N.S.W. DEPARTMENT OF EDUCATION
HIGHER SCHOOL CERTIFICATE EXAMINATION 1972
MATHEMATICS PAPER C (2S) (EQUIVALENT TO 2 UNIT)

Instructions: Time 3 hours. All questions may be attempted. In every question, all necessary working should be shown. Marks will be deducted for careless or badly arranged work.

QUESTION 1 (12 Marks)

(i) Indicate on a diagram the region in which $0 \le x + y \le 1$.

(ii) Find the equation of the tangent at the point (1, 5) on the parabola $y = x^2 + 2x + 2$.

(iii) Differentiate (a) $\dfrac{1}{1+x^2}$ (b) $(\cos x - \sin x)^2$

(iv) An integer n, $1 \le n \le 144$, is picked at random. What is the probability that n is the square of an integer?

QUESTION 2 (9 Marks)

(i) Find the second derivative of xe^{-x}.

(ii) Write down a primitive (indefinite integral) of $\dfrac{\sqrt{x}}{x^4}$.

(iii) The velocity $\dfrac{dx}{dt}$ of a particle is given by $\dfrac{dx}{dt} = 4t^3$. If $x = 5$ at $t = 0$, calculate x at $t = 2$.

QUESTION 3 (9 Marks)

(i) Given that $e^{2.3026} = 10$ (to five-figure accuracy) write down the natural logarithms of

(a) 10 (b) $\dfrac{1}{10}$ (c) 100

(ii) The curves $y = \cos x$, $y = \sin x$ intersect at $P(\dfrac{\pi}{4}, \dfrac{1}{\sqrt{2}})$. Find the gradients of these curves at P. If θ is the acute angle between their tangents at P, show that $\tan\theta = 2\sqrt{2}$.

(iii) The area under the curve $y = \sec x$, $0 \le x \le \dfrac{\pi}{4}$, is rotated about the x-axis. What is the volume of the solid of revolution thus generated?

QUESTION 4 (10 Marks)

(i) A parabola $y = 5 + 2x - x^2$ is given.

(a) Write down the coordinates of its vertex.

(b) What is the equation of its axis?

(c) Find the area under the curve between $x = -1$ and $x = 1$.

(ii) A parabola $y = ax^2 + bx + c$ is drawn through the three points A(-1, 4), B(0, 7), C(1, 8).

(a) Determine a, b and c.

(b) Show that D(3, 4) is also on the parabola.

QUESTION 5 (10 Marks)

(i) The first three terms of an arithmetic progression are 26, 23, 20.

(a) Write down the sum of the first n terms of the series.

(b) What is the least n such that the sum of the first n terms of the series is negative.

(ii) The sequence 5, 11, 29 ... has as its n-th term $3^n + 2$. Write down

(a) the fourth term;

(b) the sum of the first n terms.

QUESTION 6 (10 Marks)

(i) The position of a particle moving along the *x-axis* is given by $x = 1 - \cos 2(t - 1)$.

(a) Show that the particle is at rest when $t = 1$.

(b) When and where is the particle next at rest?

(ii) For each of the following equations, state whether it has solutions and if so find all solutions in the interval $0 \le x \le 2\pi$. (a) $2\sin x + 1 = 0$ (b) $\sin x + 2 = 0$ (c) $\sin x - 2 = 0$

QUESTION 7 (10 Marks)

(i) Sketch the curve $y = x^3 - 4x^2 + 3x$.

(ii) The origin is a common point of the line $y = mx$ and the curve $y = x^3 - 4x^2 + 3x$. Show that if $m < -1$

they have no other point in common.

(iii) Discuss the case $m = -1$ giving a geometrical interpretation.

QUESTION 8 (10 Marks)

(i) Sketch the curve $y = x^3$ and the line $y = x$, then find the area enclosed between the curve and the line in the positive quadrant.

(ii) P_1, P_2 are points on the curve $y = \frac{1}{x}$ with x-coordinates $\frac{1}{1+h}$ and $1 + h$, respectively. Show that the equation of the line through P_1, P_2 is $x + y = 1 + h + \frac{1}{1+h}$.

QUESTION 9 (10 Marks)

(i) Show that the triangle whose sides satisfy $2x - y = 0, x + 2y = 5, x - 3y = 20$ is isosceles and right-angled.

(ii) Simplify each of the following expressions. No negative or zero exponents should appear in your final answer and it is given that neither x nor y is zero.

(a) $\frac{(8x)^{-1}}{2^{-6}}$ (b) $\frac{xy^{-1} + yx^{-1}}{x^2 + y^2}$

QUESTION 10 (10 Marks)

(i) Calculate the following definite integrals:

(a) $\int_0^1 (x^{1/5} - x^{1/3})\,dx$ (b) $\int_{\pi/2}^{\pi} \sin x\,dx$ (c) $\int_0^3 \frac{x\,dx}{1+x^2}$

(ii) Show that the second derivative of $\log_e(1 + \sin x)$ is $\frac{-1}{1+\sin x}$.