

**N.S.W. DEPARTMENT OF EDUCATION  
HIGHER SCHOOL CERTIFICATE EXAMINATION 1967  
MATHEMATICS PAPER C (2S) (EQUIVALENT TO 2 UNIT)**

**Instructions:** Time 3 hours. All questions may be attempted. In every question, all necessary working should be shown. Marks will be deducted for careless or badly arranged work.

**QUESTION 1 (30 Marks)**

- (i) Draw a diagram showing the region in which both the inequalities  $x - 3y > 0$ ,  $3x + y > 0$  hold.
- (ii) Simplify  $\frac{\cos x + \sin x}{\cos x - \sin x} + \frac{\cos x - \sin x}{\cos x + \sin x}$
- (iii) P divides AB internally in the ratio 2:1, Q divides AB externally in the same ratio, where A is the point (-2, 1) and B is (1, -5). What are the coordinates of P and Q?
- (iv) Write down the derivatives of (a)  $5x^{1/5}$  (b)  $x \cos x$
- (v) Write down primitive functions of (a)  $\frac{1}{(2x+1)^2}$  (b)  $\frac{1}{2x+1}$
- (vi) Find the value of  $k$  for which  $y = e^{-2x}$  satisfies  $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + ky = 0$ .
- (vii) A number  $n$ ,  $1 \leq n \leq 299$ , is picked at random. What is the probability (expressed as a rational number) that  $n$  is divisible by 5?
- (viii) A ball is picked at random from a bag containing 2 red and 5 black balls; then a ball from a second bag containing 5 red and 2 black balls. What is the probability (expressed as a rational number) that both balls drawn are red?
- (ix) The curve  $y = \sqrt{\sin x}$ ,  $0 \leq x \leq \pi$ , is rotated about the  $x$ -axis. What is the volume of the solid thus generated?
- (x) What are the signs of (a)  $\sin 3.1$  (b)  $\sin 3.2$  (c)  $\cos 4.6$ ?

**QUESTION 2 (10 Marks)**

- (i) R is the foot of the perpendicular from the point P(-5, 10) to the line  $4x - 3y = 0$ . Find the coordinates of R.
- (ii) Find all solutions of  $2\cos^2 x - 3\cos x - 2 = 0$  with  $-\pi < x < \pi$ .

**QUESTION 3 (10 Marks)**

The sum of the first  $n$  terms of an arithmetic sequence is  $-n^2 + 21n$ .

- Find
- the first term;
  - the  $n$ -th term;
  - the common difference;

- (iv) the first value of  $n$  for which the  $n$ -th term of the sequence is negative.

**QUESTION 4 (10 Marks)**

- (i) Find from first principles the gradient of the curve  $y = 1 - x^2$  at the point (1, 0)

- (ii) Find the derivatives of (a)  $\log(x + \sqrt{x^2 - 1})$  where  $x > 1$  (b)  $e^{\frac{1}{x}}$

**QUESTION 5 (10 Marks)**

Find the stationary points of  $y = -12 - 12x^2 - 4x^3 + 3x^4$ , determine their nature and sketch the curve.

For what values of  $x$  is the function (i) monotonically increasing (ii) monotonically decreasing?

**QUESTION 6 (10 Marks)** {Suitable for 3 Unit Students}

Use the focus - directrix definition to obtain the equation of a parabola with focus (0,  $a$ ) and directrix  $y + a = 0$ .

Tangents are drawn from the point (0,  $-2a$ ) to the parabola. Find the coordinates of their points of contact.

**ALTERNATIVE QUESTION 6 (10 Marks)** {Suitable for 2 Unit Students}

Use the focus - directrix definition to obtain the equation of a parabola with focus (0,  $a$ ) and directrix  $y + a = 0$ .

The latus rectum of the parabola  $x^2 = 8y$  cuts the curve in A, B. Find the coordinates of A, B.

The tangents at A, B meet in C, determine the coordinates of C.

**QUESTION 7 (10 Marks)**

After its engine is cut off, a ship travels in a straight line with a deceleration proportional to its velocity, i.e.

$$\frac{dv}{dt} = -kv.$$

If its velocity is initially 30 cm/s and is 25 cm/s 10 seconds later, when will its velocity be 5 cm/s? (Give your answer to two significant figures.)

**QUESTION 8 (10 Marks)**

Sketch the graph of  $y = 1 + 2\cos x$  for the interval  $-\pi \leq x \leq \pi$ .

Find the area under the curve between  $x = 0$  and  $x = \frac{11\pi}{12}$  (Express the answer in terms of  $\pi$ .)