HIGHER SCHOOL CERTIFICATE EXAMINATION 1986
MATHEMATICS - 2/3 UNIT

Time allowed - Three hours (including reading time)

Direction to Candidates

All questions may be attempted. All questions are of equal value. All necessary working should be shown in every question. Marks will not be awarded for careless or badly arranged work.

Standard integrals are provided - calculators may be used.

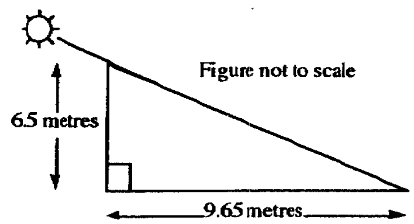
QUESTION 1

(i) (a) Find $3^{3.5}$ correct to two decimal places.

(b) The price of an article for sale at $1.60 is to increase by 20%. What will be the new price of the article?

(c) Given that $I = \dfrac{\varepsilon}{R+3r}$, and $\varepsilon = 0.6$, $R = 1.2$ and $r = -0.5$, find I.

(ii) In the figure, a vertical pole which is 6.5 metres high casts a horizontal shadow 9.65 metres long. Find the angle of elevation of the sun (correct to the nearest degree).



(iii) In a certain country town the probability of an adult catching a cold during 1987 was 0.2. Two adults in the town were selected at random. What was the probability that they both caught colds in 1987?

(iv) Solve the equation $3(2x + 1) - 2(3 - x) = 53$.

QUESTION 2

The coordinates of the points A, B, C are (0, 2), (4, 0) and (6, -4) respectively.

(a) Find the length AB, and the gradient of AB.

(b) Show that the equation of the line l, drawn through C parallel to AB is $x + 2y + 2 = 0$.

(c) Find the coordinates of D, the point where l intersects the x-axis.

(d) Find the perpendicular distance of the point A from the line l.

(e) Find the area of the quadrilateral ABCD.

QUESTION 3

(i) Differentiate (a) $(x^2 - 1)^{11}$ (b) $\tan(3x)$.

(ii) Find the equation of the tangent to the curve $t = xe^x$ at the point $(1, e)$.

(iii) Differentiate $y = \dfrac{\sin x}{1 + \cos x}$, hence show that $\dfrac{dy}{dx} = \dfrac{1}{1 + \cos x}$.

QUESTION 4

(i) Evaluate (a) $\displaystyle\int_1^2 \frac{1}{x}\,dx$, (b) $\displaystyle\int_0^{\pi/4} \cos(2x)\,dx$.

(ii) The function $p(x)$ is defined by the rule $p(x) = (x-1)(x^2-5)$.

(a) Find the real roots of the equation $p(x) = 0$.

(b) Find the coordinates of the turning points of $p(x)$, and state whether they are maxima or minima.

(c) Draw a sketch of the graph of $y = p(x)$, in the domain $-3 \le x \le 3$.

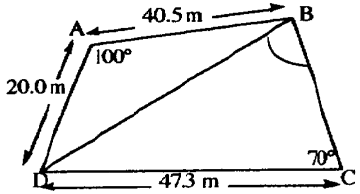
QUESTION 5

(i) The displacement x metres from the origin at time t seconds, of a particle travelling in a straight line is given by the formula $x = t^3 - 21t^2$.

(a) Find the acceleration of the particle at time t seconds.

(b) Find the times at which the particle is stationary.

(ii) In the figure, (not drawn to scale), ABCD is a quadrilateral in which AB = 40.5 m, AD = 20.0 m, DC = 47.3 m, $\angle A = 100°$ and $\angle C = 70°$.

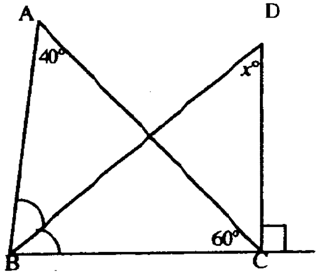


(a) Use the cosine rule for triangle ABD to find the length BD.

(b) Use the sine rule to find $\angle DBC$, correct to the nearest degree.

QUESTION 6

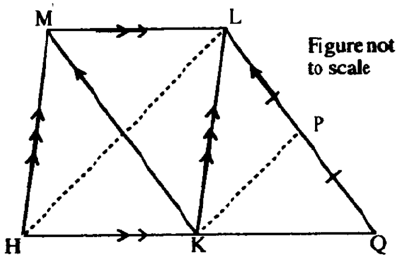
(i) In the figure (not drawn to scale), BD bisects $\angle ABC$, DC is perpendicular to BC, $\angle ACB = 60°$, $\angle BAC = 40°$, $\angle BDC = x°$.



(a) Draw a neat sketch of the diagram.

(b) Calculate x, giving reasons for each step in your calculation.

(ii) In the figure, HKLM is a parallelogram. The line through L, parallel to MK, meets HK produced at Q.



(a) Draw a neat sketch of the diagram.

(b) Prove that HK = KQ.

(c) Given that P is a midpoint of LQ, prove that $\angle PKQ = \angle LHQ$.

QUESTION 7

(i) (a) Sketch the graph of $y = \cos 2\theta$, for $0° \le \theta \le 180°$.

(b) Find all values of θ, for $0° \le \theta \le 180°$, such that $\cos 2\theta = \frac{1}{2}$.

(ii) The function $f(x)$, defined for x in the domain $0 \le x \le 2$, is given by the rule $f(x) = (4 - x^2)^{1/4}$; $0 \le x \le 2$.

(a) Draw up a table of values of $f(x)$ (correct to two decimal places) for $x = 0, \frac{1}{2}, 1, 1\frac{1}{2}, 2$.

(b) Use this table to draw a sketch of the graph of $y = f(x)$, for $0 \le x \le 2$. [Do not attempt to find the turning points.]

(c) Use the trapezoidal rule with five function values (four equal subintervals) to estimate the area between the curve, the x-axis and the y-axis.

QUESTION 8

(i) The first and last terms of an arithmetic series are 10 and 60, respectively, and the sum of the series is 3535. Find:

(a) the number of terms in the series; (b) the common difference.

(ii) Two identical perfect cubes (similar to dice) each having faces numbered 0, 1, 2, 3, 4, 5 are rolled. A score is determined as the <u>product</u> of the two numbers on the two uppermost faces.

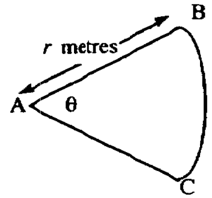
(a) The cubes are rolled once. What is the probability that the score is (α) 0? (β) at least 16?

(b) If the cubes are rolled twice and the scores for each roll are added, what is the probability of a combined score of at least 41?

QUESTION 9

(i) Find all real numbers x which satisfy the equation $4x^4 = 4x^2 + 3$.

(ii) In the figure, AB and AC are radii of length r metres, of a circle with centre A. The arc BC of the circle subtends an angle θ radians at A.



(a) Write down the formula for

(α) the length of the arc BC, (β) the area of the sector ABC.

(b) The perimeter of the above figure (i.e. the length AB + arc BC + CA) is 12 metres. Show that the area Y square metres of the sector ABC is given by

$$Y = \frac{72\theta}{(\theta + 2)^2}$$

Hence show that the maximum area of the sector is 9 square metres.

QUESTION 10

(i) The rate of increase of a population $P(t)$ of persons in a certain country is governed by the equation $\frac{dP}{dt} = kP$ where k is a constant, and t is the time in years. The population of the country doubles every twenty years.

(a) Find k.

(b) In which year will the country reach a population three times that it had at the beginning of 1960?

(c) Given that at the beginning of 1960 the population was 15.1 million, what will be the population at the beginning of the year 2010?

(ii) (a) Find the points of intersection of the curve $y = 4 - \sqrt{2x}$ with the x- and y-axes.

(b) The area enclosed by the curve $y = 4 - \sqrt{2x}$, the x-axis, and the y-axis, is rotated about the y-axis. Find the volume of the solid of revolution so formed.