)^2+(y-1)^2} = \sqrt{(x-1)^2+(y-3)^2}$

$x^2 - 6x + 9 + y^2 - 2y + 1 = x^2 - 2x + 1 + y^2 - 6y + 9$

$4x - 4y = 0$

$\therefore y = x$

straight line through the origin with gradient = 1.

d.

1st $A_1 = M \times (1.08)^{25}$

2nd $A_2 = M \times 1.08^{25} + M \times 1.08^{24}$

$\vdots$

$200\,000 = M(1.08 + 1.08^2 + .. + 1.08^{25})$

$200\,000 = M \times \frac{1.08(1.08^{25} - 1)}{0.08}$

$M = \frac{200\,000}{1.08(1.08^{25}-1)} \times 0.08$

$M = 2533.107$

$M = \$2533.11$

Question 8

a. i. π

ii. 3

iii. $y = 3\sin 2x$

b. $\frac{d}{dx}(xe^x) = xe^x + e^x$

$\therefore \int_0^2 \frac{xe^x}{2}\,dx$

$= \frac{1}{2}\int_0^2 xe^x\,dx = \frac{1}{2}\left[xe^x - e^x\right]_0^2$

$= \frac{1}{2}\left[(2e^2 - e^2) - (0-1)\right]$

$= \frac{1}{2}(e^2+1)$

c. $\int_0^2 \frac{1}{x+1}\,dx$

x	0	$\frac{1}{2}$	1	$1\frac{1}{2}$	2
y	1	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{2}{5}$	$\frac{1}{3}$

$A = \frac{\frac{1}{2}}{2}\left[1 + \frac{1}{3} + 2\left(\frac{2}{3} + \frac{1}{2} + \frac{2}{5}\right)\right]$

$= \frac{1}{4} \times \frac{67}{15}$

$= \frac{67}{60}$

$= 1.1166...$

$\doteq 1.12$

d. LHS $= \frac{\cos\theta}{1-\sin\theta} - \frac{\cos\theta}{1+\sin\theta}$

$= \frac{\cos\theta(1+\sin\theta) - \cos\theta(1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}$

$= \frac{\cos\theta + \cos\theta\sin\theta - \cos\theta + \cos\theta\sin\theta}{1 - \sin^2\theta}$

$= \frac{2\cos\theta\sin\theta}{\cos^2\theta}$

$= \frac{2\sin\theta}{\cos\theta}$

$= 2\tan\theta$

$=$ RHS

Question 9

a. $p, q, 32$

$q, 4, p$

$\frac{q}{p} = \frac{32}{q}$

$q^2 = 32p \quad \ldots ①$

$\frac{4}{q} = \frac{p}{4}$

$16 = pq$

$\therefore p = \frac{16}{q} \quad \ldots ②$

sub ② into ①

$q^2 = 32 \times \frac{16}{q}$

$q^3 = 512$

$q = 8$

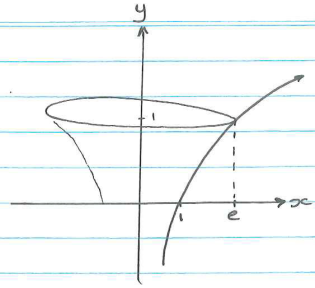
sub $q = 8$ into ②

$p = \frac{16}{8}$

$p = 2$

$\therefore p = 2,\ q = 8$

b. i



ii $y = \ln x$
$x = e^y$

$V = \pi\int_0^1 (e^y)^2 \, dy$

$= \pi\int_0^1 e^{2y} \, dy$

$= \pi\left[\frac{e^{2y}}{2}\right]_0^1$

$= \pi\left(\frac{e^2}{2} - \frac{1}{2}\right)$

$\therefore V = \frac{\pi}{2}(e^2 - 1) \ u^3$

c. $y = 2\cos\left(\frac{\pi}{2}x\right)$

$A = 2\int_0^1 2\cos\frac{\pi}{2}x \, dx$

$= 4\left[\frac{2}{\pi}\sin\frac{\pi}{2}x\right]_0^1$

$= \frac{8}{\pi}\left[\sin\frac{\pi}{2} - \sin 0\right]$

$= \frac{8}{\pi}(1 - 0)$

$\therefore A = \frac{8}{\pi} \ u^2$

Question 10

a. i. $\frac{dV}{dt} = -3e^{0.4t}$

when $t = 0$

$\frac{dV}{dt} = -3e^0$

$= -3$

ii. $V = \int -3e^{0.4t} \, dt$

$= -3\int e^{0.4t} \, dt$

$= \frac{-3}{0.4} e^{0.4t} + C$

$\therefore V = -\frac{30}{4} e^{0.4t} + C$

when $t = 0$, $V = 100$

$100 = -\frac{30}{4} e^0 + C$

$\therefore C = 107.5$

$\therefore V = -\frac{30}{4} e^{0.4t} + 107.5$

iii. $50 = -\frac{30}{4} e^{0.4t} + 107.5$

$-57.5 = -\frac{30}{4} e^{0.4t}$

$e^{0.4t} = \frac{23}{3}$

$\log_e e^{0.4t} = \log_e \frac{23}{3}$

$0.4t = \log \frac{23}{3}$

$t = \frac{\log \frac{23}{3}}{0.4}$

$t = 5.09$

$= 5h \ 6min$

b. i $\frac{x^2 - x + 2}{x^2 - x + 1} = \frac{7}{3}$

$3x^2 - 3x + 6 = 7x^2 - 7x + 7$

$4x^2 - 4x + 1 = 0$

$(2x - 1)(2x - 1) = 0$

$x = \frac{1}{2}$

ii $y = \frac{vu' - uv'}{v^2}$

$u = x^2 - x + 2$

$u' = 2x - 1$

$v = x^2 - x + 1$

$v' = 2x - 1$

$y' = \frac{(2x-1)(x^2-x+1) - (2x-1)(x^2-x+2)}{(x^2-x+1)^2}$

$y' = \frac{(2x-1)(x^2-x+1-x^2+x-2)}{(x^2-x+1)^2}$

$y' = \frac{-2x+1}{(x^2-x+1)^2}$

stat pts occur when $y' = 0$

$\frac{-2x+1}{(x^2-x+1)^2} = 0$

$-2x + 1 = 0$

$x = \frac{1}{2}$

test

x	0	$\frac{1}{2}$	1
y'	1	0	-1
slope	/	—	\

$\therefore$ max at $x = \frac{1}{2}$

$\therefore$ graph will not exceed $y = \frac{7}{3}$

iii. $\lim_{x \to \pm\infty} \frac{x^2 - x + 2}{x^2 - x + 1}$

$= \lim_{x \to \pm\infty} \frac{1 - \frac{1}{x} + \frac{2}{x^2}}{1 - \frac{1}{x} + \frac{1}{x^2}}$

$= 1$

$\therefore a = 1$