Alternative Solutions to Cambridge Extension 2 5F Q15-19, 5G Q16 and 5H Q18

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Abstract. I will use some things outside the syllabus, namely the scalar triple product and the Penrose inverse for matrix equations for non-square coefficient matrices.

I will do this out of order in the title because one of these is given an incorrect answer in the textbook (Sadler and Ward, 2020), namely I will do 5F Q17a first. Nevertheless the question can be changed to get the answer given. I have given that solution too.

Furthermore when the textbook asks to find the point of intersection of two lines in 3 dimensions I will go further and prove that they in fact do intersect first (remembering that in 3 dimensions almost all lines do not intersect - unlike in 2 dimensions where they almost always do intersect).

Firstly I will explain however the theory behind the 2 things I am using outside the syllabus, namely the scalar triple product and the Penrose inverse.

Some Theory

Scalar triple product

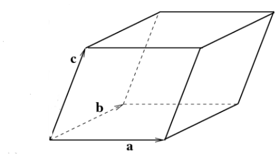
A 3×3 determinant is $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a(ei - fh) - b(di - fg) + c(dh - eg)$

If $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$ then the cross product

$\mathbf{b} \times \mathbf{c}$ is $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

The scalar triple product $\mathbf{a} \cdot \mathbf{b} \times \mathbf{c}$ could be calculated by first finding the cross product $\mathbf{b} \times \mathbf{c}$ and then taking the dot product with $\mathbf{a}$. However there is a more efficient method because $\mathbf{a} \cdot \mathbf{b} \times \mathbf{c} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

Geometrically the absolute value of the scalar triple product represents the volume of the parallelepiped spanned by the vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$:



But a consequence of this is that if the vectors are coplanar then the scalar triple product will be 0.

Consider the following statement:

If the lines intersect then the scalar triple product will be 0.

The contrapositive of this statement is

If the scalar triple product is not 0 then the lines do not intersect.

On the other hand if the scalar triple product is 0 and they are not parallel then they do intersect.

Penrose inverse

This is named after Professor Roger Penrose from Oxford University.

Generally a line can be represented by 2 intersecting planes. So 2 intersecting lines can be represented by 2 pairs of intersecting planes.

$a_{11}x + a_{12}y + a_{13}z = b_1$
$a_{21}x + a_{22}y + a_{23}z = b_2$
$a_{31}x + a_{32}y + a_{33}z = b_3$
$a_{41}x + a_{42}y + a_{43}z = b_4$

which can then be represented in a matrix equation $A\mathbf{w} = \mathbf{b}$ where $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \end{pmatrix}$, and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix}$. If the lines intersect then $\mathbf{w} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = (A^T A)^{-1} A^T \mathbf{b}$ gives the point of intersection (x, y, z), where A^T is the transpose of A, i.e., the matrix resulting from swapping rows and columns.

(Note that if A were a square matrix then we could just use $\mathbf{w} = A^{-1}\mathbf{b}$. But we can't use A^{-1} if A is not square, hence the slightly more complex generalised Penrose inverse $(A^TA)^{-1}A^T$ for when A is not square. This works because if you multiply any matrix by its own transpose then the result is a square matrix.)

Correction to textbook (5F Q17a)

5F Q17a as it appears in the textbook

The position vector given for the point of intersection (if there is one), $-\mathbf{i}+\mathbf{k}$, i.e., the point $(-1, 0, 1)$, is incorrect. There is no intersection.

Choose 4 points, 2 on each line, eg., $\lambda = 0, 1$ and $\mu = 0, 1$ and we have

$A = (3, -2, 3),\ B = (5, -3, 4),\ C = (-2, -2, 4), D = (-1, 0, 7)$ and now

$\overrightarrow{AB} \cdot \overrightarrow{AC} \times \overrightarrow{AD} = \begin{vmatrix} 2 & -1 & 1 \\ -5 & 0 & 1 \\ -4 & 2 & 4 \end{vmatrix} = -30 \neq 0$ and hence the lines are not coplanar. They do **not** intersect. They are not parallel either. They are in fact skew (as almost all pairs of lines in 3 dimensions are).

5F Q17a modified to get the answer given

We replace $\mathbf{v}_2$ with $\begin{pmatrix} -2 \\ -2 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}$ and this change will result in the answer given in the textbook.

Before we answer the question we should check that they do in fact intersect.

Choose 4 points, 2 on each line eg., $\lambda = 0, 1$ and $\mu = 0, 1$ so that

$A = (3, -2, 3),\ B = (5, -3, 4),\ C = (-2, -2, 4),\ D = (-1, 0, 1)$ and now we have that $\overrightarrow{AB} \cdot \overrightarrow{AC} \times \overrightarrow{AD} = \begin{vmatrix} 2 & -1 & 1 \\ -5 & 0 & 1 \\ -4 & 2 & -2 \end{vmatrix} = 0$ and so the lines are coplanar.

The direction vector $\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} = 2 \begin{pmatrix} 1 \\ -1/2 \\ 1/2 \end{pmatrix}$ is not a scalar multiple of $\begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}$ and so are not parallel.

Hence they intersect.

We may rewrite the lines as $\lambda = \frac{x-3}{2} = \frac{y+2}{-1} = \frac{z-3}{1}$, $\mu = \frac{x+2}{1} = \frac{y+2}{2} = \frac{z-4}{-3}$ and then again as 2 pairs of intersecting planes

$x + 2y + 0z = -1$
$0x + y + z = 1$
$2x - y + 0z = -2$
$0x + 3y + 2z = 2$

and now as a matrix equation

$\begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 2 & -1 & 0 \\ 0 & 3 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ -2 \\ 2 \end{pmatrix}$ and so using a Penrose inverse,

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \left(\begin{pmatrix} 1 & 0 & 2 & 0 \\ 2 & 1 & -1 & 3 \\ 0 & 1 & 0 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 2 & -1 & 0 \\ 0 & 3 & 2 \end{pmatrix} \right)^{-1} \begin{pmatrix} 1 & 0 & 2 & 0 \\ 2 & 1 & -1 & 3 \\ 0 & 1 & 0 & 2 \end{pmatrix} \begin{pmatrix} -1 \\ 1 \\ -2 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

and hence $(x, y, z) = (-1, 0, 1)$ with position vector $-\mathbf{i} + \mathbf{k}$.

Other Questions

5F Q15a Choose 4 points, 2 on each line eg., $\lambda = 0, 1$ and $\mu = 0, 1$ so that

$A = (4, 8, 3),\ B = (5, 10, 4),\ C = (7, 6, 5),\ D = (13, 10, 10)$ and now we have that $\overrightarrow{AB} \cdot \overrightarrow{AC} \times \overrightarrow{AD} = \begin{vmatrix} 1 & 2 & 1 \\ 3 & -2 & 2 \\ 9 & 2 & 7 \end{vmatrix} = 0$ and so the lines are coplanar.

The direction vector $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = \frac{1}{6} \begin{pmatrix} 6 \\ 12 \\ 6 \end{pmatrix}$ is not a scalar multiple of $\begin{pmatrix} 6 \\ 4 \\ 5 \end{pmatrix}$

So they are coplanar and are not parallel. Hence they intersect.

We may rewrite the lines as $\lambda = \frac{x-4}{1} = \frac{y-8}{2} = \frac{z-3}{1}$, $\mu = \frac{x-7}{6} = \frac{y-6}{4} = \frac{z-5}{5}$ and then again as 2 pairs of intersecting planes

$2x - y + 0z = 0$
$0x + y - 2z = 2$
$2x - 3y + 0z = -4$
$0x + 5y - 4z = 10$

and now as a matrix equation

$\begin{pmatrix} 2 & -1 & 0 \\ 0 & 1 & -2 \\ 2 & -3 & 0 \\ 0 & 5 & -4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ -4 \\ 10 \end{pmatrix}$ and so

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \left(\begin{pmatrix} 2 & 0 & 2 & 0 \\ -1 & 1 & -3 & 5 \\ 0 & -2 & 0 & -4 \end{pmatrix} \begin{pmatrix} 2 & -1 & 0 \\ 0 & 1 & -2 \\ 2 & -3 & 0 \\ 0 & 5 & -4 \end{pmatrix} \right)^{-1} \begin{pmatrix} 2 & 0 & 2 & 0 \\ -1 & 1 & -3 & 5 \\ 0 & -2 & 0 & -4 \end{pmatrix} \begin{pmatrix} 0 \\ 2 \\ -4 \\ 10 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$$

and hence $(x, y, z) = (1, 2, 0)$.

5FQ15b Choose 4 points, 2 on each line, e.g., $\lambda = 0, 1$ and $\mu = 0, 1$.

Then we have that $A = (7, -3, 8),\ B = (11, -4, 10),\ C = (-2, 1, 10),\ D = (3, -2, 6)$ and so

$\overrightarrow{AB} \cdot \overrightarrow{AC} \times \overrightarrow{AD} = \begin{vmatrix} 4 & -1 & 2 \\ -9 & 4 & 2 \\ -4 & 1 & -2 \end{vmatrix} = 0$ and hence the lines are coplanar.

Also the direction vector $\begin{pmatrix} 4 \\ -1 \\ 2 \end{pmatrix} = \frac{4}{5} \begin{pmatrix} 5 \\ -5/4 \\ 5/2 \end{pmatrix}$ is not a scalar multiple of $\begin{pmatrix} 5 \\ -3 \\ -4 \end{pmatrix}$ and therefore they are not parallel.

Hence they intersect.

We may rewrite the lines as $\lambda = \frac{x-7}{4} = \frac{y+3}{-1} = \frac{z-8}{2},\ \mu = \frac{x+2}{5} = \frac{y-1}{-3} = \frac{z-10}{-4}$ and then again as 2 pairs of intersecting planes

$x + 4y + 0z = -5$
$0x + 2y + z = 2$
$3x + 5y + 0z = -1$
$0x + 4y - 3z = -26$

and now as a matrix equation

$\begin{pmatrix} 1 & 4 & 0 \\ 0 & 2 & 1 \\ 3 & 5 & 0 \\ 0 & 4 & -3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -5 \\ 2 \\ -1 \\ -26 \end{pmatrix}$ and so

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \left(\begin{pmatrix} 1 & 0 & 3 & 0 \\ 4 & 2 & 5 & 4 \\ 0 & 1 & 0 & -3 \end{pmatrix} \begin{pmatrix} 1 & 4 & 0 \\ 0 & 2 & 1 \\ 3 & 5 & 0 \\ 0 & 4 & -3 \end{pmatrix} \right)^{-1} \begin{pmatrix} 1 & 0 & 3 & 0 \\ 4 & 2 & 5 & 4 \\ 0 & 1 & 0 & -3 \end{pmatrix} \begin{pmatrix} -5 \\ 2 \\ -1 \\ -26 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \\ 6 \end{pmatrix}$$

and hence the point of intersection is $(3, -2, 6)$.

5F Q16 Choose 4 points, 2 on each line, eg., $\lambda = 0, 1$ and $\mu = 0, 1$ so

$A = (1, 0, -1),\ B = (3, -1, 0),\ C = (1, 1, 0),\ D = (-3, 4, -3)$ and then

$\overrightarrow{AB} \cdot \overrightarrow{AC} \times \overrightarrow{AD} = \begin{vmatrix} 2 & -1 & 1 \\ 0 & 1 & 1 \\ -4 & 4 & -2 \end{vmatrix} = -4 \neq 0$ and so the lines are not coplanar. Therefore they do not intersect and are not parallel. Hence they are skew.

5F Q17b Choose 4 points, 2 on each line, eg., $\lambda = 0, 1$ and $\mu = 0, 1$ so

$A = (3, 1, 4),\ B = (5, 2, 3),\ C = (2, -1, 1),\ D = (1, 1, 4)$ and so

$\overrightarrow{AB} \cdot \overrightarrow{AC} \times \overrightarrow{AD} = \begin{vmatrix} 2 & 1 & -1 \\ -1 & -2 & -3 \\ -2 & 0 & 0 \end{vmatrix} = 10 \neq 0$ and so the lines are not coplanar. So they do not intersect, are not parallel and hence are skew.

5F Q18a $\ell_1 : \mathbf{r}_1 = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ 3 \\ 3 \end{pmatrix}$ and $\ell_2 : \mathbf{r}_2 = \begin{pmatrix} -1 \\ 3 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ -5 \\ 5 \end{pmatrix}$.

Check that they intersect:

Choose 4 points, 2 on each line, eg., $\lambda = 0, 1$ and $\mu = 0, 1$ so

$A = (2, 0, 1),\ B = (-1, 3, 4),\ C = (-1, 3, 0),\ D = (4, -2, 5)$ so

$\overrightarrow{AB} \cdot \overrightarrow{AC} \times \overrightarrow{AD} = \begin{vmatrix} -3 & 3 & 3 \\ -3 & 3 & -1 \\ 2 & -2 & 4 \end{vmatrix} = 0$ and so they are coplanar.

The direction vector $\begin{pmatrix} -3 \\ 3 \\ 3 \end{pmatrix} = \frac{-3}{5} \begin{pmatrix} 5 \\ -5 \\ -5 \end{pmatrix}$ is not a scalar multiple of $\begin{pmatrix} 5 \\ -5 \\ 5 \end{pmatrix}$ and so they are not parallel.

Hence they intersect.

Rewrite the lines as $\lambda = \frac{x-2}{-3} = \frac{y-0}{3} = \frac{z-1}{3},\ \mu = \frac{x+1}{5} = \frac{y-3}{-5} = \frac{z-0}{5}$ and then again as 2 pairs of intersecting planes

$x + y + 0z = 2$
$0x + y - z = -1$
$x + 0y - z = -1$
$0x + y + z = 3$

and then

$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ -1 \\ 3 \end{pmatrix}$ and so

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \left(\begin{pmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & -1 & -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{pmatrix} \right)^{-1} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & -1 & -1 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \\ -1 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$ and so the point of intersection is $(1, 1, 2)$.

5F Q18b $\theta = \cos^{-1} \left| \frac{-3\times5+3\times-5+3\times5}{\sqrt{3^2+3^2+3^2}\sqrt{5^2+5^2+5^2}} \right| = \cos^{-1} \frac{1}{3} \approx 70.5°$

5F Q19 If $\mathbf{r}_1 = 2\mathbf{i} + 9\mathbf{j} + 13\mathbf{k} + \lambda(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$ and $\mathbf{r}_2 = a\mathbf{i} + 7\mathbf{j} - 2\mathbf{k} + \mu(-\mathbf{i} + 2\mathbf{j} - 3\mathbf{k})$ intersect then with 4 points, 2 on each line we have $\overrightarrow{AB} \cdot \overrightarrow{AC} \times \overrightarrow{AD} = 0$ because they have to be coplanar in order to intersect and the direction vector $\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ is not a scalar multiple of $-\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$ and so they are not parallel.

Let $\lambda = 0, 1$ and $\mu = 0, 1$ then

$A = (2, 9, 13),\ B = (3, 11, 16),\ C = (a, 7, -2),\ D = (a - 1, 9, -5)$ and

$\overrightarrow{AB} \cdot \overrightarrow{AC} \times \overrightarrow{AD} = \begin{vmatrix} 1 & 2 & 3 \\ a-2 & -2 & -15 \\ a-3 & 0 & -18 \end{vmatrix} = 12a + 36 = 0$ and so $a = -3$.

5G Q16 $\mathbf{r} = \begin{pmatrix} -2 \\ 3 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix}$ and so $\lambda = \frac{x+2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ so

$4x - 3y + 0z = -17$
$0x + 5y - 4z = -1$
$5x + 0y - 3z = -22$ and these are to also intersect the plane
$2x + 4y - z = 55$ and so

$\begin{pmatrix} 4 & -3 & 0 \\ 0 & 5 & -4 \\ 5 & 0 & -3 \\ 2 & 4 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -17 \\ -1 \\ -22 \\ 55 \end{pmatrix}$ and so

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \left(\begin{pmatrix} 4 & 0 & 5 & 2 \\ -3 & 5 & 0 & 4 \\ 0 & -4 & -3 & -1 \end{pmatrix} \begin{pmatrix} 4 & -3 & 0 \\ 0 & 5 & -4 \\ 5 & 0 & -3 \\ 2 & 4 & -1 \end{pmatrix} \right)^{-1} \begin{pmatrix} 4 & 0 & 5 & 2 \\ -3 & 5 & 0 & 4 \\ 0 & -4 & -3 & -1 \end{pmatrix} \begin{pmatrix} -17 \\ -1 \\ -22 \\ 55 \end{pmatrix} = \begin{pmatrix} 7 \\ 15 \\ 19 \end{pmatrix}$ and so the point of intersection is $(7, 15, 19)$.

5H Q18a Check they intersect.

Choose 4 points, 2 on each line, eg., $\lambda = 0, 1$ and $\mu = 0, 1$. Then

$A = (6, 5, 3),\ B = (8, 6, 7),\ C = (-3, 7, 2),\ D = (2, 3, -5)$. Hence

$\overrightarrow{AB} \cdot \overrightarrow{AC} \times \overrightarrow{AD} = \begin{vmatrix} 2 & 1 & 4 \\ -9 & 2 & -1 \\ -4 & -2 & -8 \end{vmatrix} = 0$ and so they are coplanar.

The direction vector $\begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} = \frac{2}{5} \begin{pmatrix} 5 \\ 2.5 \\ 10 \end{pmatrix}$ is not a scalar multiple of $\begin{pmatrix} 5 \\ -4 \\ -7 \end{pmatrix}$ and so they are not parallel.

Hence they intersect.

Now $\lambda = \frac{x-6}{2} = \frac{y-5}{1} = \frac{z-3}{4}$ and $\mu = \frac{x+3}{5} = \frac{y-7}{-4} = \frac{z-2}{-7}$ and so

$x - 2y + 0z = -4$
$0x + 4y - z = 17$
$4x + 5y + 0z = 23$
$0x + 7y - 4z = 41$

and now

$\begin{pmatrix} 1 & -2 & 0 \\ 0 & 4 & -1 \\ 4 & 5 & 0 \\ 0 & 7 & -4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -4 \\ 17 \\ 23 \\ 41 \end{pmatrix}$ whereupon

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \left(\begin{pmatrix} 1 & 0 & 4 & 0 \\ -2 & 4 & 5 & 7 \\ 0 & -1 & 0 & -4 \end{pmatrix} \begin{pmatrix} 1 & -2 & 0 \\ 0 & 4 & -1 \\ 4 & 5 & 0 \\ 0 & 7 & -4 \end{pmatrix} \right)^{-1} \begin{pmatrix} 1 & 0 & 4 & 0 \\ -2 & 4 & 5 & 7 \\ 0 & -1 & 0 & -4 \end{pmatrix} \begin{pmatrix} -4 \\ 17 \\ 23 \\ 41 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ -5 \end{pmatrix}$ and so the point of intersection is $(2, 3, -5)$.

5H Q18b Check they intersect.

I will use μ instead of another λ in the second equation to avoid confusion. Choose 4 points, 2 on each line, eg., $\lambda = 0, 1$ and $\mu = 0, 1$. Then

$A = (-7, -1, 7),\ B = (-5, 2, 3),\ C = (9, -4, -16),\ D = (13, -7, -21)$. Hence

$\overrightarrow{AB} \cdot \overrightarrow{AC} \times \overrightarrow{AD} = \begin{vmatrix} 2 & 3 & -4 \\ 16 & -3 & -23 \\ 20 & -6 & -28 \end{vmatrix} = 0$ and so they are coplanar.

The direction vector $2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k} = \frac{1}{2}(4\mathbf{i} + 6\mathbf{j} - 8\mathbf{k})$ is not a scalar multiple of $4\mathbf{i} - 3\mathbf{j} - 5\mathbf{k}$ and so they are not parallel.

Hence they intersect.

Now $\lambda = \frac{x+7}{2} = \frac{y+1}{3} = \frac{z-7}{-4}$ and $\mu = \frac{x-9}{4} = \frac{y+4}{-3} = \frac{z+16}{-5}$ and so

$3x - 2y + 0z = -19$
$0x + 4y + 3z = 17$
$3x + 4y + 0z = 11$
$0x + 5y - 3z = 28$

and now

$\begin{pmatrix} 3 & -2 & 0 \\ 0 & 4 & 3 \\ 3 & 4 & 0 \\ 0 & 5 & -3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -19 \\ 17 \\ 11 \\ 28 \end{pmatrix}$ whereupon

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \left(\begin{pmatrix} 3 & 0 & 3 & 0 \\ -2 & 4 & 4 & 5 \\ 0 & 3 & 0 & -3 \end{pmatrix} \begin{pmatrix} 3 & -2 & 0 \\ 0 & 4 & 3 \\ 3 & 4 & 0 \\ 0 & 5 & -3 \end{pmatrix} \right)^{-1} \begin{pmatrix} 3 & 0 & 3 & 0 \\ -2 & 4 & 4 & 5 \\ 0 & 3 & 0 & -3 \end{pmatrix} \begin{pmatrix} -19 \\ 17 \\ 11 \\ 28 \end{pmatrix} = \begin{pmatrix} -3 \\ 5 \\ -1 \end{pmatrix}$ and so the point of intersection is $(-3, 5, -1)$.

Reference

Sadler, D. and Ward, D., CambridgeMATHS Mathematics Extension 2, Cambridge University Press, 2020.