Student Number:____________________

Student Name :____________________

Student Teacher:____________________

ABBOTSLEIGH

AUGUST 2011

YEAR 12

ASSESSMENT 4

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

General Instructions

- Reading time – 5 minutes.
- Working time – 3 hours.
- Write using blue or black pen.
- Board-approved calculators may be used.
- A table of standard integrals is provided.
- All necessary working should be shown in every question.

Total marks **(120)**

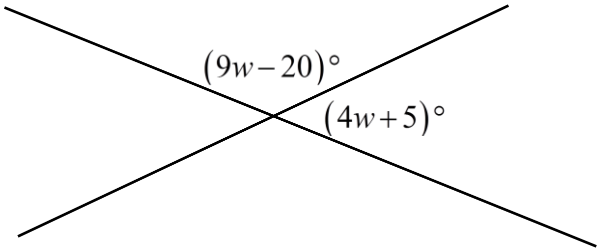
- Attempt Questions 1-10.
- All questions are of equal value.

Question 1 (12 marks) Start a new booklet **Marks**

(a) Factorise $2h^2 + 11h + 15$ **1**

(b) Find the value of w in the diagram: **2**

NOT TO SCALE



(c) Given $a = \frac{2}{7}$, $b = \frac{3}{5}$ and $c = 4\frac{1}{8}$, evaluate $\frac{b^2 - a}{2\sqrt{c}}$ in scientific notation to 3 significant figures. **2**

(d) Express $\frac{\log_3 8}{\log_3 2}$ as an integer **1**

(e) Determine the value of n to make the following expression equal to a single digit number: **1**

$$5^2 \times 2^4 \times 10^{-n}$$

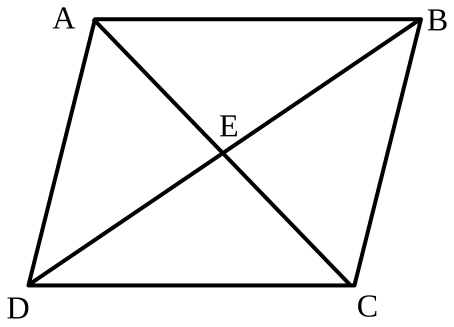
(f) Evaluate $\lim_{x \to 4} \frac{x^3 - 64}{x - 4}$ **2**

(g) Show that $\frac{21}{\sqrt{63}} - \frac{3}{\sqrt{7}+2} = 2$ **3**

End of Question 1

Question 2 (12 marks) Start a new booklet **Marks**

(a) Find the area of the rhombus ABCD given AB = 13 cm and EB = 12 cm. **2**

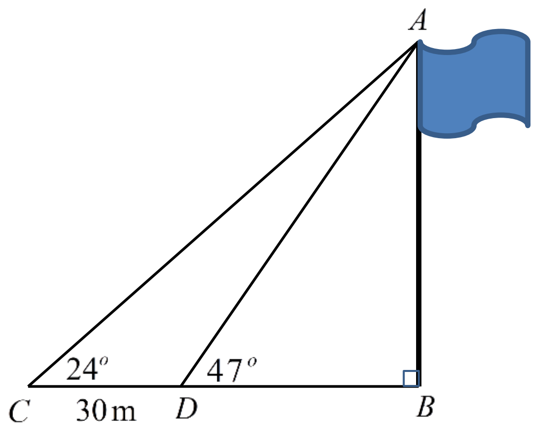


NOT TO SCALE

(b) (i) Find the point A which is the y intercept of $4x-3y-12=0$ **1**

(ii) Hence find the equation of the line passing through A, which is perpendicular to $4x-3y-12=0$ **2**

(c) Harry finds that the angle of elevation of the top of a flagpole, A from C is 24^{o} as shown in the diagram below. He walks 30 metres towards the flagpole and now finds that the angle of elevation is 47^{o}.



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(i) Use the Sine rule to show that $AD=\dfrac{30\sin 24^{o}}{\sin 23^{o}}$ **2**

(ii) Hence show that $AB=\dfrac{30\sin 24^{o}\sin 47^{o}}{\sin 23^{o}}$ **2**

(iii) Calculate the length AB correct to 2 significant figures. **1**

(d) Solve for x: $(4x-3)^2=25$ **2**

End of Question 2

Question 3 (12 marks) Start a new booklet **Marks**

(a) The points A and B have coordinates (2, 0) and (0, -2) respectively.
Draw a diagram in your assessment booklet, clearly marking A and B.

(i) Find the gradient AB. **1**

(ii) Show the equation of line l that passes through B and is perpendicular to AB is given by $x+y=-2$ **2**

(iii) Show that C, the point of intersection of l and the x-axis has coordinates (-2, 0). **1**

(iv) If D is the point (0, 2), write down the equation of the circle passing through the points A, B, C and D. **1**

(v) Show the area between the circle ABCD and the quadrilateral ABCD is $4(\pi-2)$ square units. **2**

(b) Calculate the perpendicular distance of the point (3, -1) from the line $4y=3x+2$. **2**

(c) Find the equation of the tangent to the curve $y=5\log_e x$ at $x=1$. **3**

End of Question 3

Question 4 (12 marks) Start a new booklet **Marks**

(a) Differentiate with respect to x :

(i) $e^{\cos x}$ **2**

(ii) $\dfrac{2-x}{3x+4}$ **2**

(b) Find the values of x for which the curve $y = x^3 - 6x^2 + 9x - 4$ is increasing. **2**

(c) The first three terms of an arithmetic progression are 51, 44 and 37.

(i) Write down the nth term for this sequence **2**

(ii) If the last term of the sequence is -47, how many terms are there in this series? **2**

(iii) Find the sum of this series. **2**

End of Question 4

Question 5 (12 marks) Start a new booklet **Marks**

(a) If α and β are the roots of the equation $6x^2 - 2x + 1 = 0$ find:

(i) $\alpha + \beta$ **1**

(ii) $\alpha^2 + \beta^2$ **2**

(b) Given the equation of the parabola $x^2 - 2x - 8y - 15 = 0$

(i) Show that the equation of the parabola can be expressed as: **2**

$$(x-1)^2 = 8(y+2)$$

(ii) Find the vertex. **1**

(iii) Find the focus. **1**

(iv) Find the equation of the directrix. **1**

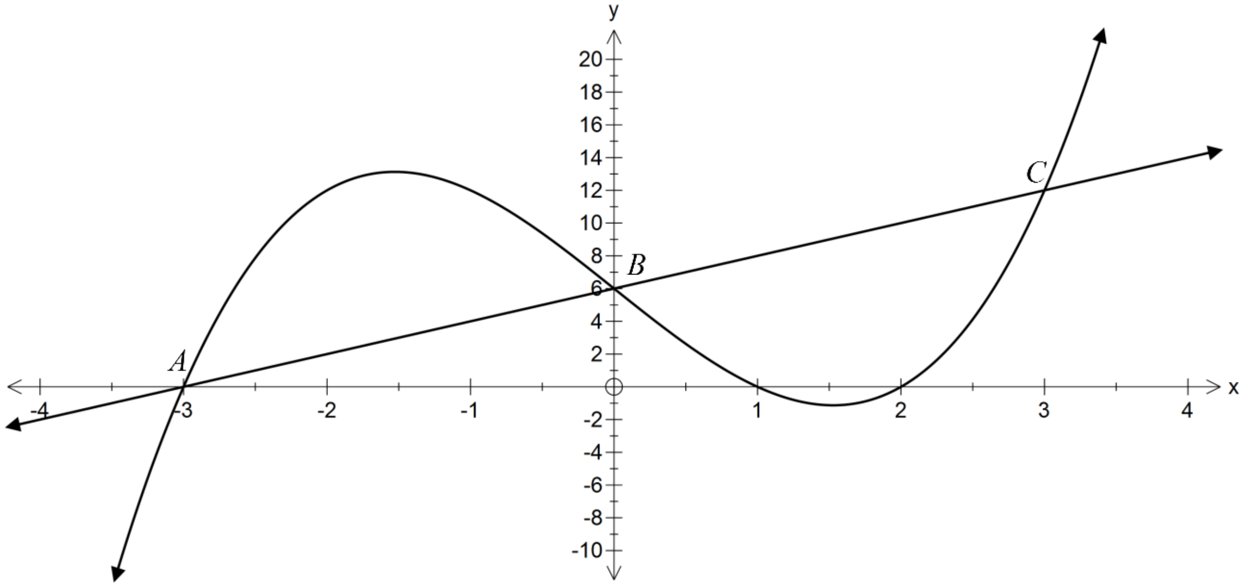
(v) Sketch the parabola showing where it crosses the y axis, the focus and the directrix. **2**

(c) The derivative of a function is given by $f'(x) = 15(5x-1)^2$. **2**
If $f(0) = 10$, find the equation of $f(x)$.

End of Question 5

Question 6 (12 marks) Start a new booklet **Marks**

(a) The following diagram shows the graphs of $y = x^3 - 7x + 6$ and $y = 2x + 6$



(i) Find the coordinates of the intersection points A, B and C. **1**

(ii) Hence determine the domain of x such that: **2**

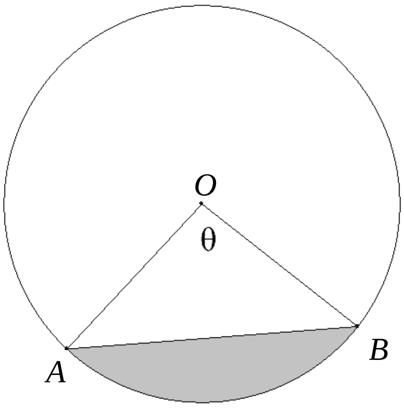
$$x^3 - 7x + 6 > 2x + 6.$$

(b) Find $\int \sqrt{5x-2}\ dx$ **2**

(c) Evaluate the definite integral $\int_{\frac{\pi}{4}}^{\pi} 3\sin 2x\ dx$ **2**

Question 6 continued on page 8

(d) A circle has a radius of 20 cm and arc AB is 32 cm.



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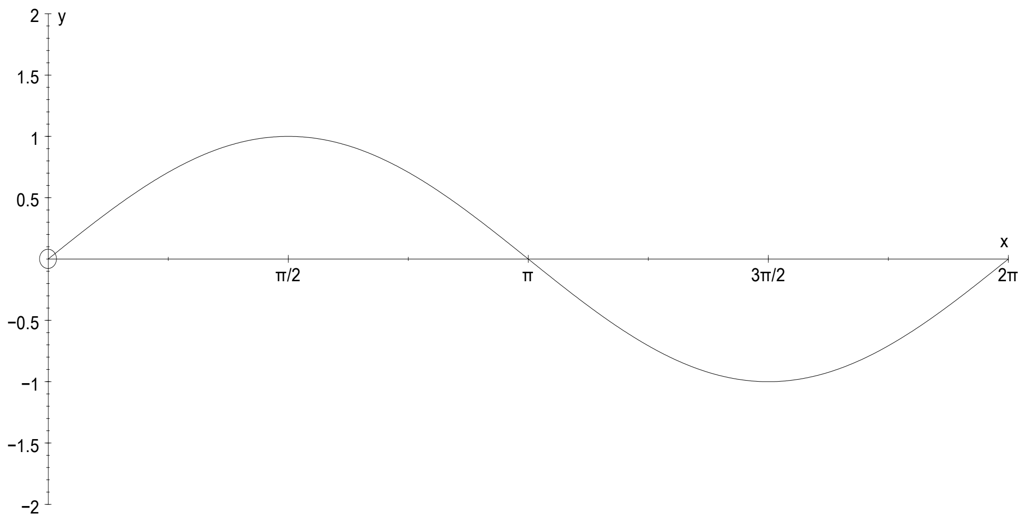
(i) Find θ, the angle subtended at the centre by chord AB. **1**
Give your answer in radians to 1 decimal place.

(ii) Hence find the shaded area. **2**

(e) The diagram shows the graph of $y = \sin x$ in the domain $0 \le x \le 2\pi$.



(i) Copy this graph into your assessment booklet **AND on the same set of axes** **1**
draw the graph of $y = \cos 2x$ in the domain $0 \le x \le 2\pi$.

(ii) How many solutions are there to the equation $\sin x = \cos 2x$ **1**
in the domain $0 \le x \le 2\pi$?

End of Question 6

Question 7 (12 marks) Start a new booklet **Marks**

(a) A geometric sequence has a second term 6 and the ratio of the sixth term to the fifth term is 3. Find the first term. **2**

(b) (i) Show that $\int_0^3 \frac{2}{x+1}\,dx = \log_e 16$ **2**

(ii) Hence use the Trapezoidal rule with four function values to find an approximation of $\log_e 16$ **3**

(c) Nancy's parents invest \$1200 each year in a superannuation fund for her. Compound interest is paid at 9% per annum on the investment. The first \$1200 is to be invested on Nancy's first birthday. The last is to be invested on her 21st birthday. To the nearest dollar:

(i) How much is the first investment worth on Nancy's 22nd birthday ? **2**

(ii) What is the total investment worth on Nancy's 22nd birthday ? **3**

End of Question 7

Question 8 (12 marks) Start a new booklet **Marks**

This question considers the function defined as $f(x) = e^{-\frac{x^2}{2}}$.

(i) State the domain of $f(x)$. **1**

(ii) Show that $f(x)$ is an even function. **1**

(iii) Show that $f'(x) = -xe^{-\frac{x^2}{2}}$ **1**

(iv) Find the stationary point of $y = f(x)$ and determine its nature. **2**

(v) Use the product rule to show that $f''(x) = (x^2 - 1)e^{-\frac{x^2}{2}}$ **2**

(vi) Find the two points at which $f''(x) = 0$ and show they are points of inflexion. **2**

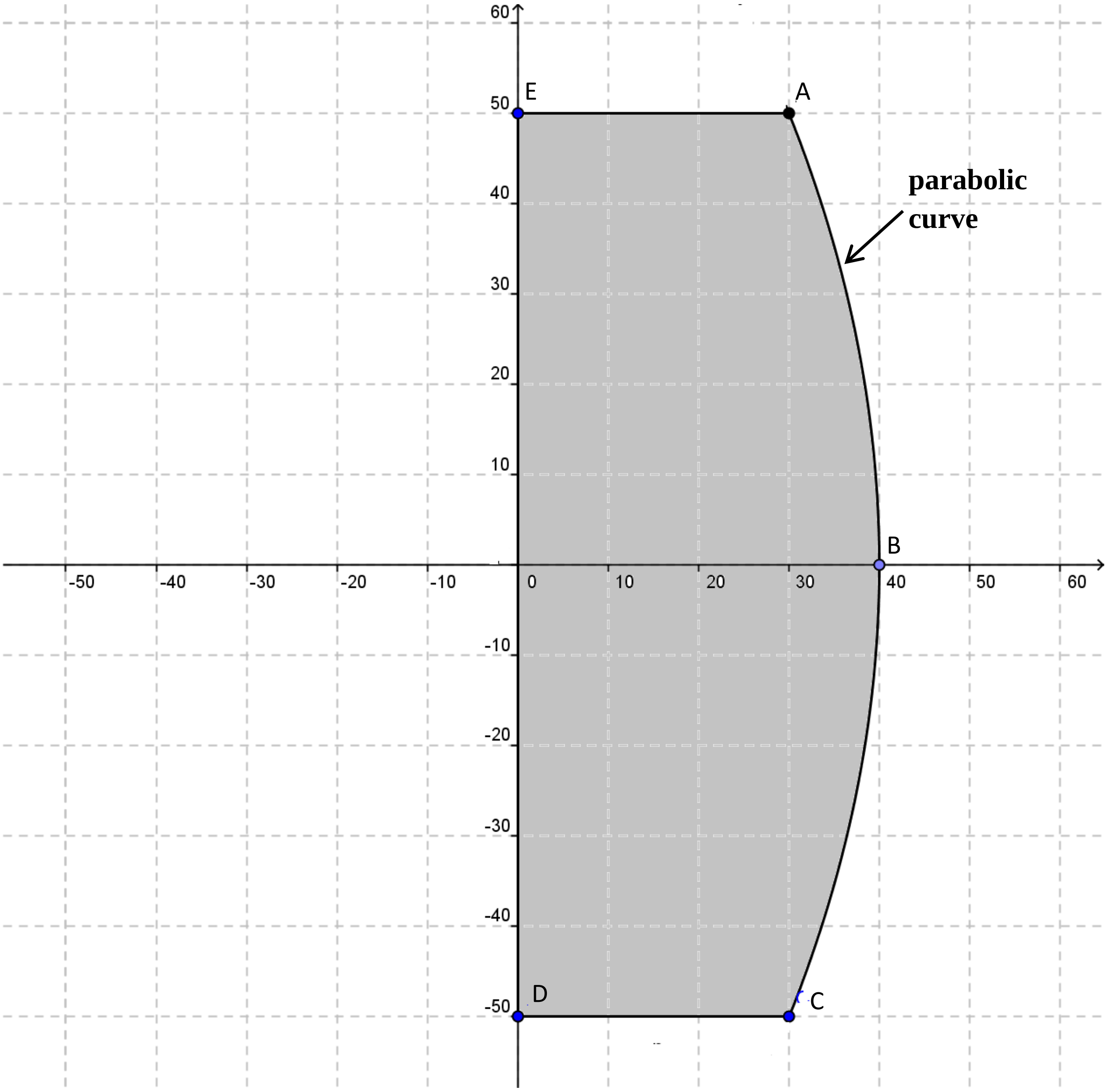
(vii) By considering the value that $f(x)$ approaches as x becomes large, state the range of $f(x)$. **1**

(viii) Sketch $y = f(x)$ showing the information found above. **2**

End of Question 8

Question 9 (12 marks) Start a new booklet **Marks**

(a) A wine barrel has been designed by rotating the shape ABCDE (shown in the following diagram) about the y axis. The curve ABC is parabolic. The point B is the vertex of this parabolic curve. All units on the graph are shown in cm.



(i) Using the formulae $(y-k)^2 = -4a(x-h)$ show that the equation of the parabolic curve in the diagram is $y^2 = -250x + 10000$ **3**

(ii) All units on the graph are shown in cm. By rotating the shaded area **around the y-axis**, find the volume of the barrel in Litres, where $1\text{cm}^3 = 1\text{mL}$. **3**

Question 9 continued on page 12

(b) A particle is moving in a straight line. Its velocity, v as a function of time t ($t \geq 0$) is given by $v = \dfrac{4}{t+1} - 2t$.

(i) Find when the particle changes direction. **2**

(ii) Find the exact distance travelled in the first two seconds. **2**

(iii) What is the acceleration of the particle as $t \to \infty$ **2**

End of Question 9

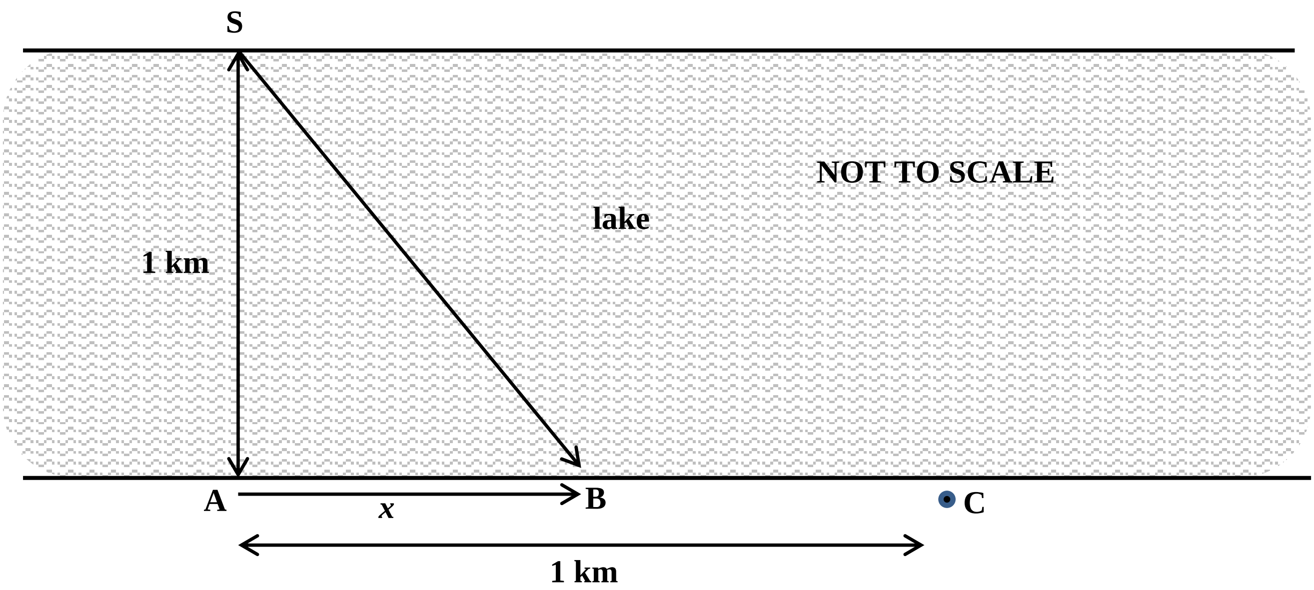
Question 10 (12 marks) Start a new booklet **Marks**

(a) Show that $\dfrac{(1+\tan^2\theta)\cot\theta}{\text{cosec}^2\theta}=\tan\theta$ **3**

(b) By using the identity $\cos\alpha = 2\cos^2\dfrac{\alpha}{2}-1,$ (**do not prove this**) **3**

find the solutions to the equation $\cos\dfrac{\alpha}{2}=1+\cos\alpha$ for the domain $0\le\alpha\le 2\pi$

(c) Suzy wishes to return to camp. She is standing at S, on the edge of a lake, which is 1km wide. The camp (at C) is one km from the direct opposite side (Point A) from where Suzy is currently standing, as shown in the diagram. She knows she walks at 3km/h and swims at 2km/h and wonders to herself at what distance, x, from the point opposite, should she swim to, in order to minimise the time to get to camp.
Note that point B is a distance of x from point A.



(i) Using $\text{time}=\dfrac{\text{distance}}{\text{speed}}$, show that the total elapsed time, T in swimming to point B and walking from there to camp is given by $T=\dfrac{3\sqrt{1+x^2}+2-2x}{6}$ **2**

(ii) Knowing that Suzy wants to take the least amount of time getting back to camp, show that $x=\dfrac{2}{\sqrt{5}}\ km$ AND determine her travel time in hours (correct to one decimal place). **4**

End of assessment