HIGHER SCHOOL CERTIFICATE EXAMINATION 1991
MATHEMATICS - 2/3 UNIT

Direction to Candidates

Time allowed - Three hours (includes reading time)

All questions may be attempted. All questions are of equal value. All necessary working should be shown in every question. Marks may not be awarded for careless or badly arranged work.

Standard integrals are provided; approved calculators may be used.

QUESTION 1

(a) Calculate $\frac{\sqrt{4.8+7.3}}{9.6}$. Round off your answer to one decimal place.

(b) Factorize $9x^2 - y^2$.

(c) Express $\frac{1}{3-\sqrt{2}}$ with a rational denominator.

(d) If $S = \frac{a}{1-r}$, find the value of a when $S = 60$ and $r = \frac{1}{2}$.

(e) Solve $\frac{2x}{3} - \frac{x}{5} = 1$.

(f) Mark on a number line the value of x for which $|x+1| \leq 3$.

QUESTION 2.

(a) On a number plane, mark the origin O and A(5, 0), B(8, 4), and C(0, 10). Join A to B, B to C and C to A.

(b) Show that the line AB has equation $3y = 4x - 20$.

(c) Show that the gradient of the line BC is $-\frac{3}{4}$.

(d) Show that AB and BC are perpendicular.

(e) Show that the length AB is 5 units.

(f) Show that the triangles ACO and ACB are congruent. Give reasons.

(g) Find the area of the quadrilateral ABCO.

(h) If D is the point (8, 0), calculate the perpendicular distance of D from the line AB.

QUESTION 3.

(a) Differentiate: (i) $3x^2 + 5x$ (ii) $\tan 3x$ (iii) $\frac{\ln x}{x}$.

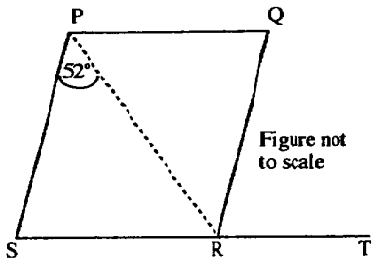
(b) Find $\int \frac{dx}{3x+5}$.

(c) Evaluate $\int_0^1 e^{4x}dx$.

(d) In the diagram PQRS is a rhombus where ∠SPR = 52° and SR is produced to T. Copy this diagram.

(i) What is the value of ∠SPQ? Give reasons

(ii) What is the value of ∠QRT? Give reasons.



QUESTION 4

(a) The focus of a parabola is S(0, 2) and its directrix is the line y = -2.

(i) Sketch the parabola and indicate the coordinates of the vertex V.

(ii) Write down the focal length of the parabola.

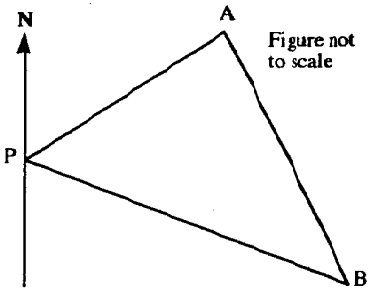
(iii) Find the equation of the parabola.

(b) Ship A is 20 nautical miles from a port P and is on a bearing of 055°. Ship B is 27 nautical miles from P and is on a bearing of 115°.

(i) Copy the diagram. On your diagram indicate the given information.

(ii) Show that ∠APB = 60°.

(iii) Use the cosine rule to determine the distance between the two ships, giving your answer correct to one decimal place.



(c) An insurance company has calculated that the probability of a woman being alive in 40 years time is 0.8 and that the probability of her husband being alive in 40 years time is 0.7. What is the probability that in 40 years time:

(i) both will be alive; (ii) only one of them will be alive?

QUESTION 5

(a) Let α and β be the roots of the equation $x^2 - 5x + 2 = 0$. Find the values of

(ii) α + β (ii) αβ (iii) (α + 1)(β + 1).

(b) The tenth term of an arithmetic sequence is 29 and the fifteenth term is 44.

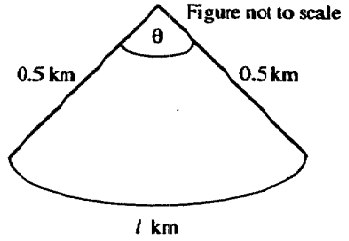
(i) Find the value of the common difference and the value of the first term.

(ii) Find the sum of the first 75 terms.

(c) A car travels at 45 km / h on a circular curve whose radius is 0.5 km.

(i) Find the distance l km, that the car travels in one minute.

(ii) Calculate the size of the angle θ through which the car turns in one minute. Give your answer to the nearest degree.



QUESTION 6

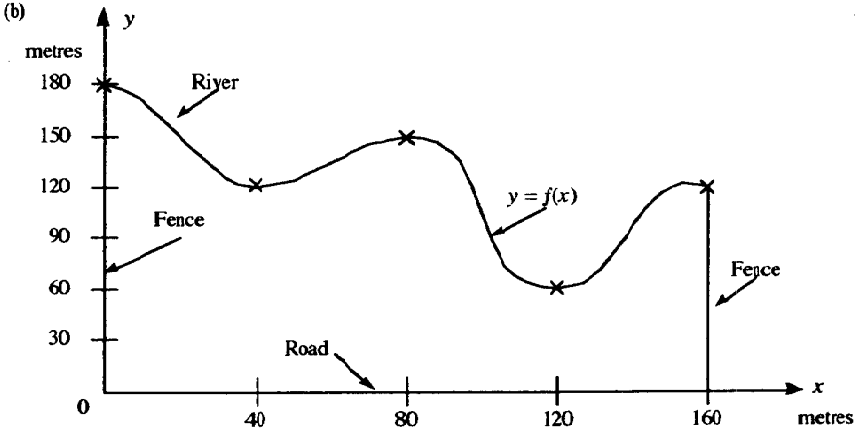
(a) Consider the curve given by $y = 3x^2 - x^3$.

(i) Find the stationary points and determine their nature.

(ii) Sketch the curve, indicating where it crosses the x-axis.

(iii) Find the equation of the tangent to the curve at the point R(-1, 4).

(b)



The diagram is a scale drawing of a paddock bounded by a river, a road, and two fences perpendicular to the road.

A farmer wishes to calculate the area of this paddock and has measured the perpendicular distances of the river from the road at intervals of 40 metres. These distances can be read off the diagram.

(i) Take the road as the x-axis, the fences as the y-axis and the line $x = 160$, and the river as $y = f(x)$. Copy and complete the following table of values.

x	0	40	80	120	160
$y = f(x)$					

(ii) Estimate the area of the paddock using Simpson's Rule with five function values.

QUESTION 7

(a) In the diagram, ABC is an isosceles triangle where AB = AC and DE is parallel to BC.

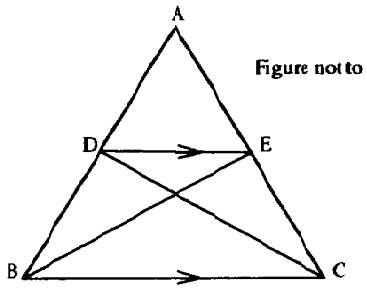


Figure not to scale

(i) Show that ADE is an isosceles triangle.

(ii) Show that DB = EC.

(iii) Show that the triangles DBC and ECB are congruent.

(b) The amount A grams of a given carbon isotope in a dead tree trunk is given by $A = A_0e^{-kt}$ where A_0 and k are positive constants, and where the time t is measured in years from the death of the tree.

(i) Show that A satisfies the equation $\frac{dA}{dt} = -kA$.

(ii) Find the value of k if the amount of isotope is halved every 5500 years.

(iii) For a particular dead tree trunk the amount of isotope is only 15% of the original amount in the living tree.

How long ago did the tree die? Give your answer to the nearest 1000 years.

QUESTION 8

(a) The shaded region in the diagram is bounded by the line $y=3x+1$, the x-axis, the y-axis, and the line $x=2$.

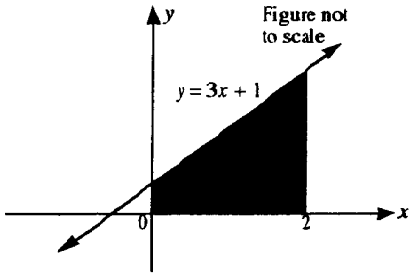


Figure not to scale

Calculate the volume of the solid of revolution formed when this region is rotated about the x-axis.

(b) A particle moves along a straight line so that its distance x, in metres, from a fixed point O is given by $x = 1-2\sin 2t$ where the time t is measured in seconds from $t = 0$.

(i) Where is the particle initially?

(ii) When, and where, does the particle first come to rest?

(iii) Where does the particle next come to rest?

(iv) What is the acceleration of the particle when $t = \frac{\pi}{12}$?

QUESTION 9

(a) (i) Graph $y = \cos x$, for $0 \le x \le 2\pi$.

(ii) On your diagram shade the regions bounded by the curve $y = \cos x$, the x-axis, and the lines $x = 0$ and $x = \pi$.

Calculate the total area of these regions.

(iii) Solve $\cos x = \frac{\sqrt{3}}{2}$ for $0 \le x \le 2\pi$.

(b) Benjamin Franklin left a will in which he established a fund of $1000 for the citizens of Boston. His instructions were that his money was to be invested at 5% interest, compounded annually.

(i) If Franklin's instructions were followed, how much money would have been in the fund 100 years after it was established?

(ii) Suppose that at the beginning of each subsequent year after establishment, a further $1000 has been added to the fund and had also earned 5% interest, compounded annually.

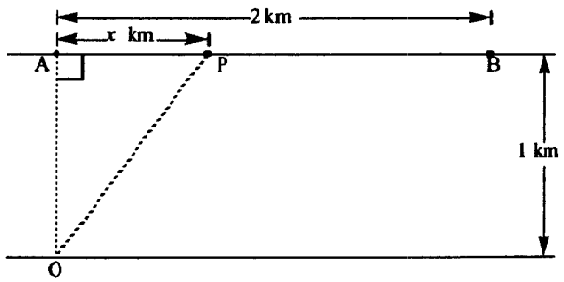
How much money would have been in the fund after 200 years, just before the next $1000 would have been added to the fund?

QUESTION 10

(a) Consider the equation $x^2 + (k+2)x + 4 = 0$. For what values of k does the equation have:

(i) equal roots; (ii) distinct real roots?

(b) The diagram shows a straight section of a river, one kilometre wide. Adrienna is at a point O on one bank and she wishes to reach a point B on the opposite bank. The point A is directly opposite O and the distance from A to B is two kilometres.



Adrienna can row at 6 km/h and jog at 10 km/h. She intends to row in a straight line to a point P on the opposite bank and then jog directly from P to B.

Let the distance AP be x kilometres.

(i) Show that the time T, in hours, that Adrienna takes to reach B is given by $T = \frac{\sqrt{x^2+1}}{6} + \frac{2-x}{10}$.

(ii) Show that if Adrienna wishes to minimize the time taken to complete the journey then she should row to a point P, $\frac{3}{4}$ kilometre from A.