N.S.W. DEPARTMENT OF EDUCATION
HIGHER SCHOOL CERTIFICATE EXAMINATION 1973
MATHEMATICS PAPER C (2S) (EQUIVALENT TO 2 UNIT)

Instructions: Time 3 hours. All questions may be attempted. In every question, all necessary working should be shown. Marks will be deducted for careless or badly arranged work.

QUESTION 1 (12 Marks)

(i) Write down the exact values of: (a) $\sin 30°$ (b) $\sin 150°$ (c) $\sin 210°$

(ii) What is the equation of the circle whose centre is at the origin and which passes through the point $(5, -7)$?

(iii) Simplify: $x - \frac{1}{x}$ if $x = \sqrt{2} + 1$

(iv) Find all the solutions of $|x - 3| < 2$

QUESTION 2 (9 Marks)

(i) Differentiate (a) $\frac{1}{x+5}$ (b) $\cos 3x$

(ii) Write down primitive functions of (a) $\frac{1}{x+5}$ (b) $\cos 3x$

(iii) Find the area of the triangle formed by the line $3x + 8y - 4 = 0$ and the coordinate axes.

QUESTION 3 (9 Marks)

(i) Find the area of a sector of $36°$ cut from a circle of radius 10 cm.

(ii) Evaluate $\lim_{x \to 4} \frac{x^2 - 16}{x - 4}$

(iii) In a triangle ABC, AB = 8 cm, BC = 5 cm and the angle ABC is $60°$. Find the length of AC.

QUESTION 4 (10 Marks)

(i) In an English speaking country ten people are in a room; five speak only French, three speak only German and two speak only English. Two are selected at random. What is the probability that:

(a) they both speak French;

(b) one speaks French and one speaks German;

(c) neither speaks a foreign language?

(ii) It is given that the series $\log 4 + \log 8 + \log 16 + \log 32 + \ldots$ is either arithmetic or geometric.

(a) Which is it? Justify your answer.

(b) Find also the sum of the first 50 terms of this series. (Give your answer in the form $c \log 2$.)

QUESTION 5 (10 Marks)

(i) Evaluate (a) $\int_0^1 \sqrt{x}\,dx$ (b) $\int_1^2 \frac{x^2 - 2x + 1}{x}\,dx$

(ii) Without differentiating, find the range of the function f given by $f(x) = \frac{1}{1 + x^2}$ over the domain of all real numbers.

§(iii) Use the mid-ordinate rule to find an approximate value of: $\int_{1.5}^{2.5} \frac{1}{1 + x^2}\,dx$

QUESTION 6 (10 Marks)

(i) Find the equation of the tangent to the curve $y = x^3 - 9x^2 + 20x - 8$ at the point $(1, 4)$.

(ii) At what points on this curve are the tangents parallel to the line $y = -4x + 3$?

(iii) Prove (using calculus or otherwise) that if the sum of the radii of two circles is constant, then the sum of the areas of the two circles is least when the circles have equal radii.

QUESTION 7 (10 Marks)

(i) A particle moves in a straight line and after t seconds its velocity v m/s is given by $v = 12t - 3t^2$.

(a) When is the particle at rest?

(b) Find the acceleration at $t = 1$.

(c) Find the distance travelled in the fourth second.

(ii) Find the volume of the solid generated when the region bounded by the curve $y = x^2 - 3$ and the x-axis, is rotated about the y-axis.

QUESTION 8 (10 Marks)

Sketch the following curves showing the main features (do not use squared paper):

(i) $y = e^{-x}$ (ii) $y = \sin 2x \quad (-2\pi \le x \le 2\pi)$ (iii) $y = |x| + 1$

QUESTION 9 (10 Marks)

(i) For what values of k does the equation $x^2 + (k-3)x + k = 0$ have real roots?

(ii) For what values of k is the expression $x^2 + (k-3)x + k$ positive for all values of x.

(iii) The parabola $y = 3x^2 + bx + 10$ is symmetrical about the line $x = 4$. Find the value of b.

QUESTION 10 (10 Marks) **{Suitable for 3 Unit Students}**

$P(2ap, ap^2)$ and $Q(2aq, aq^2)$ are the points of the parabola $x^2 = 4ay$.

(i) Derive the equation of the tangent at P.

(ii) Determine the coordinates of T, the point of intersection of the tangents at P and Q.

(iii) Find the coordinates of M, the midpoint of PQ.

(iv) Show that the curve bisects the interval TM.

ALTERNATIVE QUESTION 10 **(Suitable for 2 Unit Students - based on above)**

$P(6, 9)$ and $Q(-2, 1)$ are points on the parabola $x^2 = 4y$.

(i) Derive the equation of the tangent at P.

(ii) Determine the coordinates of T, the point of intersection of the tangents at P and Q.

(iii) Find the coordinates of M, the midpoint of PQ.

(iv) Show that the curve bisects the interval TM.