N.S.W. DEPARTMENT OF EDUCATION
HIGHER SCHOOL CERTIFICATE EXAMINATION 1977
MATHEMATICS - 2 UNIT COURSE

TIME 3 HOURS

Instructions All questions may be attempted. All questions are of equal value. In every question, all necessary working should be shown. Marks will be deducted for careless or badly arranged work.

QUESTION 1

(i) (a) Express $\sqrt{45} + \sqrt{80}$ in the form $a\sqrt{5}$, and find a

(b) Express $\dfrac{4+\sqrt{3}}{2+\sqrt{3}}$ in the form $b - 2\sqrt{3}$, and find b

(ii) Find the discriminant of the following equation and state whether the roots are real: $2x^2 + 3x + 5 = 0$

(iii) In the Cartesian plane, indicate, by shading, the region whose points simultaneously satisfy the inequalities: $x + y \leq 3$, $x - y \geq 1$.

QUESTION 2

(i) Find the axis of symmetry and the vertex of the parabola $y = 5x^2 + 10x + 2$.

(ii) Find the acute angle between the lines $2x + 3y = 13$, $2x - 5y = 5$.

(iii) Find the perpendicular distance from the point (2, -6) to the line $4x - 3y + 9 = 0$.

QUESTION 3

(i) Differentiate with respect to x: (a) $2x^2 - 6x + 7$ (b) $\sqrt{x^2 + 5}$ (c) $\sin\left(\frac{x}{2}\right)$

(ii) Evaluate $\lim\limits_{x \to 0} \dfrac{\sin 3x}{x}$

(iii) Find the minimum value of $x \log_e x$ (where $x > 0$) correct to 3 decimal places.

QUESTION 4

(i) If $\log_e 3 = 1.0986123$ and $\log_e 2 = 0.693\,147\,2$ evaluate $\log_e 12$ correct to 5 decimal places.

(ii) Solve the inequality $|x - 5| > 9$

(iii) If $f(x) = ax^2 + bx + c$ find the value of $f(x) - f(-x)$

(iv) The area under the curve $y = 2x - x^2$ between $x = 0$ and $x = 2$ is rotated about the x-axis through one complete revolution. Find the volume of the solid so formed.

QUESTION 5

(i) Find primitives (i.e. indefinite integrals) of: (a) $x^{1/3}$ (b) $x^{-1/3}$ (c) e^{2x}

(ii) Evaluate (a) $\int_2^7 \frac{4}{x+3}dx$ (b) $\int_0^{\pi/3} \sin 2x\, dx$

§ (iii) Use the mid-ordinate rule with one strip to find an approximate value of $\int_2^4 \frac{\log_e x}{x}dx$

QUESTION 6

¶ (i) If $\sin A + \sin 40° = 2 \sin 50° \cos 10°$ and A is acute, find A.

(ii) A triangular piece of land is enclosed by three sections of straight road. Two of these sections are 2.1 km and 3.4 km, and meet at an angle of 67°. Find the length of the third side to two significant figures.

(iii) Draw separate sketches (not on graph paper) of:

(a) $y = 2\cos x$ for $-2\pi \le x \le 2\pi$ (b) $y = 2 + 2\cos x$ for $-2\pi \le x \le 2\pi$

QUESTION 7

(i) Show that the points A(3, -1), B(7, 2) and C(1, 10) are the vertices of a right angled triangle. Also find the area of the triangle ABC.

(ii) The tangent at the point L(3, 4) on the curve $y = \frac{12}{x}$ cuts the x-axis at P and the y-axis at Q. Find the coordinates of P and Q. In what ratio does L divide the interval PQ?

QUESTION 8

(i) A particle moves along the x-axis with acceleration at time t equal to $12(t+3)^2$. If the particle is initially at rest at the origin, (i.e. $x = \frac{dx}{dt} = 0$ at $t = 0$), find its position when $t = 2$.

(ii) Show that of all rectangles with perimeter 36 cm, the one which has maximum area is a square.

QUESTION 9

(i) Comment on the following reasoning:

"When two coins are tossed they can either fall as two heads or two tails or as a head and a tail. As there are three possibilities, the probability of two heads is $\frac{1}{3}$."

(ii) If three coins are tossed, what is the probability of their falling as two heads and a tail?

(iii) In a certain school the senior students are divided into two classes A and B. Class A contains 11 female and 9 male students and class B contains 8 female and 12 male students. A certain occasion requires two speakers, one from each class. These are selected at random. Find the probabilities that:

(a) both speakers are male;

(b) both speakers are female;

(c) one speaker is male and one is female.

QUESTION 10

(i) (a) Sum the arithmetic series $1 + 3 + 5 + \ldots$ to n terms.

(b) Sum the arithmetic series $2 + 4 + 6 + \ldots$ to n terms.

(c) Find the sum of the first 2000 terms of the series $1 - 2 + 3 - 4 + 5 - 6 + \ldots + (-1)^{n+1} n + \ldots$

(ii) If $\frac{dy}{dt} = -4y$ and $y = 10$ when $t = 0$

(a) express y as a function of t;

(b) find the value of t for which $y = 0.1$;

(c) if $y = \frac{dx}{dt}$ and $x = 10$ when $t = 0$, find and expression for x in terms of t.