

HIGHER SCHOOL CERTIFICATE EXAMINATION 1987
MATHEMATICS - 2/3 UNIT

Direction to Candidates

Time allowed - Three hours (includes reading time)

All questions may be attempted. All questions are of equal value. All necessary working should be shown in every question. Marks may not be awarded for careless or badly arranged work.

Standard integrals are provided; approved calculators may be used.

QUESTION 1

- Find, correct to two decimal places, the value of $\frac{5.6 \times 4.9}{7.3 + 4.1}$.
- Solve the equation $3(x - 2) = x - 5$.
- Factorize $3x^2 - 12$.
- Given that $V = \frac{1}{3}\pi R^2 H$ and that $R > 0$, find R if $V = 2000$ and $H = 12$. Give your answer correct to one decimal place.
- In 1987 Council rates increased by $7\frac{1}{2}\%$. The new rate for a property was \$1735. What was the old rate for this property? Give your answer correct to the nearest dollar.

QUESTION 2

The points P and Q have coordinates (3, -2) and (1, 3) respectively.

- The line k has equation $4x + 5y - 2 = 0$. Verify that P lies on k .
- The line l through Q has gradient $\frac{1}{3}$. Show that the equation of l is $x - 3y + 8 = 0$.
- The point of intersection of k and l is R. Find the coordinates of R.
- Draw a neat sketch on a number plane showing P, Q, R, k , and l .
- Find the perpendicular distance of P from l . Leave your answer as a surd.
- Find the area of the triangle PQR.

QUESTION 3

- Differentiate: (a) $5(4 - x)^9$ (b) $\frac{\log_e x}{x}$
- Find the exact value of: (a) $\int_0^1 e^{2x} dx$ (b) $\int_1^2 \frac{1}{1+x} dx$
- A parabola has equation $y = x^2 - 4x + 6$. Find:
 - the equation of its axis of symmetry;
 - the coordinates of its vertex;
 - its focal length.

QUESTION 4

- ABCD is a quadrilateral, $\angle ADC = \angle BCD$ and $AD = BC$.
 - Draw a neat sketch and mark on it all the given information.
 - Prove that triangle DAC is congruent to triangle CBD.
 - Why are angles ACD and BDC equal?
 - Prove that $\angle ADB = \angle ACB$.
- In the figure, the lines UZ and WY intersect at X. $UW \parallel YZ$, UW is 10 cm, WX is 8 cm, UX is 7 cm and XZ = 12 cm.
 - Draw a neat sketch and mark on it all the given information.
 - Prove that the triangles UWX and XYZ are similar.
 - Hence, or otherwise, find the length ZY.

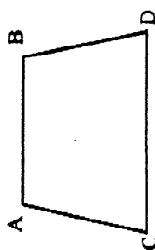


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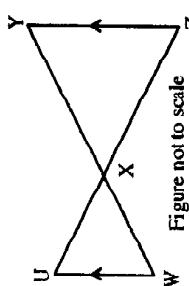


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QUESTION 5

- Find the equation of the tangent to the curve $y = 3x + e^x$ at the point where $x = 0$.
- The function $f(x)$ is given by $f(x) = x(x - 3)^2$.
 - Find the coordinates of the points at which the graph of $y = f(x)$ meets the x -axis.
 - Find the coordinates of the turning points of $f(x)$ and state whether they are maxima or minima.
 - Draw a sketch of $y = f(x)$ in the domain $-1 \leq x \leq 4$.

QUESTION 6

- Find the values of x for which $|2x - 1| \leq 5$.
- Find the volume of the solid of revolution formed by rotating the curve $y = x + \frac{1}{x}$ about the x -axis between $x = 1$ and $x = 3$.
- Shade on a number plane the region R bounded by the curve $y = 4x - x^2$ and the line $y = 5$, between $x = 0$ and $x = 4$.
 - Find the area of R.

QUESTION 7

- Find the exact value of m for which the equation $4x^2 - mx + 9 = 0$ has
 - exactly one real root;
 - real roots.
- Michele buys five tickets in a raffle in which 20 tickets are sold. Three different tickets are to be drawn out for first, second and third prizes. Using a tree diagram find the probability that:
 - Michele wins all three prizes;
 - Michele does not win a prize;
 - Michele wins at least one prize;
 - Michele wins exactly one prize.

QUESTION 8

- (i) From a lighthouse L, a ship S bears 053°T and is at a distance of 8 nautical miles. From L, a boat B bears 293°T and is at a distance of 6 nautical miles.

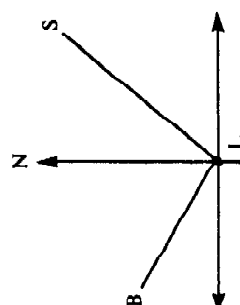


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- (a) Draw a diagram marking on it the information supplied.
 (b) Find the distance of ship S from boat B. Give your answer as a surd.
 (c) Find the bearing of ship S from boat B. Give your answer to the nearest degree.
 (ii) A particle moves in a straight line. At time t seconds its displacement x metres from a fixed point O on the line is given by $x = 3 - \cos 2t$, $0 \leq t \leq 2\pi$.
 (a) Sketch the graph of x as a function of t .
 (b) Write down the times when the particle is at rest and the position of the particle at each of these times.
 (c) Find the time when the particle first reaches its maximum speed.

QUESTION 9

- (i) A person invests \$800 at the beginning of each year in a superannuation fund. Compound interest is paid at 10% per annum on the investment. The first \$800 is to be invested at the beginning of 1988 and the last is to be invested at the beginning of 2017. Calculate to the nearest dollar:
 (a) the amount to which the 1988 investment will have grown by the beginning of 2018.
 (b) the amount to which the total investment will have grown by the beginning of 2018.
 (ii) Tom took a bottle which had 2000 millilitres of water in it. He poured more water into it for twenty seconds until it was full. During this time the volume flow rate R of water, in millilitres per second, into the bottle was given by $R = 4(20 - t)$.
 (a) Find a formula for the volume V of water in the bottle after t seconds where $t \leq 20$.
 (b) How many millilitres of water were in the bottle when it was full?
 (c) How long did it take before the bottle was half full?

QUESTION 10

- (i) Find the number which when added to each of 2, 6 and 13 will give a set of three numbers in geometric progression.
 (ii) A children's picture book is being designed so that each page contains 320 square centimetres of print and pictures, surrounded completely by a white border as illustrated in the figure:
 Each page is to have a border of width 2 cm at the bottom and on each side, as well as a border of width 3 cm at the top.
 Let the width of a page be x cm and its length be y cm.
 (a) Show that the area A square centimetres of one such page is given by $A = x \left(5 + \frac{320}{x-4} \right)$.

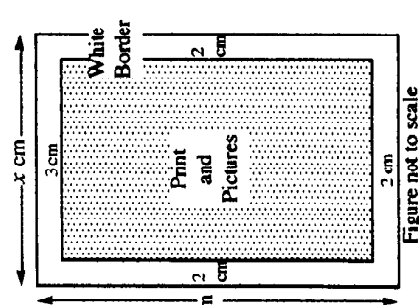


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Prove that the page which fulfils all printing requirements given and which has the smallest area is 20 cm wide and 25 cm long.