YEAR 12 MATHEMATICS TRIAL 2011

QUESTION 1

a) $\cos\frac{5\pi}{6} = \cos 150° = -\frac{\sqrt{3}}{2}$ (1)

b) i) $|x-3| < 4$

$-4 < x-3 < 4$

$-1 < x < 7$ (2)

-1 mark if $x > -1$ or $x < 7$

ii) $x^2 = 6x$

$x^2 - 6x = 0$

$x(x-6) = 0$

$x = 0$ or 6 (2)

iii) $2 - \frac{x-1}{10} = \frac{x}{5}$

$20 - x + 1 = 2x$

$-3x = -21$

$x = 7$ (2)

iv) $2\ln x = \ln(2x-1)$

$x^2 = 2x-1$

$x^2 - 2x + 1 = 0$

$(x-1)^2 = 0$

$x = 1$ (2)

c) $(x-3)^2 = 8y$

Vertex $= (3, 0)$

$4a = 8 \quad a = 2$

Focus $= (3, 2)$ (2)

d) $(x+1)^2 + (y-2)^2 = 25$ (1)

QUESTION 2

a) i) $\frac{d}{dx} x^2\cos x = \cos x \times 2x + x^2 \times -\sin x$

$= 2x\cos x - x^2\sin x$ (2)

ii) $\frac{d}{dx}\frac{3x^3-1}{x} = \frac{d}{dx} 3x^2 - x^{-1}$

$= 6x + x^{-2}$

$= 6x + \frac{1}{x^2}$ (2)

iii) $\frac{d}{dx}\ln 2 = 0$ (1)

b) i) $\int \sqrt[3]{e^{2x}}\,dx = \int e^{\frac{2x}{3}}\,dx$

$= \frac{e^{\frac{2x}{3}}}{\frac{2}{3}} + c$

$= \frac{3}{2} e^{\frac{2x}{3}} + c$

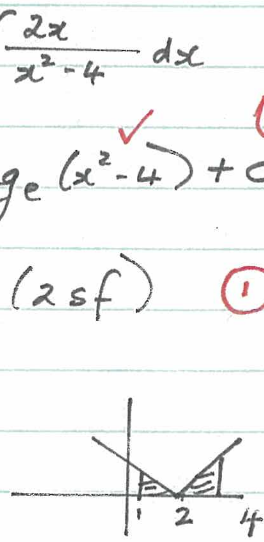
last line not necessary $\left(= \frac{3}{2}\sqrt[3]{e^{2x}} + c\right)$

ii) $\int \frac{3x}{x^2-4}\,dx = \frac{3}{2}\int \frac{2x}{x^2-4}\,dx$ (2)

$= \frac{3}{2}\log_e(x^2-4) + c$

c) $\sin\frac{1}{\sqrt{2}} = 0.65$ (2 sf) (1)

d) $\int_1^4 |x-2|\,dx$



$= \left(\frac{1}{2}\times 1 \times 1\right) + \frac{1}{2}(2\times 2)$

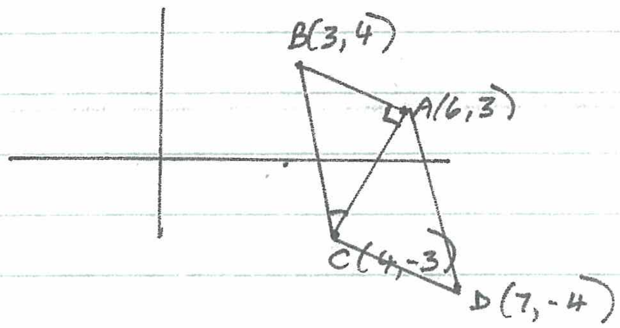
$= \frac{1}{2} + 2$

$= 2\frac{1}{2}$ (2)

no marks if use integration unless 2 lines used.

QUESTION 3 or other methods

a)



i) $m_{AB} = \frac{1}{-3}$ ✓

$m_{AC} = \frac{6}{2}$
$= 3$ ✓

$m_{AB} \times m_{AC} = -\frac{1}{3} \times 3$
$= -1$ ✓ (3)

$\therefore AB \perp AC$

ii) $AB = \sqrt{3^2 + 1^2}$
$= \sqrt{10}$ ✓
$AC = \sqrt{2^2 + 6^2}$
$= \sqrt{40}$
$= (2\sqrt{10})$ ✓

$\frac{AB}{AC} = \tan \angle ACB$

$\frac{\sqrt{10}}{2\sqrt{10}} = \tan \angle ACB$

$\tan \angle ACB = \frac{1}{2}$ ✓ (3)

$\angle ACB = 26°34'$ (to n. min)

iii) $m_{AB} = -\frac{1}{3}$
$m_{CD} = \frac{-1}{3}$
$\therefore AB \parallel CD$ ✓

$m_{BC} = \frac{7}{-1}$
$m_{AD} = \frac{7}{-1}$
$\therefore BC \parallel AD$ ✓

$\therefore ABCD$ is a parallelogram
(2 pairs opposite sides are parallel) (2)

iv) Area of parallelogram ABCD
$= 2 \times$ Area of ΔABC
$= 2 \times \frac{1}{2} AB \times AC$
$= \sqrt{10} \times \sqrt{40}$
$= \sqrt{400}$
$= 20u^2$ (1)

v) E is midpoint of AC
(diagonals bisect each other)
$\therefore E = (5, 0)$ (1)

b) $y' = 6\sin 2x$
$y = \frac{-6\cos 2x}{2} + c$

$y = -3\cos 2x + c$ ✓
Sub $(\frac{\pi}{2}, 3)$
$3 = -3\cos \pi + c$
$3 = -3 \times -1 + c$
$3 = 3 + c$
$c = 0$ ✓

(2)

$\therefore y = -3\cos 2x$

must show $c = 0$

QUESTION 4

a) $\sum_{r=4}^{24} (4r-7)$

$T_1 = 9$

$T_2 = 13$

$T_3 = 17$ ✓

$\therefore a = 9 \quad d = 4 \quad n = 21$

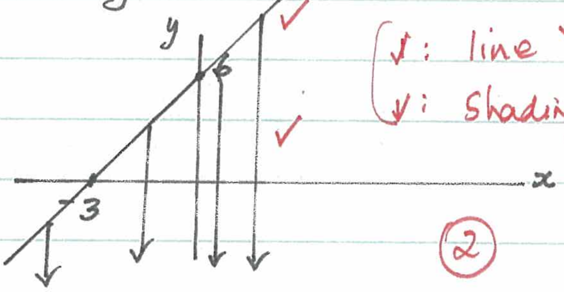
$S_{21} = \frac{21}{2}\{2 \times 9 + 20 \times 4\}$ ✓

$= \frac{21}{2}\{18 + 80\}$

$= \frac{21}{2} \times 98$

$= 1029$ ✓ 3

b) $2x - y + 6 \geq 0$ ✓



✓: line

✓: shading

2

c) $C = 12\pi$

$2\pi r = 12\pi$

$2r = 12$

$r = 6$ ✓

Area of minor segment

$= \frac{1}{2}r^2(\theta - \sin\theta)$

$= \frac{1}{2}36\left(\frac{\pi}{3} - \sin\frac{\pi}{3}\right)$ ✓

$= 18\left(\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right)$

$= 6\pi - 9\sqrt{3}\ \text{cm}^2$ ✓ 3

d) $T_3 = 36 \quad T_8 = \frac{4}{27}$

$ar^2 = 36$ — ① ✓

$ar^7 = \frac{4}{27}$ — ② ✓

② ÷ ① $r^5 = \frac{1}{243}$

$r = \frac{1}{3}$ ✓

Sub in ① $a \times \frac{1}{9} = 36$

$a = 324$

$\therefore$ First term is 324 ✓ 4

QUESTION 5

a) $y = x^3 - 3x^2 - 9x + 2$

i) $y' = 3x^2 - 6x - 9$

$y'' = 6x - 6$

For stationary points $y' = 0$

$3(x^2 - 2x - 3) = 0$

$(x-3)(x+1) = 0$

$x = 3, -1$

$y = -25, 7$

$\therefore$ stationary points are

$(3, -25)$ ✓, $(-1, 7)$ ✓

Nature of stationary points 4

If $x = 3 \quad y'' = 12 > 0$

$\therefore (3, -25)$ is a minimum turning pt ✓

If $x = -1 \quad y'' = -12 < 0$

$\therefore (-1, 7)$ is a maximum turning pt ✓

ii) For possible points of inflexion

$y'' = 0$

$6x - 6 = 0$

$x = 1$

$y = -9$ ✓

$\therefore (1, -9)$ is a possible point of inflexion

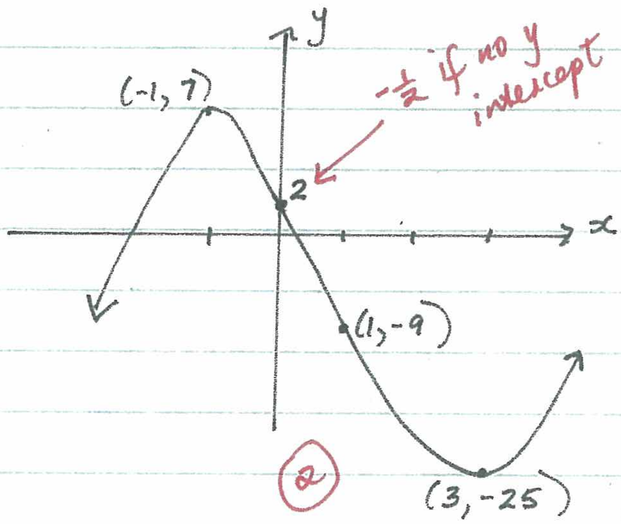
Test for change in concavity

x	0	1	2
y''	-6	0	6
	$\cap$		$\cup$

$\therefore$ There is a change in concavity (2)

$\therefore (1, -9)$ is a point of inflexion ✓

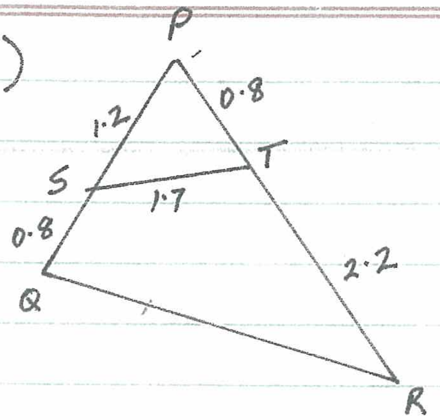
iii)



(2)

iv) $-1 < x < 1$ (1) no part marks

b)



i) In Δs PST, PRQ

$\angle P$ is common

$$\frac{PS}{PR} = \frac{1 \cdot 2}{3} = \frac{2}{5}$$

$$\frac{PT}{PQ} = \frac{0 \cdot 8}{2} = \frac{2}{5}$$ (2)

$\therefore \Delta PST \parallel\!\parallel \Delta PRQ$ (sides around equal $\angle$ in same ratio)

ii) $\frac{ST}{QR} = \frac{2}{5}$ (matching sides of similar Δs)

$$\frac{1 \cdot 7}{QR} = \frac{2}{5}$$

$2QR = 8 \cdot 5$

$QR = 4 \cdot 25$ ✓ (1)

QUESTION 6

a) $\int_3^7 \log_e(x-3)\,dx$

x	5	5·5	6	6·5	7
$\log_e(x-3)$	ln 2	ln 2·5	ln 3	ln 3·5	ln 4
	y_0	y_1	y_2	y_3	y_4

✓

$\int_5^7 \log_e(x-3)\,dx$

$\doteq \frac{0.5}{3}\{\ln 2 + \ln 4 + 4(\ln 2.5 + \ln 3.5) + 2\ln 3\}$ ✓

$\doteq 2.1588\ldots$

$\doteq 2.159$ (3dp) ✓ 3

b) $\frac{dM}{dt} = -kM$

i) $M = M_0 e^{-kt}$

$\frac{dM}{dt} = M_0 e^{-kt} \times -k$

$= M \times -k$

$= -kM$ 1

ii) $\frac{1}{2}M_0 = M_0 e^{-k\times 8}$ ✓

must be M_0 not M

$\frac{1}{2} = e^{-8k}$

$\ln\frac{1}{2} = -8k$

$k = \frac{\ln\frac{1}{2}}{-8}$

$= \frac{-\ln 2}{-8}$ show steps

$= \frac{\ln 2}{8}$ ✓

2

iii) $\frac{3}{4}$ decayed.

$\therefore \frac{1}{4}$ left

$\frac{1}{4}M_0 = M_0 e^{-\frac{\ln 2}{8}\times t}$

✓ $\frac{1}{4} = e^{-\frac{\ln 2}{8}\times t}$

$\ln\frac{1}{4} = -\frac{\ln 2}{8}\times t$

$t = \ln\frac{1}{4} \div \left(-\frac{\ln 2}{8}\right)$

$= 16$ ✓

2

$\therefore$ it will take 16 days for 75% to have decayed.

c) $x^2 + (m-3)x + (m-1) = 0$

Let roots be α and 2α

$3\alpha = -m+3$ — (1)

$2\alpha^2 = m-1$ — (2) ✓

From (1) $m = -3\alpha + 3$

Sub in (2): $2\alpha^2 = -3\alpha + 3 - 1$

$2\alpha^2 + 3\alpha - 2 = 0$

$(2\alpha - 1)(\alpha + 2) = 0$

$\alpha = \frac{1}{2}$ or -2 ✓

Sub in (1) $m = -3\times\frac{1}{2}+3$ or $-3\times -2+3$

$= 1\frac{1}{2},\quad 9$ ✓

$\therefore m = 1\frac{1}{2}$ or 9

roots are $\frac{1}{2}$ & 1

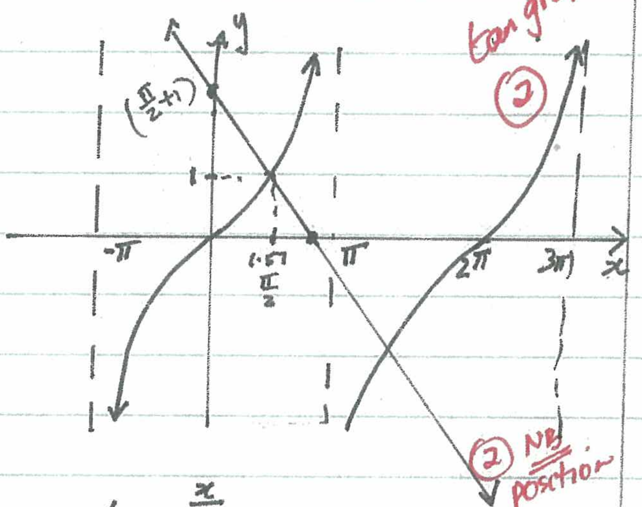
or -2 and -4 ✓

4

QUESTION 7

a) i) $y = \tan\frac{x}{2}, \; -\pi \le x \le 3\pi$

Period $= \frac{\pi}{n} = \frac{\pi}{\frac{1}{2}} = 2\pi$



tan graph ②

② NB position of x & y int

ii) $y = \tan\frac{x}{2}$

$y' = \sec^2\frac{x}{2} \times \frac{1}{2}$

$= \frac{1}{2}\sec^2\frac{x}{2}$ ✓

Sub $x = \frac{\pi}{2}$

$y' = \frac{1}{2}\left(\sec\frac{\pi}{4}\right)^2$

$= \frac{1}{2}(\sqrt{2})^2$

$= 1$

$\therefore$ gradient of normal is -1 ✓

$x = \frac{\pi}{2} \quad y = \tan\frac{\pi}{4}$

$= 1$

$y - 1 = -1\left(x - \frac{\pi}{2}\right)$

$y - 1 = -x + \frac{\pi}{2}$

$x + y - 1 - \frac{\pi}{2} = 0$

is eqn of normal ✓

x int: $1 + \frac{\pi}{2} \doteqdot 2.57$

y int $= 1 + \frac{\pi}{2} \doteqdot 2.57$ ③

b) $x = 2\sin t + t\sqrt{3}$

i) $v = 2\cos t + \sqrt{3}$

Particle at rest when $v = 0$

$2\cos t = -\sqrt{3}$

$\cos t = -\frac{\sqrt{3}}{2}$

(relt $= 30^\circ : \frac{\pi}{6}$) II, III

$t = \frac{5\pi}{6}, \frac{7\pi}{6}$ ✓ ✓ ②

$\therefore$ particle at rest at $\frac{5\pi}{6}$ & $\frac{7\pi}{6}$ seconds

ii) $t = 0 \quad x = 2\sin 0 + 0$

$= 0$ ½

$t = \frac{5\pi}{6} \quad x = 2\sin\frac{5\pi}{6} + \frac{5\pi}{6}\sqrt{3}$

$= 2 \times \frac{1}{2} + \frac{5\sqrt{3}\pi}{6}$

$= 1 + \frac{5\sqrt{3}\pi}{6}$ ✓

$t = \pi \quad x = 2\sin\pi + \pi\sqrt{3}$

$= 0 + \pi\sqrt{3}$

$= \pi\sqrt{3}$ ½

$\therefore$ Distance $= 2\left(1 + \frac{5\sqrt{3}}{6}\pi\right) - \pi\sqrt{3}$

$= 2 + \frac{5\sqrt{3}}{3}\pi - \pi\sqrt{3}$

$= \frac{6 + 5\sqrt{3}\pi - 3\sqrt{3}\pi}{3}$

$= \frac{6 + 2\sqrt{3}\pi}{3}$ M ✓

0 $\quad \pi\sqrt{3} \quad 1 + \frac{5\sqrt{3}\pi}{6}$

QUESTION 8

a) i) $t = 2, 10$ 1

ii) $2 < t < 10$ 1

iii) $t = 5$ 1

iv) $t = 6$ $v > 0$

$a < 0$ 1

$\therefore$ particle slowing down

b) i) $y = e^x$ — (1)

$y = e^{2x} - 2$ — (2)

$e^{2x} - 2 = e^x$

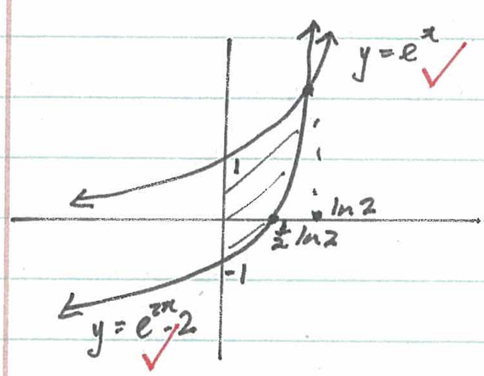
$e^{2x} - e^x - 2 = 0$ ✓

$(e^x + 1)(e^x - 2) = 0$ 2

$e^x = -1$ or $e^x = 2$

no solution $x = \ln 2$ ✓

ii)



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$e^{2x} = 2$

$\ln 2 = 2x$

$x = \frac{1}{2}\ln 2$

iii) Area $= \int_0^{\ln 2} e^x - (e^{2x} - 2)\,dx$

$= \int_0^{\ln 2} e^x - e^{2x} + 2\,dx$ ✓

$= \left[e^x - \frac{e^{2x}}{2} + 2x\right]_0^{\ln 2}$ ✓

$= e^{\ln 2} - \frac{e^{2\ln 2}}{2} + 2\ln 2 - (e^0 - \frac{e^0}{2}$ ✓

$= 2 - \frac{4}{2} + 2\ln 2 - 1 + \frac{1}{2}$

$= -\frac{1}{2} + 2\ln 2$ ✓

$= (2\ln 2 - \frac{1}{2})u^2$ 4

QUESTION 9

a) i) $y = -x^2 + 6x - 8$

$x^2 - 6x + 8 = 0$

$(x - 2)(x - 4) = 0$

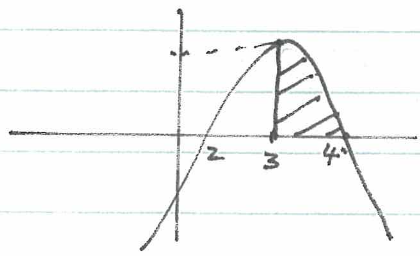
$x = 2, 4$ ✓ 1

ii) $x = 3 \pm \sqrt{1 - y}$

$x = 3$

$y = -9 + 18 - 8$

$= 1$



$$\text{Volume} = \pi \int_0^1 \left((3+\sqrt{1-y})^2 - 3^2 \right) dy$$

$$= \pi \int_0^1 9 + 6\sqrt{1-y} + 1 - y - 9 \, dy$$

$$= \pi \int_0^1 6\sqrt{1-y} + 1 - y \, dy$$

$$= \pi \left[\frac{6(1-y)^{1\frac{1}{2}}}{1\frac{1}{2} \times -1} + y - \frac{y^2}{2} \right]_0^1$$

$$= \pi \left[0 + 1 - \frac{1}{2} - \left(\frac{6}{-1\frac{1}{2}} + 0 - 0 \right) \right]$$

$$= \left(\frac{1}{2} + 4 \right) \pi$$

4

$$= 4\tfrac{1}{2} \pi u^3 = \frac{9}{2} \pi u^3 = \frac{9\pi}{2} u^3$$

b) Let A_n be value of nth investment

$A_1 = 2000(1.06)^4$

$A_2 = 2000(1.06)^3$

$\vdots$

$A_4 = 2000(1.06)^1$

Total at end of 2013

$= 2000(1.06 + \dots 1.06^4)$

$= 2000 \times 1.06 \dfrac{(1.06^4 - 1)}{1.06 - 1}$

$= \$9274.19$

3

$A_5 = 2000(1.08)^7$

$A_6 = 2000(1.08)^6$

$\vdots$

$A_{11} = 2000(1.08)^1$

$\therefore$ Total =

$9274.19(1.08)^7 +$
$2000(1.08 + 1.08^2 + \dots 1.08^7)$

$= 9274.19(1.08)^7 + 2000 \times 1.08 \times \dfrac{(1.08^7 \cdot}{1.08 - \cdot}$

$= 15894.33 + 19273.26$

$= \$35167.59$

4

QUESTION 10

a) $y = \dfrac{mx^3}{3} + \dfrac{3x^2}{2} - 4x + 7$

$y' = \dfrac{3mx^2}{3} + \dfrac{6x}{2} - 4$

$= mx^2 + 3x - 4$ ½

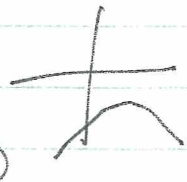
$\therefore$ curve to be decreasing

$y' < 0$ ½ Also $m < 0$

ie. $mx^2 + 3x - 4 < 0$ for all x

$\Delta = 9 - 4 \times m \times -4$

$= 9 + 16m$



$9 + 16m < 0$ (no roots)

$16m < -9$

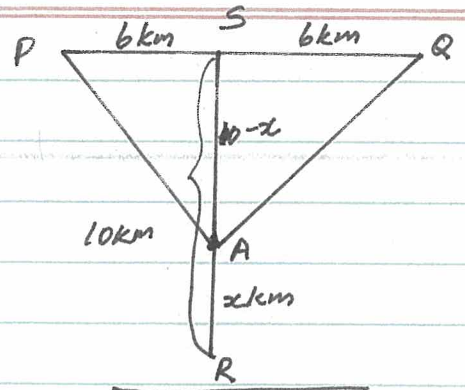
$m < -\dfrac{9}{16}$

3

m is < 0

$\therefore m < -\dfrac{9}{16}$

b)



i) $AP = \sqrt{(10-x)^2 + 6^2}$

$= \sqrt{100 - 20x + x^2 + 36}$

$= \sqrt{x^2 - 20x + 136}$

$AQ = \sqrt{x^2 - 20x + 136}$

$\therefore L = 2\sqrt{x^2 - 20x + 136} + x$

2

ii) For min L, $L' = 0$ + $L'' > 0$

$L' = 2 \times \frac{1}{2}(x^2 - 20x + 136)^{-\frac{1}{2}}(2x - 20) + 1$

$= \dfrac{2(x-10)}{\sqrt{x^2 - 20x + 136}} + 1$

$L' = 0$

$2x - 20 = -\sqrt{x^2 - 20x + 136}$

$4x^2 - 80x + 400 = x^2 - 20x + 136$

$3x^2 - 60x + 264 = 0$

$x^2 - 20x + 88 = 0$

$x = \dfrac{20 \pm \sqrt{400 - 4 \times 88}}{2}$

$= \dfrac{20 \pm \sqrt{48}}{2}$

$= \dfrac{20 \pm 4\sqrt{3}}{2}$

$= 10 \pm 2\sqrt{3}$

But $x < 10$

$\therefore x = 10 - 2\sqrt{3}$

5

iii) $L = 2\sqrt{x^2 - 20x + 136} + x$

$= 2\sqrt{(10-2\sqrt{3})^2 - 20(10 - 2\sqrt{3}) + 136} + (10 - 2\sqrt{3})$

$= 2\sqrt{100 - 40\sqrt{3} + 12 - 200 + 40\sqrt{3} + 136} + 10 - 2\sqrt{3}$

$= 2\sqrt{48} + 10 - 2\sqrt{3}$

$= 8\sqrt{3} + 10 - 2\sqrt{3}$

$= (10 + 6\sqrt{3})$ km

2

Check $x = 10 - 2\sqrt{3}$

(6.5)

x	5	$10-2\sqrt{3}$	7
L'	-0.8	0	0.11

must be numbers here not just + or -

$x = 5$ $L' = \dfrac{-10}{\sqrt{31}} + 1 = -0.8$

$x = 7$ $L' = \dfrac{-6}{\sqrt{45}} + 1 = 0.11$

$\therefore$ MIN