Question 1

(a) using calculator,

$e^{-3.5} = 0.030197\ldots$

$= 0.0302$ to 3 sig figs

(b) let $y = 3x + \tan x$

$\frac{dy}{dx} = 3 + \sec^2 x$

(c) $\frac{\theta}{360^\circ} = \frac{5}{2\pi(3)}$ ⇐ circumference

$\theta = \frac{5}{6\pi} \times 360^\circ$

$= 95^\circ$

Alternatively,

$l = r.\theta$

$5 = 3\theta$

$\theta = \frac{5}{3}$ radians

$= 95^\circ$

(d) the cost of the meal = 100%

tip = $12\frac{1}{2}\%$ (of the 100% / the cost of meal)

payment = $112\frac{1}{2}\%$

cost of meal $= \frac{100}{112\frac{1}{2}} \times \$315.00$

$= \$280.00$

(e) $\int 3x^2 - 8$

$= \frac{3x^3}{3} - 8x + C = x^3 - 8x + C$

(f) $|x-3| = 7$

$(x-3) = 7$ OR $(x-3) = -7$

$x = 10$ OR $x = -4$

Question 2

(a) $y = 2\log_e x$

$\frac{dy}{dx} = \frac{2}{x}$

at $(e, 2)$,

$\frac{dy}{dx} = \frac{2}{e}$

$= m_T$

$m_T \times m_N = -1$

$m_N = -\frac{e}{2}$

$\therefore$ normal: $y - y_1 = m_N(x - x_1)$

$y - 2 = -\frac{e}{2}(x - e)$

$2y - 4 = -ex + e^2$

$ex + 2y - (4 + e^2) = 0$

Question 2

(b) (i) $OA^2 = (x_2-x_1)^2 + (y_2-y_1)^2$
$= (-1-0)^2 + (1-0)^2$
$= 2$
$OA = \sqrt{2}$

(ii) $m_{OA} = \dfrac{y_2-y_1}{x_2-x_1} = \dfrac{1-0}{-1-0} = -1$

(iii) let $\angle AOC = \alpha$.
$\tan\alpha = m_{OA}$
$\alpha = \tan^{-1} -1$
$= -45°$
but $\alpha \neq -45°$. $\alpha = 180° - 45°$
$= 135°$

(iv) $m_{BC} = m_{OA} = -1$.
BC: $y - y_1 = m(x-x_1)$
$y-6 = -1(x-4)$
$y + x - 10 = 0$
now put $y=0$: $x = 10$
$\therefore C = (10, 0)$

(v) $p = \dfrac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$ point (x_1, y_1) line $ax+by+c=0$
$= \dfrac{|10+0-10|}{\sqrt{1+1}}$
$= \dfrac{10}{\sqrt{2}} = 5\sqrt{2}$

If you can't remember the formula:
let the perpendicular line meet BC at E.
$m_{OE} \times m_{BC} = -1$
then you can find the equation of line OE and where it intersects BC, then use distance formula.

(vi) Well firstly it doesn't look like a very good trapezium. But it is a trapezium, with height $5\sqrt{2}$.
Area $= \frac{1}{2}(a+b)h = \frac{1}{2}(OA+BC)5\sqrt{2}$ $BC^2 = (10-4)^2+(0-6)^2$
$= \frac{1}{2}(\sqrt{2}+6\sqrt{2})5\sqrt{2} = \mathbf{35}$ units^2. $BC = \sqrt{72} = 6\sqrt{2}$

Question 3

(a) (i) let $y = (2e^x - 4)^9$

$\frac{dy}{dx} = 9(2e^x-4)^8 \times 2e^x$ → the derivative of the thing inside the brackets

$= 18e^x(2e^x-4)^8$

(ii) let $y = x^2 \sin x$

$\frac{dy}{dx} = (\sin x)(2x) + (x^2)(\cos x)$

$= 2x\sin x + x^2\cos x$

(b) Approach #1: $AB = DC$ (opposite sides are equal)

$AP = CP$ (diagonals bisect each other)
$BP = DP$

$\therefore \triangle ABP \equiv \triangle CDP$

actually, they are alternate angles in parallel lines ⟸ $\angle DCP = \angle BAP = 55°$ (corresponding $\angle$'s)

$\therefore \angle DPC = 180° - 36° - 55°$ (sum of $\angle$'s in $\triangle DPC$)

$= 89°$

Approach #2: since $AB \parallel DC$,

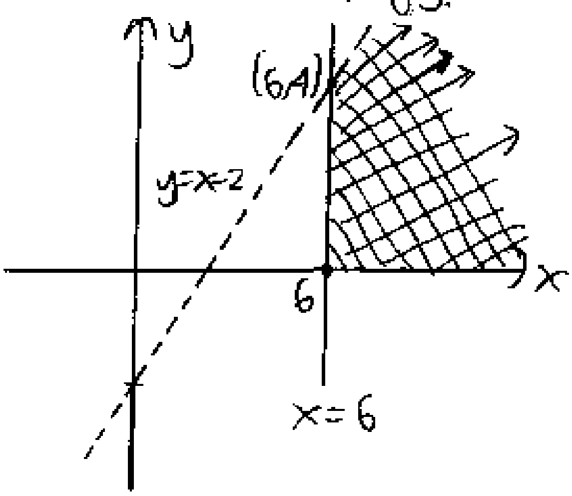
$\angle BAD + \angle CDA = 180°$

$\angle DAP + \angle ADP = 180° - 55° - 36° = 89°$

but $\angle DPC = \angle DAP + \angle ADP$ (external angle of $\triangle DPA$)

$= 89°$

(c)



you need to show the boundary points.

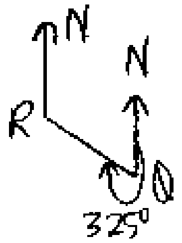
(d) (i) $\int \frac{2x}{x^2+5}\,dx$ we have the derivative of (x^2+5) on the top.

$= \ln(x^2+5) + C$

(ii) $\int_{\pi/4}^{\pi/3} \sec^2 x\,dx = \left[\tan x\right]_{\pi/4}^{\pi/3} = \tan\frac{\pi}{3} - \tan\frac{\pi}{4} = \sqrt{3} - 1$

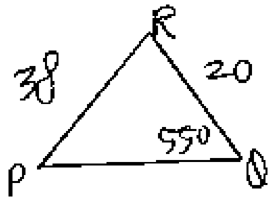
Question 4

(a) (i)



$\angle PQR \quad 325^\circ - 270^\circ$
$= 55^\circ$

(ii)



using sine rule:

$$\frac{\sin\angle RPQ}{20} = \frac{\sin 55^\circ}{38}$$

$$\angle RPQ = \sin^{-1} 0.4311326$$
$$= 25^\circ 32'$$

Bearing of R from P,

$$90^\circ - 25^\circ 32' = 64^\circ 28'\ T.$$

(b) (i) $P(\text{both stop on } 1) = \frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$

$P(\text{both stop on } 3) = \frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$

$P(\text{both stop on } 1 \text{ or on } 3) = \frac{1}{12} + \frac{1}{12} = \frac{1}{6}$

(ii) $P = 1 - P(\text{neither stops on } 3)$
$= 1 - \frac{2}{3} \times \frac{3}{4} = \frac{1}{2}$

Alternatively,
$P = \frac{1}{3} \times \frac{3}{4} + \frac{2}{3} \times \frac{1}{4} + \frac{1}{3} \times \frac{1}{4}$

(c) (i) $x - 4 = x^2 - 4x$

① $- \ x^2 - 5x + 4 = 0$

② $- \ (x-4)(x-b) = 0$

By equating constants in ① and ②:

$4b = 4$
$b = 1$

$A = (1, y_A)$. $\quad y_A = x_A - 4$
$= 1 - 4 = -3$

$A = (1, -3)$

(ii)

$$\int_1^4 (x^2 - 5x + 4)\,dx$$
$$= \left[\frac{x^3}{3} - \frac{5x^2}{2} + 4x\right]_1^4$$
$$= (64/3 - 40 + 16) - (1/3 - 5/2 + 4)$$
$$= -4.5$$

Area $= |-4.5|$
$= 4.5$ units^2

Question 5

(a) (i) $f'(x) = 4x^3 - 4.3x^2 = 4x^2(x-3)$

(ii) put $f'(x) = 0$: $4x^2(x-3) = 0$

either $x^2 = 0$ or $(x-3) = 0$

points: $\begin{pmatrix} x = 0 \\ y = 0 \end{pmatrix}$ or $\begin{pmatrix} x = 3 \\ y = -27 \end{pmatrix}$

Question 5

(a) (ii) for $(0,0)$, horizontal point of inflexion

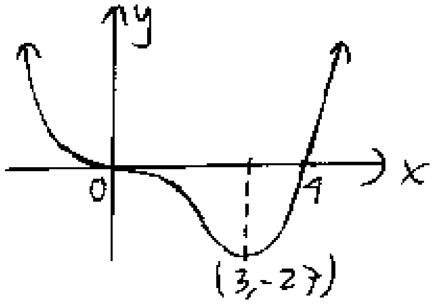
x	-1	0	1
$f'(x)$	$-ve$	0	$-ve$

for $(3,-27)$, minimum

x	1	0	4
$f'(x)$	$-ve$	0	$+ve$

tip:
don't calculate $f'(x)$.
$f'(x) = 4x^2(x-3)$:
the x^2 must be +ve.
so we only need to look at $(x-3)$.

(iii) $f(x) = x^3(x-4)$. zeroes at $x=0$ and $x=4$



(iv) $f'(x) = 4x^3 - 12x^2$ ⇐ the expanded form is easier to differentiate
$f''(x) = 12x^2 - 24x$
put $f''(x) = 0$: $0 = 12x^2 - 24x$
$0 = x(x-2)$
from graph we know there's a change in concavity so we don't need to test it.
$\therefore$ concave down for $0 < x < 2$ (inequality)

(b) (i) This forms an AP. $a = \frac{180}{1.5} = 120$ (blocks)
$d = -3$.
$n = 20$

$T_n = a + (n-1)d$
$T_{20} = 120 + 19(-3) = 63 \Rightarrow$ there are 63 blocks.

(ii) Consider: rows 21 and above: they also form an AP.
$a = 62$
$l = 10$, l is the last term.
$d = -1$.

$l = a + (n-1)d$ | $(n-1) = 62-10$ | $\therefore$ Total number of rows
$10 = 62 + (n-1)(-1)$ | $n = 53$ | $= 20 + 53 = 73$.

(iii) In the first AP, $S_n = \frac{1}{2}(20)(120+63) = 1830$ using
in the second AP, $S_n = \frac{1}{2}(53)(62+10) = 1908$ $S_n = \frac{1}{2}n(a+l)$
total number of blocks $= 1830 + 1908 = 3738$.

Question 6

(a) $\log_2(3x-4)=5$

$(3x-4)=2^5$

$3x=32+4$

$x=12$

(b) (i) they're alternate angles in parallel lines BE and CD.

(ii) $\angle ABD = \angle BDC + \angle BCD$ (external angle of $\triangle BCD$)

$2\theta = \theta + \angle BCD$, $\theta = \angle BDC$.

$\therefore \angle BCD = \angle BDC = \theta$

hence $\triangle BCD$ is isosceles.

(iii) show: $\frac{AE}{ED} = \frac{AB}{BD}$. By locating where the sides are, it looks like we need to prove $\triangle AEB \,|||\, \triangle EDB$.

But, no matter how long you try, you won't be able to do it because they're not similar.

Looking at the word hence, we need to use the fact that $\triangle BCD$ is isosceles: either the 2 angles are equal or the 2 sides are equal. Using the 2 sides are equal, we're proving

$AE : ED = AB : BC$, which will be true if

$\triangle AEB \,|||\, \triangle ADC$.

$\angle A$ is common

$\angle BEA = \angle CDA$ (corresponding $\angle$'s, $BE \parallel CD$).

$\therefore \triangle AEB \,|||\, \triangle ADC$ (2 angles equal)

(c) (i) at $t=0$, $C=5 \Rightarrow$ $C = C_0 e^{-kt}$

$5 = C_0 e^0 \Rightarrow C_0 = 5$

at $t=1$, $C=2.8 \Rightarrow$ $2.8 = 5e^{-k}$ | $-k = \ln\left(\frac{2.8}{5}\right)$

$\frac{2.8}{5} = e^{-k}$ | $k = \ln\left(\frac{5}{2.8}\right)$ using $a\ln b = \ln b^a$

(ii) put $C=0.2$: $0.2 = 5e^{-kt}$ ⇐ leave k because it's too ugly to write down.

$0.04 = e^{-kt}$

$-kt = \ln 0.04$

$\therefore t = -\frac{\ln 0.04}{k} = 5.55\ldots = 5.6$ years

Question 7

(a) (i) $a=2$. $r = 1/(\sqrt{2}+1)$. $S_\infty = \frac{a}{1-r} = \frac{2}{1-\frac{1}{\sqrt{2}+1}}$

$S_\infty = \frac{2}{1-\frac{1}{\sqrt{2}+1}} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} = \frac{2(\sqrt{2}+1)}{\sqrt{2}+1-1} = \sqrt{2}(\sqrt{2}+1)$

$= 2+\sqrt{2}$.

Note: if you forget $S_\infty = \frac{a}{1-r}$ you can still use the normal S_n formula. Put $n=\infty$, remembering $r^\infty \to 0$

Question 7 2003 HSC M2U p7

(a)(ii) $r = \frac{1}{\sqrt{2}-1} = 2.414$ (using calculator)

$|r| > 1$ so it doesn't have limiting sum

(b)(i) rest: $v = 0$.

$0 = 2 - 4\cos t \Rightarrow 4\cos t = 2$

$\cos t = \frac{1}{2}$

related angle of $t = \cos^{-1}\frac{1}{2} = \pi/3$.

for $0 \leq t \leq 2\pi$, $t = \frac{\pi}{3}$ or $\frac{5\pi}{3}$ seconds.

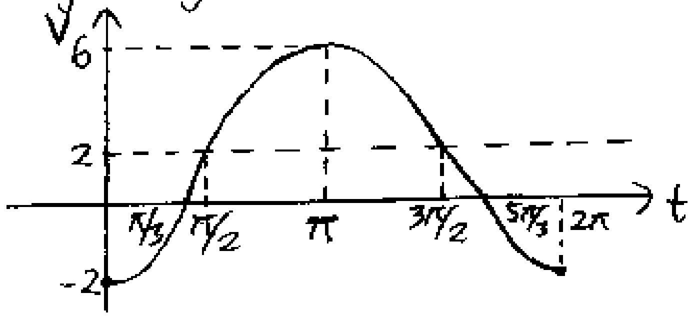
(ii) $v = 2 - 4\cos t$, $-1 \leq \cos t \leq 1$

v is max when $\cos t = -1$

$v = 2 - 4(-1)$

$= 6$ m/s

(iii) It will be $v = \cos t$, only it will be flipped upside down and enlarged by a factor of 4, then shifted up by 2 units.



(iv) distance $\Rightarrow$ area under curve (not integral)

$= \left|\int_0^{\pi/3}(2-4\cos t)\,dt\right| + \int_{\pi/3}^{\pi}(2-4\cos t)\,dt$

$= \left[2t - 4\sin t\right]_{\pi/3}^{0} + \left[2t - 4\sin t\right]_{\pi/3}^{\pi}$

$= \left(-\frac{2\pi}{3} + 4.\frac{\sqrt{3}}{2}\right) + \left(2\pi - 4.0 - \frac{2\pi}{3} + 4.\frac{\sqrt{3}}{2}\right)$

$= -\frac{2\pi}{3} + 2\sqrt{3} + \frac{6\pi}{3} - \frac{2\pi}{3} + 2\sqrt{3}$

$= \left(4\sqrt{3} + \frac{2\pi}{3}\right)$ metres.

Question 8

(a) A parabola is the locus of all points equidistant from the focus and the directrix.

$x^2 = -8y \Rightarrow y = -\ldots x^2$, i.e. concave down (in case you didn't know)

$x^2 = -4.2y$ has focus $(0,-2)$ $\Leftarrow$ compare with $x^2 = -4ay$.

Look at the distance at $(0,0)$.

distance to focus = distance to directrix. | directrix: $y = 2$.

Question 8 2003 HSC M2U p8

(b) $y = e^x$
$\ln y = x$.
at $x=0$, $y = e^0 = 1$ | $V = \pi \int_1^l (\ln y)^2 dy$

but we can find l.
put $x = \log_e 5$. $y = e^{\log_e 5} = 5 \Rightarrow l = 5$.
$V = \pi \int_1^5 (\ln y)^2 dy$

(c) $f(x) = \dfrac{x}{\ln x}$

$f(2) = \dfrac{2}{\ln 2}$. $f(4) = \dfrac{4}{\ln 4}$. $f(6) = \dfrac{6}{\ln 6}$

Area $= \dfrac{6-2}{3(2)}\left(f(2) + 4f(4) + f(6)\right)$
$= 11.85...$
$= 11.9$ by calculator

Simpson's rule:
$\frac{1}{3} \times$ width $\left(f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \ldots + f(x_n)\right)$
n is even.

(d)(i) put $y = mx - 3m^2$ into $x^2 = 12y$
(or the other way around):
$x^2 = 12(mx - 3m^2)$
$x^2 - 12mx + 36m^2$.
discriminant $= b^2 - 4ac = 144m^2 - 4(36m^2)$
$= 0$

$\therefore \Delta = 0$ for all $m \Rightarrow$ they always touches, for any m.

(ii) $y = mx - 3m^2$ passes $(5,2)$:
$2 = m(5) - 3m^2$
$3m^2 - 5m + 2 = 0$
$\dfrac{(3m-3)(3m-2)}{3} = 0$
| $(m-1) = 0 \Rightarrow m = 1$
$(3m-2) = 0 \Rightarrow m = 2/3$.

(iii) $y = mx - 3m^2$ is the tangent.
The tangents are: $y = x - 3$
and $y = 2/3\,x - 3(\frac{2}{3})^2 = \frac{2}{3}x - \frac{4}{3}$

Question 9

(a) this is a quadratic in $\sin x$. let $u = \sin x$.
$2u^2 - 3u - 2 = 0$
$\dfrac{(2u+1)(2u-4)}{2} = 0$
| $u = -1/2 \Rightarrow \sin x = -1/2 \Rightarrow x = \frac{7\pi}{6}, \frac{11\pi}{6}$
$u = 2 \Rightarrow \sin x = 2 \Rightarrow$ no solutions.

Question 9

(b) (i) let $\angle BAO = \theta$. using cosine rule:

$$OB^2 = AB^2 + AO^2 - 2(AB)(AO)\cos\theta$$
$$r^2 = 3r^2 + r^2 - 2r^2\sqrt{3}\cos\theta$$
$$-3r^2 = -2r^2\sqrt{3}\cos\theta$$
$$\frac{3}{2\sqrt{3}} = \cos\theta \Rightarrow \cos\theta = \frac{\sqrt{3}}{2}$$
$$\theta = \frac{\pi}{6} \text{ or } 30^\circ$$

(iii) I accidentally did this first.

$\angle BAO = \angle ABO$ (isosceles ΔBAO)

$\angle AOB = 180^\circ - 2\theta$

$= \angle AOC$ ($\Delta AOB \equiv \Delta AOC$)

$\angle BOC = 360^\circ - (\angle AOB + \angle AOC)$

$= 4\theta = 120^\circ$.

area of sector is proportional to the angle.
(sector is the region between radii and arc)

$$\frac{120^\circ}{360^\circ} = \frac{\text{area of sector}}{\text{area of circle}}$$
$$\frac{1}{3} = \frac{\text{area of sector}}{\pi r^2}$$

area of sector $= \frac{1}{3}\pi r^2$ units^2

(ii) C_2 has centre at A.

$\angle BAC = 2\theta$ since $\Delta AOB \equiv \Delta AOC$ (all sides equal)

$$\frac{2\theta}{360^\circ} = \frac{\text{area of segment}}{\text{area of circle}}$$
$$\frac{60^\circ}{360^\circ} = \frac{\text{area of segment}}{\pi(r\sqrt{3})^2}$$

area of segment
$= \frac{1}{6} \times \pi r^2 (3)$
$= \frac{1}{2}\pi r^2$ units^2

(iii) area = area in (iii) − [area in (ii) − area of ΔAOB and ΔAOC]

(try to visualise it).

Area of ΔAOB = area of ΔAOC
$= \frac{1}{2} \times r \times r\sqrt{3} \times \sin\theta$
$= \frac{1}{4} r^2\sqrt{3}$

area of shaded region $= \frac{1}{3}\pi r^2 - \left[\frac{1}{2}\pi r^2 - \frac{2}{4} r^2\sqrt{3}\right]$
$= \left(\frac{1}{3}\pi - \frac{1}{2}\pi + \frac{1}{2}\sqrt{3}\right) r^2$
$= \frac{1}{6}(3\sqrt{3} - \pi)\, r^2$ units^2

Question 9

(c) (i) distance = speed × time

$L = (v-u)t$

$t = \dfrac{L}{v-u}$

put this into $E = av^3t$:

$E = \dfrac{aLv^3}{v-u}$

(ii) differentiating : $\dfrac{dE}{dv} = \dfrac{(v-u)(3aLv^2) - (aLv^3)(1)}{(v-u)^2}$

put $\dfrac{dE}{dv} = 0$: numerator = 0

$(v-u)3aLv^2 - aLv^3 = 0$

$\div aLv^2$: $(v-u)3 - v = 0$

$3v - v = 3u$

$v = \frac{3}{2}u$

v	$1.1u$	$1.5u$	$2u$
$\frac{dE}{dv}$	$-ve$	0	$+ve$

minimum.

$v = 1.1u$,

$\dfrac{dE}{dv} = \dfrac{(0.1u)(3.63aLu^2) - 1.331aLu^3}{(0.1u)^2}$

$v = 2u$,

$\dfrac{dE}{dv} = \dfrac{(u)(12aLu^2) - 8aLu^3}{u^2}$

Question 10

(a) (i) Let A_n be the amount owing after n repayments.

$A_1 = 120\,000\left(1+\frac{0.06}{k}\right) - F$

$A_2 = \underbrace{120000\left(1+\frac{0.06}{k}\right)^2}_{\text{principal (has been compounded twice)}} - \left(\underbrace{F}_{\text{second payment (hasn't received any interest)}} + \underbrace{F\left(1+\frac{0.06}{k}\right)}_{\text{first payment (has received interest once)}}\right)$

$\frac{0.06}{k}$ because money is paid k times a year

Alternatively $A_2 = A_1\left(1+\frac{0.06}{k}\right) - F$.

(ii) Continuing this pattern,

$A_n = 120\,000\,\alpha^n - (F + F\alpha + F\alpha^2 + \dots + F\alpha^{n-1})$

$= 120\,000\,\alpha^n - F(\underbrace{1+\alpha+\alpha^2+\dots+\alpha^{n-1}}_{GP,\ a=1,\ r=\alpha})$. $S_n = \dfrac{a(r^n-1)}{r-1} = \dfrac{\alpha^n-1}{\alpha-1} = \dfrac{k\alpha^{n-1}}{0.06}$

$= 120\,000\,\alpha^n - \dfrac{kF(\alpha^n-1)}{0.06}$

(a) (iii) we want it to be gone in 25 years.

i.e. we make $4 \times 25 = 100$ payments.

$A_{100} = 0$

$\alpha = 1 + \frac{0.06}{k} = 1 + \frac{0.06}{4} = 1.015$

using the formula in (ii):

$$0 = 120\,000 \times 1.015^{100} - \frac{4F(1.015^{100} - 1)}{0.06}$$

$$\times 0.06: \quad 0 = 0.06 \times 120\,000 \times 1.015^{100} - 4F(1.015^{100} - 1)$$

$$4F(1.015^{100} - 1) = 0.06 \times 120\,000 \times 1.015^{100}$$

$$F = \frac{0.06 \times 120\,000 \times 1.015^{100}}{4(1.015^{100} - 1)}$$

Now there is no other choice except to use this wonderful thing called "calculator". $F = 2\,324$ dollars (2324.468548)

(iv) $k = 12$. $n = 12 \times 25 = 300$.

$\alpha = 1 + \frac{0.06}{12} = 1.005$

$$0 = 120\,000 \times 1.005^{300} - \frac{12F(1.005^{300} - 1)}{0.06}$$

$$F = \frac{120\,000 \times 1.005^{300} \times 0.06}{12(1.005^{300} - 1)} = 773.1616818$$

$= 773$ dollars.

when $k = 4$, total payments $= nF$

$= 100F$

$= 232446.8548$

when $k = 12$, total payments $= 300F$

$= 231948.5045 \Leftarrow$ don't do 300×773.

$\therefore$ Barbara would have saved

$\$ 498$

(b) (i) $l = XP + PM$

so let's find XP.

from ΔOXP, $XP = \sqrt{r^2 - x^2}$, and this appears in the given result so it's a good sign.

now, $PM = 2 - PA$ (length of rope is 2)

using ΔAXP, $PA = \sqrt{XP^2 + XA^2}$, $XA = 1 - x$ (length $OA = 1$)

$= \sqrt{(r^2 - x^2) + (1-x)^2}$

$= \sqrt{1 - 2x + r^2}$

$l =$ the thing given.

Question 10 2003HSC M2U p12

(b)(ii) The given result looks so scary. Let's differentiate the terms in l one by one.

$\frac{d}{dx}(2) = 0$

$\frac{d}{dx}\sqrt{r^2-x^2} = \frac{d}{dx}(r^2-x^2)^{1/2}$; r is a constant

$= \frac{1}{2}(r^2-x^2)^{-1/2} \times -2x$

$\frac{d}{dx}\left(-\sqrt{1-2x+r^2}\right) = -\frac{1}{2}(1-2x+r^2)^{-1/2} \times -2$

$\therefore \frac{dl}{dx} = \frac{-x}{\sqrt{r^2-x^2}} + \frac{1}{\sqrt{1-2x+r^2}}$ which doesn't look bad at all...

$= \frac{-x\sqrt{1-2x+r^2}+\sqrt{r^2-x^2}}{\sqrt{r^2-x^2}\sqrt{1-2x+r^2}}$ (combining the fractions)

By inspection of the denominator in the given result, we multiply top and bottom by $\left(\sqrt{r^2-x^2}+x\sqrt{1-2x+r^2}\right)$.

top $\Rightarrow$ $\left(-x\sqrt{1-2x+r^2}+\sqrt{r^2-x^2}\right)\left(\sqrt{r^2-x^2}+x\sqrt{1-2x+r^2}\right)$

this is of the form $(-a+b)(b+a)$ or $(b-a)(b+a)$
$= b^2-a^2$.

$= (r^2-x^2)-x^2(1-2x+r^2)$ as required.

(iii) M is closest to floor when l is greatest, i.e. at max TP.

$\frac{dl}{dx} = 0$ when the numerator in the given result in (ii) equals zero.

In (iii) we're given the factored form of this numerator.

$(x-1)(2x^2-r^2x-r^2) = 0$

either $(x-1)=0$ or $(2x^2-r^2x-r^2)=0$

$x=1$

but $x \leq r$ from ΔOXP and $r<1$ (given). so $x \neq 1$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{r^2 \pm \sqrt{r^4-4(2)(-r^2)}}{2(2)}$

$= \frac{r}{4}\left(r \pm \sqrt{r^2+8}\right)$

but $r < \sqrt{r^2+8}$, so if it's minus, $x<0$.

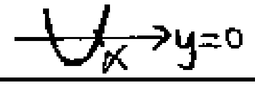
$\therefore x = \frac{r}{4}\left(r+\sqrt{r^2+8}\right)$
$= \alpha$

denominator in $\frac{dl}{dx} \geq 0$

numerator is $(x-1)(2x^2-r^2x-r^2)$

$(x-1)<0$ since $x<1$

$y=(2x^2-r^2x-r^2)$ is concave up parabola :



$x=\alpha$ is a max TP because when $x=\alpha-\epsilon$, $\frac{dl}{dx}>0$ when $x=\alpha+\epsilon$, $\frac{dl}{dx}<0$.