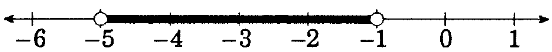
2001 Higher School Certificate Solutions
MATHEMATICS

QUESTION 1

(a) $\sqrt{\dfrac{3^2+12^2}{231-12^2}} = \sqrt{\dfrac{153}{87}}$
$= 1{\cdot}326\,12$
$= 1{\cdot}33$ (correct to 3 sig. figs).

(b) $|x+3| < 2$
$-2 < x+3 < 2$
$-5 < x < -1.$

OR $x+3 < 2$ or $-x-3 < 2$
$x < -1$ or $-x < 5$
$x > -5.$
$\therefore\ -5 < x < -1.$



(c) $x^2-2x-8 = 0$
$(x-4)(x+2) = 0$
$x = 4$ or $x = -2.$

OR $x = \dfrac{2 \pm \sqrt{4+32}}{2}$
$= \dfrac{2 \pm 6}{2}$
$= 4$ or $-2.$

(d) Primitive of $3 + \dfrac{1}{x} = 3x + \ln x + c.$

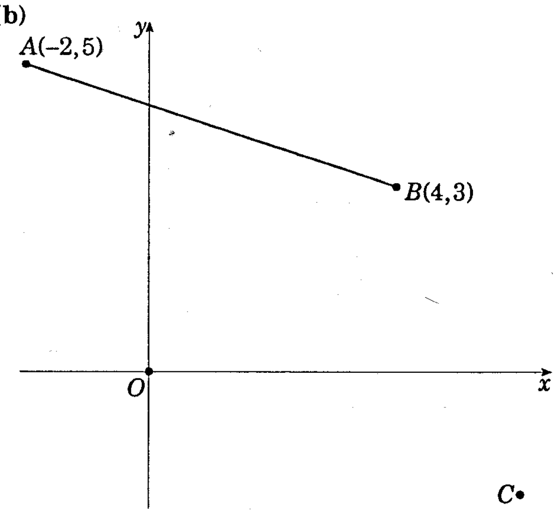
(e) $\dfrac{x}{x^2-4} + \dfrac{2}{x-2} = \dfrac{x}{(x-2)(x+2)} + \dfrac{2}{x-2}$
$= \dfrac{x+2(x+2)}{(x-2)(x+2)}$
$= \dfrac{3x+4}{(x-2)(x+2)}$
$= \dfrac{3x+4}{x^2-4}.$

(f) Let $\$x$ be the original cost.
$x + 10\%$ of $x = 979$
$x + 0{\cdot}1x = 979$
$1{\cdot}1x = 979$
$x = 890.$
$\therefore$ The original cost is \$890.

QUESTION 2

(a) $y = x^2+3x$
$\dfrac{dy}{dx} = 2x+3.$
At $x = 1$, $\dfrac{dy}{dx} = 5,$
$\therefore$ gradient of tangent $= 5.$
The tangent is $y-4 = 5(x-1)$
$y = 5x-1.$

(b)



(i) $m_{AB} = \dfrac{5-3}{-2-4} = -\dfrac{1}{3}.$
The equation of AB is
$y-3 = -\frac{1}{3}(x-4)$
$3y-9 = -x+4$
$x+3y-13 = 0.$

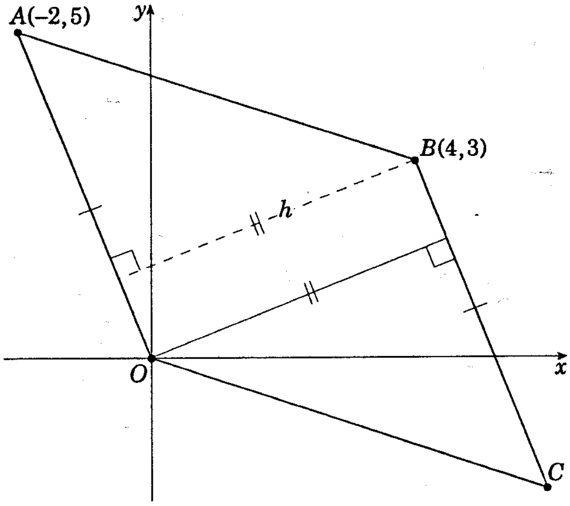
(ii) $AB = \sqrt{(-2-4)^2+(5-3)^2}$
$= \sqrt{40}$
$= 2\sqrt{10}.$

(iii) Perpendicular distance
$= \dfrac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$

/continued . . .

$$= \frac{|1\times 0 + 3\times 0 - 13|}{\sqrt{1^2+3^2}}$$
$$= \frac{13}{\sqrt{10}}.$$

(iv) Area $=$ base $\times$ height
$$= 2\sqrt{10}\times\frac{13}{\sqrt{10}}$$
$= 26$ square units.



The perpendicular distance from O to BC equals the distance from B to OA.

$CB = OA$

$$OA = \sqrt{(-2-0)^2+(5-0)^2}$$
$$= \sqrt{29}.$$

Area $=$ base $\times$ height

$26 = \sqrt{29}\times h$

$$\therefore\ h = \frac{26}{\sqrt{29}}.$$

That is, perpendicular distance from O to $BC = \dfrac{26}{\sqrt{29}}.$

QUESTION 3

(a)
$$\int_0^1 \frac{dx}{x+4} = \Big[\ln(x+4)\Big]_0^1$$
$$= \ln 5 - \ln 4$$
$$= \ln\frac{5}{4}.$$

(b) $S = kM^{\frac{2}{3}}$, $M = 70$, $S = 18\ 600$.

$$18\ 600 = k\times 70^{\frac{2}{3}}$$
$$k = \frac{18\ 600}{70^{\frac{2}{3}}}$$
$= 1095{\cdot}08$ (2 decimal places).

For $M = 60$,
$$S = k\times 60^{\frac{2}{3}}$$
$$= 1095{\cdot}08\times 60^{\frac{2}{3}}$$
$$= 16\ 783{\cdot}463\ \ldots$$
$= 16\ 783\ \text{cm}^2$ (nearest square centimetre).

(c) (i)
$$\frac{d}{dx}\left[\ln\left(x^2-9\right)\right] = \frac{2x}{x^2-9}.$$

(ii)
$$\frac{d}{dx}\left(\frac{x}{e^x}\right) = \frac{e^x.1 - xe^x}{(e^x)^2} \quad \text{(using quotient rule)}$$
$$= \frac{e^x(1-x)}{e^{2x}}$$
$$= \frac{1-x}{e^x}.$$

(d)
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$13^2 = 7^2 + x^2 - 2\times 7\times x\times\cos 60^\circ$$
$$169 = 49 + x^2 - 7x, \quad \left(\cos 60^\circ = \frac{1}{2}\right)$$
$$\therefore\quad x^2 - 7x = 120.$$
$$x^2 - 7x - 120 = 0$$
$$(x-15)(x+8) = 0$$
$\therefore\ x = 15$ or $x = -8$.

Since x is a length, $x > 0$.

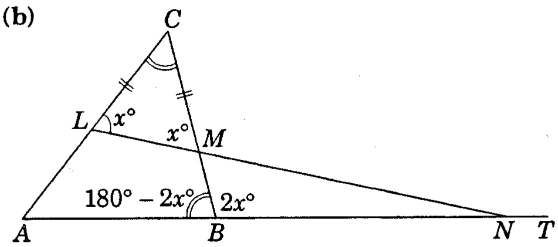
$\therefore\ x = 15.$

QUESTION 4

(a) $3x^2 + 2x + k = 0$
$$\Delta = b^2 - 4ac$$
$$= 2^2 - 4\times 3\times k$$
$$= 4 - 12k.$$

For no real roots $\Delta < 0$,
$$\therefore\quad 4 - 12k < 0$$
$$-12k < -4$$
$$k > \frac{1}{3}.$$

(b)



(i) In ΔCLM, with $CL = CM$,

$\angle CML = \angle CLM$

$= x^\circ$ (Base $\angle$s of isosceles Δ)

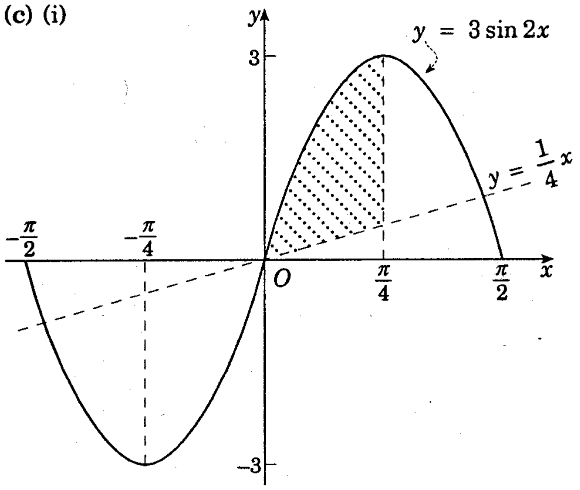
$\therefore\ \angle LCM = 180^\circ - 2x^\circ$ ($\angle$ sum of ΔLCM)

$\angle ABC = \angle LCM$ (Given, base $\angle$s of ΔABC)

$= 180^\circ - 2x^\circ.$

(ii) $\angle NBM = 2x°$ (Adj. $\angle$s on a straight line.)
$\angle BMN = \angle MMC$ (Vertically opp. $\angle$s.)
$= x°$
$\therefore\ \angle TNL = x° + 2x°$ (Ext. $\angle$ of ΔMBN)
$= 3x°$.

(c) (i)



(ii) See shaded region on the diagram.

(iii) $\int_0^{\frac{\pi}{4}} 3\sin 2x - \frac{1}{4}x \ dx$

$$= -\frac{3}{2}\cos 2x - \frac{1}{8}x^2\Big]_0^{\frac{\pi}{4}}$$

$$= \left[-\frac{3}{2}\cos\left(2\times\frac{\pi}{4}\right) - \frac{1}{8}\times\left(\frac{\pi}{4}\right)^2\right] - \left[-\frac{3}{2}\cos(2\times 0) - \frac{1}{8}\times 0^2\right]$$

$$= -\frac{3}{2}\cos\frac{\pi}{2} - \frac{\pi^2}{128} + \frac{3}{2}\cos 0$$

$$= -\frac{\pi^2}{128} + \frac{3}{2}.$$

QUESTION 5

(a) Domain: $-5 \le x \le 5$
Range: $0 \le y \le 10$

(b) (i) $\log_{10} 2^{1000} = 1000\log_{10} 2$
$= 301{\cdot}030$ (3 decimal places).

(ii) $10^1 = 10$ (2 digits)
$10^2 = 100$ (3 digits)
$10^3 = 1000$ (4 digits)

From (i),
$2^{1000} = 10^{301{\cdot}030}$ $\left[\text{If } \log_a x = y,\ x = a^y\right]$.

$\therefore$ From the pattern above,
2^{1000} has 302 digits.

(c) $\ell = r\theta$; $\ell = 8$, $\theta = \frac{\pi}{6}$

$$8 = r\times\frac{\pi}{6}$$
$$r = \frac{48}{\pi}$$
$$= 15{\cdot}278\ \ldots$$
$= 15{\cdot}3$ cm (153 mm).

(d) (i)

x	0	4	8	12
y	0	1·3	1·7	0

$$A \doteqdot \frac{4-0}{2}(0+1{\cdot}3) + \frac{8-4}{2}(1{\cdot}3+1{\cdot}7) + \frac{12-8}{2}(1{\cdot}7+0)$$
$$\doteqdot 2\times 1{\cdot}3 + 2\times 3{\cdot}0 + 2\times 1{\cdot}7$$
$\doteqdot 12\ \text{m}^2$.

(ii) Rate $= 0{\cdot}5$ m/s
$= 30$ m/min
$= 1800$ m/h.

$\therefore$ Volume in 1 hour $= 12\times 1800$
$= 21\,600\ \text{m}^3$.

QUESTION 6

(a) $a = -1$, $d = 5$, $n = 60$.

(i) $T_n = a + (n-1)d$
$T_{60} = -1 + 59\times 5$
$= 294$.

(ii) $S_n = \frac{n}{2}(a+\ell)$
$S_{60} = \frac{60}{2}(-1+294)$
$= 8790$.

(b) $100(1{\cdot}23)^t = 100e^{\alpha t}$
$1{\cdot}23^t = e^{\alpha t}$
$\ln(1{\cdot}23)^t = \ln e^{\alpha t}$
$t.\ln(1{\cdot}23) = \alpha t.\ln e$ ($\ln e = 1$)
$\ln 1{\cdot}23 = \alpha$
$\therefore\ \alpha = 0{\cdot}207\ 014\ 1694\ \ldots$
$= 0{\cdot}207$ cm (3 sig. figs).

(c) (i) $y = x^3 + x^2 - x + 2$
$\frac{dy}{dx} = 3x^2 + 2x - 1$.

Since A and B are turning points, $\frac{dy}{dx} = 0$.
$3x^2 + 2x - 1 = 0$
$(x+1)(3x-1) = 0$
$\therefore\ x = -1$ or $x = \frac{1}{3}$.

/continued . . .

When $x = -1$, $y = (-1)^3 + (-1)^2 - (-1) + 2$
$= 3$.

$\therefore$ A is the point $(-1, 3)$.

When $x = \frac{1}{3}$, $y = \left(\frac{1}{3}\right)^3 + \left(\frac{1}{3}\right)^2 - \left(\frac{1}{3}\right) + 2$
$= 1\frac{22}{27}$.

$\therefore$ B is the point $\left(\frac{1}{3}, 1\frac{22}{27}\right)$.

(ii) Concavity is shown by values of $\frac{d^2y}{dx^2}$.

$$\frac{dy}{dx} = 3x^2 + 2x - 1$$

$$\frac{d^2y}{dx^2} = 6x + 2.$$

The graph is concave up when $\frac{d^2y}{dx^2} > 0$.

$$6x + 2 > 0$$
$$x > -\frac{1}{3}.$$

The curve is concave up for $x > -\frac{1}{3}$.

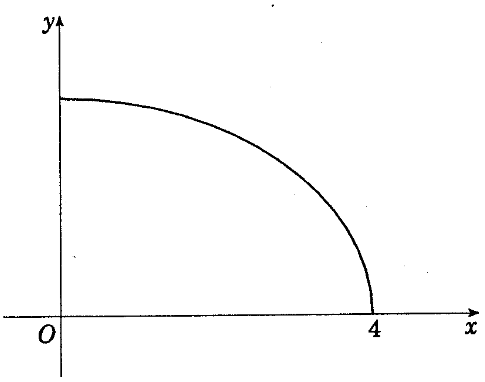
(iii) $x^3 + x^2 - x + 2 = k$ represents the intersection of $y = x^3 + x^2 - x + 2$ and $y = k$. There will be 3 real solutions for

$$1\frac{22}{27} < k < 3.$$

N.B. If you count double roots as 2 roots, this would be $1\frac{22}{27} \le k \le 3$.

QUESTION 7

(a)



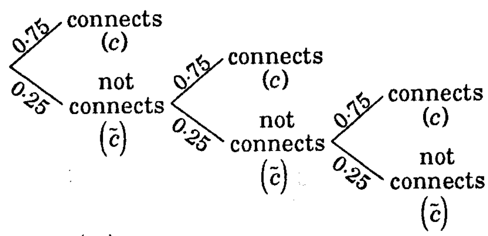
$$\frac{x^2}{2} + y^2 = 8$$

$$\therefore \quad y^2 = 8 - \frac{x^2}{2}.$$

When $y = 0$, $x = 4$.

$$\text{Volume} = \pi\int_a^b y^2\, dx$$
$$= \pi\int_0^4 8 - \frac{x^2}{2}\, dx$$
$$= \pi\left[8x - \frac{x^3}{6}\right]_0^4$$
$$= \frac{64\pi}{3} \text{ cubic units.}$$

(b)



(i) $P(\bar{c}c) = 0{\cdot}25 \times 0{\cdot}75$
$= 0{\cdot}1875$.

(ii) $P(\bar{c}\bar{c}\bar{c}) = 0{\cdot}25 \times 0{\cdot}25 \times 0{\cdot}25$
$= 0{\cdot}015\,625$.

(c) $x = \frac{t-2}{t+2}$

(i) When $t = 0$, $x = \frac{0-2}{0+2} = -1$.

(ii) $1 - \frac{4}{t+2} = \frac{t+2}{t+2} - \frac{4}{t+2}$
$= \frac{t-2}{t+2}$.

$\therefore \quad x = 1 - \frac{4}{t+2}$
$= 1 - 4(t+2)^{-1}$.

$$\text{Velocity} = \frac{dx}{dt}$$
$$= 0 - 4 \times (-1) \times (t+2)^{-2}$$
$$= 4(t+2)^{-2} \qquad \text{—①}$$
$$= \frac{4}{(t+2)^2}.$$

$$\text{Acceleration} = \frac{dv}{dx}$$
$$= 4 \times (-2) \times (t+2)^{-3}, \text{ from ①}$$
$$= \frac{-8}{(t+2)^3}.$$

(iii) No. The particle is never at rest because $v = \frac{4}{(t+2)^2}$ is always greater than zero.

(iv) $\lim_{t \to \infty} \frac{4}{(t+2)^2} = 0$,

$\therefore$ the limiting velocity is 0 m s^{-1}.

QUESTION 8

(a) $N = N_0 e^{kt}$

At $t = 0$, $N = 18$

$$\therefore \quad 18 = N_0 e^{k\times 0} = N_0 e^0 = N_0.$$

At November 1993, $t = 70$, $N = 5000$.

$$N = 18\times e^{kt}$$
$$5000 = 18e^{70k}$$
$$e^{70k} = \frac{5000}{18}$$
$$70k = \ln\left(\frac{5000}{18}\right)$$
$$k = \frac{1}{70}\ln\left(\frac{5000}{18}\right)$$
$$k = 0{\cdot}080\ 3163.$$

At November 2001, $t = 78$.

$$N = 18e^{k\times 78} = 18e^{0{\cdot}080\,383\,163\times 78} = 9511{\cdot}52.$$

$\therefore$ There will be 9512 koalas in November 2001.

(b) (i) $\mathrm{P}(A \text{ drawn first}) = \dfrac{1}{5}$.

(ii) There are $5\times 4\times 3\times 2\times 1 = 120$ possible arrangements,

$\therefore \mathrm{P}(\text{one particular arrangement}) = \dfrac{1}{120}$.

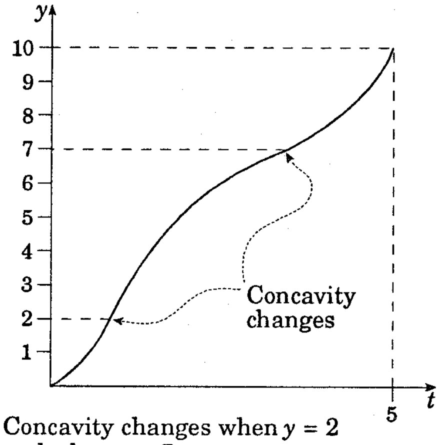
OR $\mathrm{P}(A \text{ then } B \text{ then } C \text{ then } D \text{ then } E)$

$$= \mathrm{P}(A\ 1\text{st})\times\mathrm{P}(B\ 2\text{nd})\times\mathrm{P}(C\ 3\text{rd})\times\mathrm{P}(D\ 4\text{th})\times\mathrm{P}(E\ 5\text{th})$$
$$= \frac{1}{5}\times\frac{1}{4}\times\frac{1}{3}\times\frac{1}{2}\times 1 = \frac{1}{120}.$$

(c) (i) $\dfrac{dy}{dt}$ is a maximum at $y_1 = 2$ (approximately).

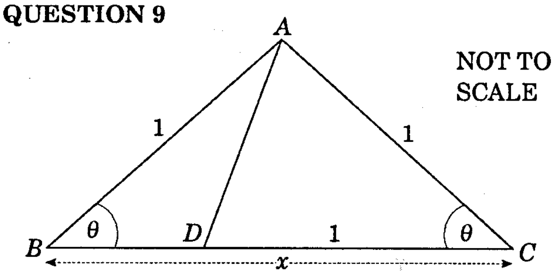
$\dfrac{dy}{dt}$ is a minimum at $y_2 = 7$ (approximately).

(ii)



Concavity changes when $y = 2$ and when $y = 7$.

QUESTION 9



(a) (i) In ΔADC,

$\angle DAC = \angle ADC$ (Opp. equal sides)

$\therefore\ 2\angle ADC + \theta = \pi$ ($\angle$ sum of a Δ is π)

$$\angle ADC = \frac{\pi-\theta}{2}.$$

But $\theta = \dfrac{\pi}{5}$,

$\therefore\ 5\theta = \pi$

$$\angle ADC = \frac{5\theta-\theta}{2} = 2\theta.$$

In ΔDBA,

$\angle BDA = \pi - 2\theta$ (BDC is a straight line)

$= 3\theta$ (Since $\pi = 5\theta$).

In ΔBAC,

$\angle BAC = \pi - 2\theta$ ($\angle$ sum of ΔABC is π)

$= 3\theta$.

$\therefore$ In ΔDBA and ΔABC,

$\angle DBA = \angle ABC$ (Common)

$\angle BDA = \angle BAC$ (Shown above)

$\Delta DBA \,|||\, \Delta ABC$ (Two $\angle$s of one Δ equal two $\angle$s of the other Δ).

(ii) In similar triangles, corresponding sides are in the same ratio.

$$\therefore \quad \frac{DB}{AB} = \frac{BA}{BC}$$
$$\therefore \quad \frac{x-1}{1} = \frac{1}{x}$$
$$x(x-1) = 1,$$

that is, $x^2 - x - 1 = 0$.

(iii) Solving the equation in (ii),

$$x^2 - x - 1 = 0$$
$$x = \frac{-(-1)\pm\sqrt{(-1)^2 - 4\times 1\times(-1)}}{2\times 1} = \frac{1\pm\sqrt{5}}{2}.$$

But x is the length of a side,

$$\therefore\ x = \frac{1+\sqrt{5}}{2} \text{ since } \frac{1-\sqrt{5}}{2} < 0.$$

In ΔABC, $\angle ACB = \dfrac{\pi}{5}$.

By the cosine rule in ΔABC,

$$\cos\angle ACB = \frac{1^2 + x^2 - 1^2}{2\times 1\times x} = \frac{x}{2}.$$

/continued . . .

$$\therefore \quad \cos\frac{\pi}{5} = \frac{1}{2} \times \frac{1+\sqrt{5}}{2},$$

that is, $\cos\frac{\pi}{5} = \frac{1+\sqrt{5}}{4}$.

(b) $\frac{dV}{dt} = 2e^t + 2e^{-t}$

(i) At $t = 0$, $\frac{dV}{dt} = 2e^0 + 2e^0 = 4.$

$\therefore$ The initial rate is 4 litres per hour.

(ii) $\frac{dV}{dt} = 2e^t + 2e^{-t}$

$$V = 2e^t - 2e^{-t} + c.$$

At $t = 0$, $V = 0$.

$$\therefore \quad 0 = 2e^0 - 2e^0 + c$$
$$c = 0$$
$$\therefore \quad V = 2e^t - 2e^{-t}.$$

(iii) When $V = 3$, $3 = 2e^t - 2e^{-t}$.

Multiply by e^t: $3e^t = 2e^{2t} - 2$.

That is, $2e^{2t} - 3e^t - 2 = 0$.

(iv) Solve for t, $2\left(e^t\right)^2 - 3\left(e^t\right) - 2 = 0$.

Let $e^t = x$.

$$2x^2 - 3x - 2 = 0$$
$$(2x+1)(x-2) = 0$$
$$x = -\frac{1}{2} \quad \text{or} \quad x = 2,$$
$$\therefore e^t = -\frac{1}{2} \quad \text{or} \quad e^t = 2.$$
$$t = \ln\left(-\frac{1}{2}\right) \quad \text{or} \quad t = \ln 2$$

(no solution) $\quad t = 0{\cdot}693$ hours $= 41{\cdot}58$ minutes $\doteqdot 42$ minutes.

QUESTION 10

(a) (i) Balance at the beginning of the second year

$$= 1000 + 6\% \text{ of } 1000 - 72$$
$$= 1000 \times 1{\cdot}06 - 72$$
$$= \$988.$$

(ii)
$$B_1 = 1000 \times 1{\cdot}06 - 72.$$
$$B_2 = \left(1000 \times 1{\cdot}06 - 72\right) \times 1{\cdot}06 - 72$$
$$= 1000(1{\cdot}06)^2 - 72(1{\cdot}06) - 72$$
$$= 1000(1{\cdot}06)^2 - 72(1{\cdot}06 + 1).$$
$$B_3 = \left[1000(1{\cdot}06)^2 - 72(1{\cdot}06 + 1)\right] \times 1{\cdot}06 - 72$$
$$= 1000(1{\cdot}06)^3 - 72\left(1{\cdot}06^2 + 1{\cdot}06\right) - 72$$
$$= 1000(1{\cdot}06)^3 - 72\left(1{\cdot}06^2 + 1{\cdot}06 + 1\right).$$

$$\therefore \; B_n = 1000(1{\cdot}06)^n - 72\left(1{\cdot}06^{n-1} + 1{\cdot}06^{n-2} + \cdots + 1{\cdot}06^2 + 1{\cdot}06 + 1\right)$$
$$= 1000(1{\cdot}06)^n - 72\left(1 + 1{\cdot}06 + 1{\cdot}06^2 + \cdots + 1{\cdot}06^{n-1}\right)$$
$$= 1000(1{\cdot}06)^n - 72 \times \frac{a(r^n - 1)}{r - 1}, \quad \left[a = 1,\ r = 1{\cdot}06\right],$$
$$= 1000(1{\cdot}06)^n - 72 \times 1 \times \frac{(1{\cdot}06^n - 1)}{1{\cdot}06 - 1}$$
$$= 1000(1{\cdot}06)^n - 72\left[\frac{1{\cdot}06^n - 1}{0{\cdot}06}\right]$$
$$= 1000(1{\cdot}06)^n - 1200\left(1{\cdot}06^n - 1\right).$$
$$\therefore \; B_n = 1200 - 200(1{\cdot}06)^n.$$

(iii) Balance at the end of the 10th year:

$$B_{10} = 1200 - 200(1{\cdot}06)^{10}$$
$$= \$841{\cdot}83.$$

At the end of 1 more year:

$$B_{11} = 841{\cdot}83(1{\cdot}06) - 90$$

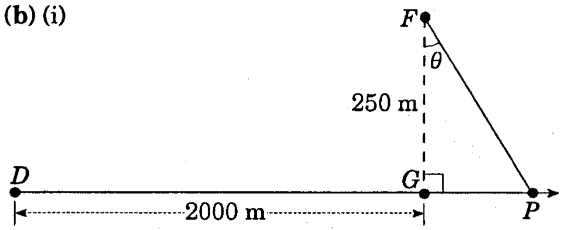
and
$$B_{12} = \left[841{\cdot}83(1{\cdot}06) - 90\right] \times 1{\cdot}06 - 90$$
$$= 841{\cdot}83(1{\cdot}06)^2 - 90(1{\cdot}06 + 1).$$
$$B_{10+n} = 841{\cdot}83(1{\cdot}06)^n - 90\left(1{\cdot}06^{n-1} + 1{\cdot}06^{n-2} + \cdots + 1{\cdot}06 + 1\right)$$
$$= 841{\cdot}83(1{\cdot}06)^n - 90\left(1 + 1{\cdot}06 + \cdots + 1{\cdot}06^{n-1}\right)$$
$$= 841{\cdot}83(1{\cdot}06)^n - 90 \times 1 \times \left(\frac{1{\cdot}06^n - 1}{1{\cdot}06 - 1}\right)$$
$$= 841{\cdot}83(1{\cdot}06)^n - 90\left(\frac{1{\cdot}06^n - 1}{0{\cdot}06}\right)$$
$$= 841{\cdot}83(1{\cdot}06)^n - 1500\left(1{\cdot}06^n - 1\right).$$
$$\therefore \; B_{10+n} = 1500 - 658{\cdot}17(1{\cdot}06)^n.$$

To find n, let $B_{10+n} = 0$.

$$0 = 1500 - 658{\cdot}17(1{\cdot}06)^n$$
$$(1{\cdot}06)^n = \frac{1500}{658{\cdot}17}$$
$$\ln(1{\cdot}06)^n = \ln\left(\frac{1500}{658{\cdot}17}\right)$$
$$n\ln(1{\cdot}06) = \ln\left(\frac{1500}{658{\cdot}17}\right)$$
$$n = \ln\left(\frac{1500}{658{\cdot}17}\right) \div \ln 1{\cdot}06$$
$$= 14.$$

$\therefore$ The prize can be awarded for 14 more years.

(b) (i)



$$\frac{250}{FP} = \cos\theta$$
$$\frac{FP}{250} = \sec\theta$$
$$FP = 250\sec\theta$$
$$GP = 250\tan\theta$$
$$DP = DG + GP$$
$$\therefore\ DP = 2000 + 250\tan\theta.$$

Let T_1 be the time taken for the bus to travel from D to P.

$$T_1 = \frac{2000+250\tan\theta}{15} \text{ seconds}$$
$$= \frac{250(8+\tan\theta)}{15\times 60} \text{ minutes}$$
$$= \frac{5(8+\tan\theta)}{18} \text{ minutes.}$$

Let T_2 be the time taken for Claire to travel from F to P.

$$T_2 = \frac{250\sec\theta}{4} \text{ seconds}$$
$$= \frac{250\sec\theta}{4\times 60} \text{ minutes}$$
$$= \frac{25\sec\theta}{24} \text{ minutes.}$$

(ii) The time the bus arrives at P is $\frac{5(8+\tan\theta)}{18}$ minutes past 8 am.

The time that Claire can leave the farm-house is $(T_1 - T_2)$ minutes past 8 am.

$$T_1 - T_2 = \left[\frac{5(8+\tan\theta)}{18} - \frac{25\sec\theta}{24}\right] \text{minutes past 8 am.}$$

Let $T = \frac{5}{18}(8+\tan\theta) - \frac{25}{24}(\sec\theta)$.

$$\frac{dT}{d\theta} = 0 + \frac{5}{18}\sec^2\theta - \frac{25}{24}\sec\theta\tan\theta$$
$$= \frac{5}{6}\sec\theta\left(\frac{1}{3}\sec\theta - \frac{5}{4}\tan\theta\right).$$

But $\frac{dT}{d\theta} = 0$ for maximum or minimum T,

$$\therefore\ \frac{5}{6}\sec\theta\left(\frac{1}{3}\sec\theta - \frac{5}{4}\tan\theta\right) = 0.$$

$\therefore\ \sec\theta = 0$ (no solution), **or**

$$\frac{1}{3}\sec\theta - \frac{5}{4}\tan\theta = 0$$
$$\frac{1}{3}\cdot\frac{1}{\cos\theta} = \frac{5}{4}\cdot\frac{\sin\theta}{\cos\theta}$$
$$\therefore\ \sin\theta = \frac{4}{15}$$
$$\theta = 0{\cdot}269\ 932\ 759 \text{ radians.}$$

$\therefore$ The time after 8 am. that Claire may leave the farmhouse is

$$\left[\frac{5}{18}(8\tan 0{\cdot}269\ 93) - \frac{25}{24}\times\frac{1}{\cos 0{\cdot}269\ 93}\right]\text{min.}$$
$$= 1{\cdot}2185 \text{ min.}$$
$$= 1 \text{ minute } 13 \text{ seconds (to the nearest second).}$$

To show that this time is a maximum,

$$\frac{dT}{d\theta} = \frac{5}{18}\sec^2\theta - \frac{25}{24}\sec\theta{\cdot}\tan\theta$$
$$\frac{d^2T}{d\theta^2} = \frac{10}{18}\sec\theta\cdot\sec\theta\cdot\tan\theta - \frac{25}{24}\sec\theta\cdot\sec^2\theta - \frac{25}{24}\sec\theta\cdot\tan^2\theta.$$

Substitute $\theta = 0{\cdot}27$ (2 decimal places):

$$\frac{d^2T}{d\theta^2} = \frac{10}{18}\times(1{\cdot}04)^2\times(0{\cdot}28) - \frac{25}{24}\times(1{\cdot}04)^3 - \frac{25}{24}\times(1{\cdot}04)\times(0{\cdot}28)^2$$
$$\doteqdot -1{\cdot}09.$$

$\therefore$ T is a maximum value (latest time).

The latest time that Claire can leave home in order to catch the bus is 1 minute 13 seconds past 8 am.

END OF MATHEMATICS SOLUTIONS