HIGHER SCHOOL CERTIFICATE EXAMINATION 1993
MATHEMATICS - 2/3 UNIT

Direction to Candidates

Time allowed - Three hours (includes reading time)

All questions may be attempted. All questions are of equal value. All necessary working should be shown in every question. Marks may not be awarded for careless or badly arranged work

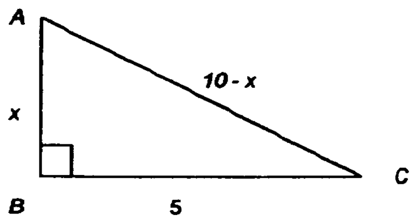
Standard integrals are provided; approved calculators may be used.

QUESTION 1.

(a) Find the value of $4\pi\sqrt{\frac{a}{g}}$ if $a = 4.1$ and $g = 9.8$. Give your answer to 2 significant figures.

(b) The line $6x - ky = 4$ passes through the point $(3, 2)$. Find the value of k.

(c)



In the diagram $\angle ABC$ is a right angle.

Find the value x.

(d) There are 7 routes from P to Q and 5 routes from Q to R. How many routes are there from P to R through Q?

(e) Solve the pair of simultaneous equations: $2x + y = 7$, $x - 2y = 1$.

(f) Find those values of x which satisfy the inequality $5 - 3x < 7$.

QUESTION 2

(a) (i) On a number plane, mark the origin O and the points $A(2,1)$ and $B(3, -1)$.

(ii) Find the gradients m_1 of OA and m_2 of AB.

(iii) Show that OA is perpendicular to AB.

(iv) Show that $OA = AB$.

(v) Find the midpoint D of the interval OB.

(vi) Find the coordinates of the point C such that D is the midpoint of AC.

(vii) What shape best describes the geometric figure $OABC$?

(b) Seventy-five tagged fish were released into a dam known to contain fish. Later a sample of forty-two fish was netted from this dam and then released. Of these forty-two fish it was noted that five were tagged. Estimate the total number of fish in the dam.

QUESTION 3.

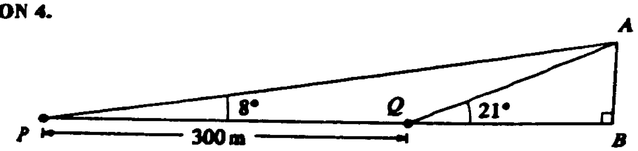
(a) Differentiate: (i) $\sin(3x + 1)$ (ii) $\frac{1}{x^3}$ (iii) $x^2 \ln x$.

(b) 'Although the amount of pollution is increasing, the Government's policies to reduce pollution seem to be taking effect.' Given that P is the amount of pollution, what does the above statement imply about $\frac{dP}{dt}$ and $\frac{d^2P}{dt^2}$?

(c) Find: (i) $\int (x-1)^3 dx$ (ii) $\int_0^{\frac{\pi}{2}} \sin 2x dx$.

QUESTION 4.

(a)



From position P, Anne finds that the angle of elevation of the top A of a rock pillar AB is 8°. After walking 300 m directly towards the pillar to the point Q she finds that the angle of elevation of A is 21°.

(i) Copy the diagram and find $\angle PAQ$.

(ii) Calculate the length AQ.

(iii) Find the height of the rock pillar AB.

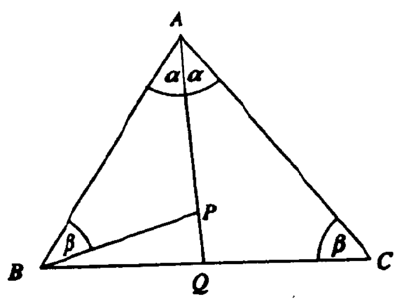
(b) Under certain climatic conditions the number N of blue-green algae satisfies the equation $N(t) = Ae^{0.15t}$, where t is measured in days and A is a constant.

(i) Show that the number of algae increases at a rate proportional to the number present.

(ii) When $t = 3$ the number of algae was estimated to be 1.7×10^8. Evaluate A.

(iii) The number of algae doubles every x days. Find x.

(c)



In the diagram $\angle BAQ = \angle QAC = \alpha$ and $\angle ABP = \angle QCA = \beta$

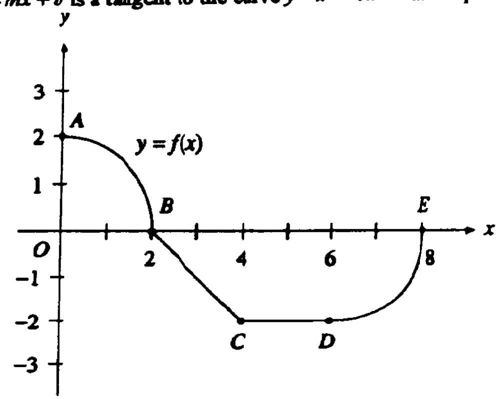
(i) Copy the diagram.

(ii) Prove that $BP = BQ$

QUESTION 5

(a) The line $y = mx + b$ is a tangent to the curve $y = x^3 - 3x + 1$ at the point (-2, -1). Find m and b.

(b)

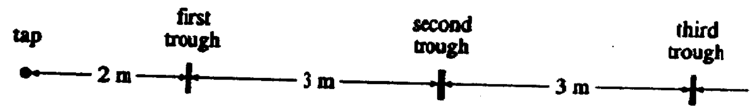


The graph of the function f consists of a quarter circle AB, a straight line segment BC, a horizontal straight line segment CD, and a quarter circle DE as shown above.

(i) Evaluate $\int_0^8 f(x)dx$.

(ii) For what values of x satisfying $0<x<8$ is the function f NOT differentiable?

(c)



A tap and n water troughs are in a straight line. The tap is first in line, 2 metres from the first trough, and there is 3 metres between consecutive troughs. A stable hand fills the troughs by carrying a bucket of water from the tap to each trough and then returning to the tap. Thus she walks $2 + 2 = 4$ metres to fill the first trough, 10 metres to fill the second trough, and so on.

(i) How far does the stable hand walk to fill the kth trough?

(ii) How far does the stable hand walk to fill all n troughs?

(iii) The stable hand walks 1220 metres to fill all the troughs. How many water troughs are there?

QUESTION 6

(a) Consider the curve given by $y = \frac{1}{4}x^4 - x^3$.

(i) Find any turning points and determine their nature.

(ii) Find any points of inflexion.

(iii) Sketch the curve for $-1.5 \le x \le 4.5$, indicating where the curve crosses the x axis.

(iv) For what values of x is the curve concave down?

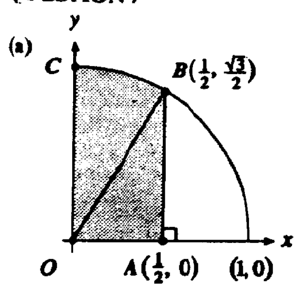
(b) The region beneath the curve $y = e^{-x}$ which is above the x axis and between the lines $x = 0$ and $x = 1$ is rotated about the x axis.

(i) Sketch the region.

(ii) Find the volume of the resulting solid of revolution.

QUESTION 7

(a)



The diagram shows the first quadrant of the circle $x^2 + y^2 = 1$. The point A has coordinates $\left(\frac{1}{2}, 0\right)$ and AB is perpendicular to the x axis.

(i) What is the exact value of $\angle COB$?

(ii) Show that the exact value of the shaded area $OABC$ is $\frac{2\pi + 3\sqrt{3}}{24}$.

(b) (i) Copy and complete the table of values for $y = \sqrt{1-x^2}$

x	0	0.125	0.25	0.375	0.5
y	1		0.968		0.866

(ii) By using Simpson's rule with 5 function values, estimate the integral $\int_0^{\frac{1}{2}} \sqrt{1-x^2}\,dx$.

(iii) By referring to (ii) of part (a), find an approximate value for π.

(c) The die used in a new game has 20 faces. Each face has a different letter of the alphabet marked on it, however the letters Q, U, V, X, Y and Z have not been used.

(i) The die is rolled twice. What is the probability that the same letter appears on the upper face twice?

(ii) The die is rolled three times. What is the probability that the letter E appears on the upper face exactly twice?

QUESTION 8

(a) Let A and B be the fixed points (-1, 0) and (2, 0) and let P be the variable point (x, y).

(i) Write down expressions for PA^2 and PB^2 in terms of x and y.

(ii) Suppose that P moves so that $PA = 2\,PB$.
Deduce that P moves on a circle.

(iii) Find the centre and radius of this circle.

(b) Ezzat invests \$50 000 in an account which earns 8% interest, compounded annually. He intends to withdraw \$$M$ at the end of each year, immediately after the interest has been paid. He wishes to be able to do this for exactly 20 years, so that the account will then be empty.

(i) How much money does he have in the account immediately after he has made his first withdrawal?

(ii) Write an expression in terms of M for the amount of money in the account, immediately after his 20th withdrawal.

(iii) Calculate the value of M which leaves his account empty after the 20th withdrawal.

(iv) Suppose Ezzat wished to be able to withdraw \$8000 per year for the 20 years. By using your calculator alone, estimate, to the nearest per cent, the interest rate he would then need to earn on his account.

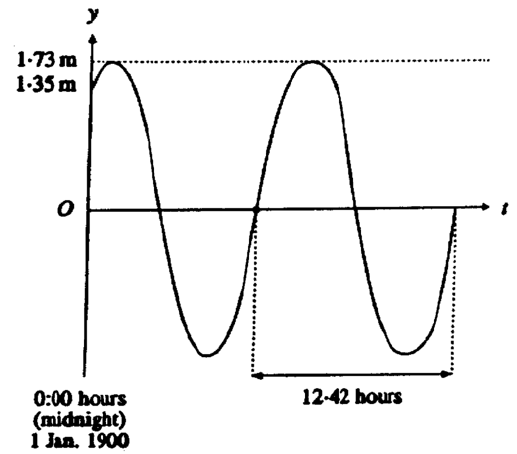
QUESTION 9

(a) 'The car moved away from where it had stopped, its speed increasing at a constant rate, and after exactly 10 seconds it was travelling at 25 m/s. It continued at a constant speed for a further 20 seconds. Then the brakes were applied causing it to slow down at a constant rate, so that 5 seconds later it was travelling at 5 m/s.'

(i) Let the car's speed be v m/s. Graph v as a function of time t, measured in seconds.

(ii) Let the distance travelled by the car be s metres from where it had stopped. On a separate diagram, graph s as a function of time t.

(b)



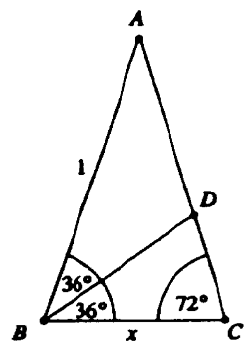
The diagram shows the tidal effect due to the Moon at Port Hedland on 1 January 1900. The water level can be approximated by a sine curve of the form $y = A \sin(at + b)$, where y is the water level in metres measured as on the diagram and t is the time in hours after 0:00 hours.

(i) Find the amplitude A. (ii) Estimate b by letting $t = 0$. (iii) Estimate a.

(c) Solve the equation $2 \ln x = \ln(5 + 4x)$.

QUESTION 10.

(a)



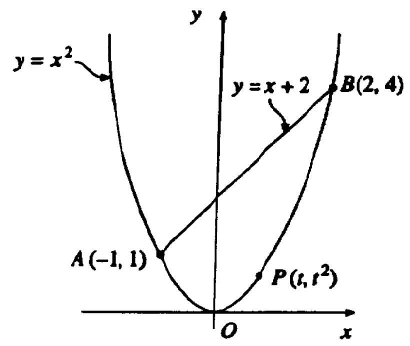
In the diagram ABC is an isosceles triangle where $\angle ABC = \angle BCA = 72^{o}$ and $AB = AC = 1$.
Angle ABC is bisected by BD, and $BC = x$.

(i) Copy the diagram.

(ii) Show that triangles ABC and BCD are similar.

(iii) By using (ii) find the exact value of x.

(b) In the diagram below, $A(-1, 1)$ and $B(2, 4)$ are the points of intersection of the parabola $y = x^2$ with the line $y = x + 2$. The point $P(t, t^2)$ is the variable point on the parabola below the line.



(i) Find the area of the parabolic segment APB, i.e. the area below the line and above the parabola.

(ii) Show that the maximum area of triangle APB is three-quarters of the area of the parabolic segment APB.