HIGHER SCHOOL CERTIFICATE 1994
MATHEMATICS 2/3 UNIT (COMMON)

DIRECTIONS TO CANDIDATES

Time allowed - Three hours (plus 5 minutes' reading time).
Attempt ALL questions.
ALL questions are of equal value.
All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
Standard integrals are given.
Board-approved calculators may be used.

QUESTION 1.

(a) Find the value of $\dfrac{4.23}{\sqrt{6.14 - 1.78}}$. Give your answer correct to 2 decimal places.

(b) Simplify $2 - 3(x - 5)$.

(c) Differentiate $3x^5 - 4x + 7$.

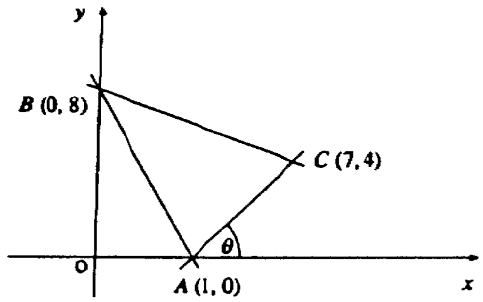
(d) Solve $|x - 1| = 4$.

(e) The distance from Earth to the Sun is 149 492 000 km. Write this number in scientific notation, correct to 4 significant figures.

(f) Kim invests \$1000 at 8% per year compound interest, compounded quarterly. Calculate the value of the investment after 5 years.

QUESTION 2.

NOT TO SCALE



The points A, B and C have coordinates (1,0), (0,8), and (7,4), as shown in the diagram
The angle between the line AC and the x-axis is θ.

(a) Find the gradient of the line AC.

(b) Calculate the size of the angle θ in degrees.
(c) Find the equation of the line AC.

(d) Find the coordinates of D, the midpoint of AC.

(e) Show that AC is perpendicular to BD.

(f) What does part (e) show about ΔABC?

(g) Find the area of ΔABC.

(h) Write down the coordinates of a point E such that ABCE is a rhombus.

QUESTION 3.

(a) Differentiate: (i) $\dfrac{1}{x^4}$ (ii) $\cos(x^2)$ (iii) $x\tan x$.

(b)

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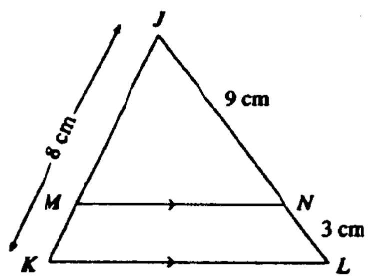
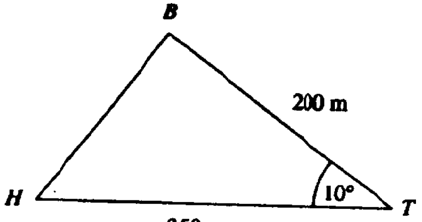


The diagram shows a triangle JKL. MN || KL, JK = 8 cm, JN = 9 cm, and NL = 3 cm.
(i) Prove that ΔJMN is similar to ΔJKL.
(ii) Find the length of MK.

(c) Find: (i) $\int e^{2x}\,dx$ (ii) $\int_1^9 (1 + \sqrt{x})\,dx$.

QUESTION 4.

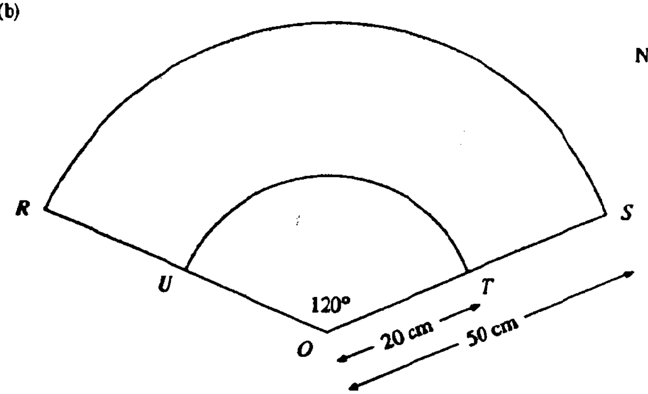
(a)



NOT TO SCALE

On a golf course, the distance from a tee T to the hole H is 350 m. A golfer's ball comes to rest at point B, 200 m from T. Angle HTB is 10°, as shown in the diagram.
How far is B from H?

(b)



NOT TO SCALE

A car windscreen wiper traces out the area RSTU where RS and UT are arcs of circles centre O, radii 50 cm and 20 cm respectively, as shown in the figure.
Calculate the area RSTU.

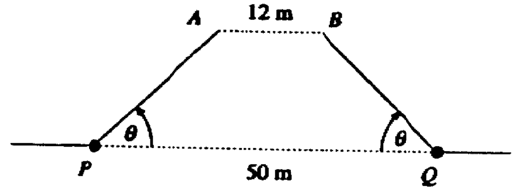
(c) Given that $\frac{d}{dx}(e^{x^2}) = 2xe^{x^2}$, evaluate $\int_0^1 xe^{x^2}\,dx$.

(d) The positive multiples of 7 are 7, 14, 21,....
(i) What is the largest multiple of 7 less than 1000?
(ii) What is the sum of the positive multiples of 7 which are less than 1000?

QUESTION 5.

(a) Find the values for m for which $12 + 4m - m^2 > 0$.

(b)



NOT TO SCALE

The figure shows the side view of a bridge opened to let boats pass underneath. When the equal arms of the bridge PA and QB are lowered, they meet exactly to form the straight roadway PQ, which is 50 m long. When the arms PA and QB are raised through an angle θ as shown, the 'corridor' AB is 12 m wide.
Calculate the size of angle θ.

(c) A particle moves along a straight line so that its distance x metres from a fixed point O is given by

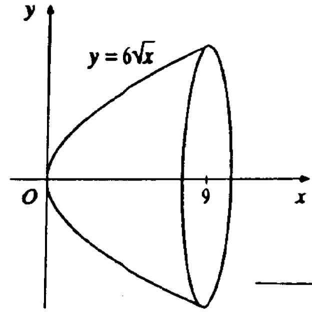
$$x = 6 - 2t + 8\ \ln(t+3),$$

where the time t is measured in seconds.

(i) What is the position of the particle when $t = 0$?
(ii) Find expressions for the velocity and acceleration of the particle at time t.
(iii) Find the time t when the velocity of the particle is zero.

(d) The region enclosed by the curve $y = 6\sqrt{x}$ and the x-axis between $x = 0$ and $x = 9$ is rotated about the x-axis as shown.

Find the volume of the solid of revolution.

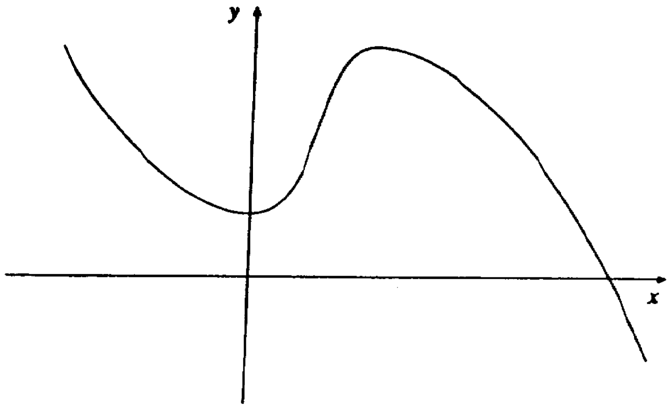


QUESTION 6.

(a) 'A bag contains green, black, and red jelly beans. Therefore, if I choose one jellybean at random from the bag, the probability that it is black is $\frac{1}{3}$.'

Is this statement true or false? Explain why, in no more than one sentence.

(b) The diagram shows the graph of a certain function $f(x)$.

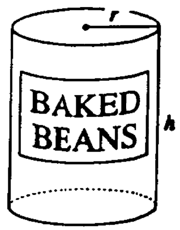


(i) Copy this graph
(ii) On the *same* set of axes, draw a sketch of the derivative $f'(x)$ of the function.

(c) Consider the curve given by $y = x^3 - 6x + 4$.
(i) Find the coordinates of the stationary points and determine their nature.
(ii) Find the coordinates of any point of inflexion.
(iii) Sketch the curve for the domain $-3 \le x \le 3$.
(iv) What is the maximum value of $x^3 - 6x + 4$ in the domain $-3 \le x \le 3$?

QUESTION 7.

(a) A can of baked beans is in the shape of a closed cylinder with height h cm and radius r cm, as shown in the diagram.



(i) The volume of the can is 500 cm^3. Find an expression for h in terms of r.

(ii) Show that the surface area, S cm^2, of the can is given by

$$S = 2\pi r^2 + \frac{1000}{r}$$

(iii) If the area of metal used to make the can is to be minimized, find the radius of the can.

(b) Solve $2\log_5 3 = \log_5 x - \log_5 6$.

(c) A Geiger counter is taken into a region after a nuclear accident and gives a reading of 10 000. One year later, the same Geiger counter gives a reading of 9000. It is known that the reading T is given the formula

$$T = T_0 e^{-kt}$$

where T_0 and k are constants and t is the time measured in years.
(i) Evaluate the constants T_0 and k.
(ii) It is known that the region will become safe when the reading reaches 40. After how many years will the region become safe?

QUESTION 8.

(a) The number unemployed people u at time t was studied over a period of time. At the start of this period, the number of unemployed was 800 000.

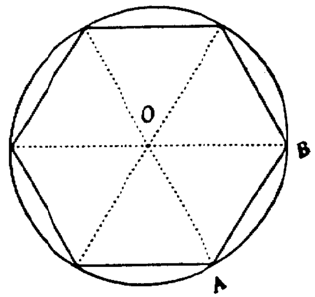
(i) Throughout the period, $\frac{du}{dt} < 0$.

What does this say about the number of unemployed during the period?

(ii) It is also observed that, throughout the period, $\frac{d^2u}{dt^2} > 0$.

Sketch the graph of u against t.

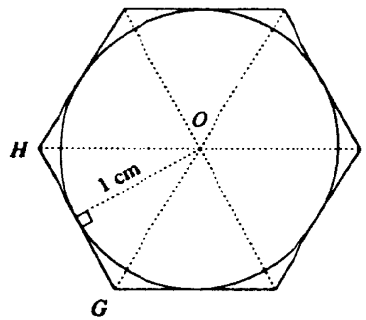
(b) A regular hexagon is drawn inside a circle with centre O so that its vertices lie on the circumference, as shown in the diagram. The circle has radius 1 cm.



(i) Prove that ΔOAB is equilateral.

(ii) Find the area of ΔOAB and hence find the area of this hexagon. Leave your answer in surd form.

Another regular hexagon is drawn outside the circle, as shown. The altitude of ΔOGH is 1 cm.



(iii) Find the area of ΔOGH and hence find the area of this outer hexagon. Leave your answer in surd form.

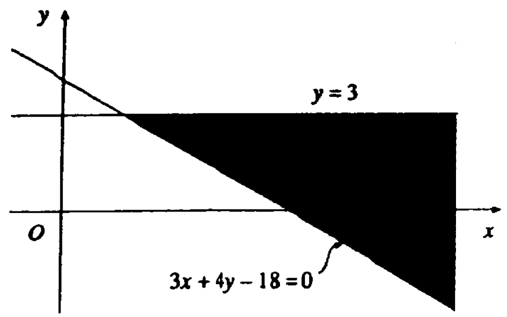
(iv) By considering the results in (ii) and (iii), show that $\frac{3\sqrt{3}}{2} < \pi < 2\sqrt{3}$.

QUESTION 9.

(a) Prove that the line $y = x + 2$ is a tangent to the parabola $y = x^2 - 5x + 11$.

(ii) Let Q be the point where the line $y = x + 2$ touches the parabola $y = x^2 - 5x + 11$.
Show that the normal to the parabola at Q is $y = -x + 8$.
(iii) Find the area of the region enclosed between the parabola and the line $y = -x + 8$.

(b)



The point $P(x, y)$ is equidistant from the line $y = 3$ and $3x + 4y - 18 = 0$, and lies in the shaded region of the diagram.
Find the equation of the locus of P.

QUESTION 10.

(a) David has invented a game for one person. He throws two ordinary dice repeatedly until the sum of the two numbers shown is either 7 or 9. If the sum is 9, David wins. If the sum is 7, David loses. If the sum is any other number, he continues to throw until it is 7 or 9.

(i) Show the probability that David wins on his first throw of the dice is $\frac{1}{9}$.

(ii) Calculate the probability that a second throw is needed.
(iii) What is the probability that David wins on his first, second, or third throw?
Leave your answer in unsimplified form.
(iv) Calculate the probability that David wins the game.

(b) A triangle ABC is right-angled at B. D is the point on AC such that BD is perpendicular to AC.
Let $\angle BAC = \theta$.
(i) Draw a diagram showing this information.

You are given that $6AD + BC = 5AC$.

(ii) Show that $6\cos\theta + \tan\theta = 5\sec\theta$.
(iii) Deduce that $6\sin^2\theta - \sin\theta - 1 = 0$

(iv) Find θ.