

**N.S.W. DEPARTMENT OF EDUCATION
HIGHER SCHOOL CERTIFICATE EXAMINATION 1974
MATHEMATICS PAPER C (2S) (EQUIVALENT TO 2 UNIT)**

Instructions: Time 3 hours. All questions may be attempted. In every question, all necessary working should be shown. Marks will be deducted for carelessness or badly arranged work.

QUESTION 1 (12 Marks)

- (i) Arrange the following numbers in order of increasing magnitude: $\sqrt{2}$; $\frac{7}{5}$; $1\frac{3}{7}$; 1.4 .
- (ii) A circle, of radius 5 units, has its centre at the point $(-3, 4)$. What are the coordinates of the two points at which the circle cuts the y-axis?
- (iii) What is the area of the square on the hypotenuse of a triangle with the other sides 1 and $(1 + \sqrt{2})$ units? (Give the answer in surd form.)
- (iv) Comment briefly on the following statement, giving justification for your view:
- "As there are 26 letters in the alphabet, if I choose any letter on a page of print at random, the probability that it is an 'e' is $\frac{1}{26}$."

QUESTION 2 (9 Marks)

- (i) Differentiate (a) $x^3 - 5x + 2$ (b) $(2x - 3)^5$ (c) $x^2 \cos x$
- (ii) Write down a primitive function of (a) $x^{1/2} + 1$ (b) x^{-13}
- (iii) Evaluate $\int_1^4 (3x - 2) dx$

QUESTION 3 (9 Marks)

- (i) The first two terms of a geometric sequence are 9 and 6.
- (a) What is the third term?
- (b) What is the n th term?
- (ii) For the arithmetic sequence 1, 4, 7, ...
- (a) find the 50th term
- (b) calculate the sum of the first 50 terms.

- (iii) A plant is 50 cm high when first observed. In the first week of observation it grows 10 cm, and in each succeeding week the growth in height is 80% of the previous week's growth. If this pattern of growth continues, what will be its ultimate height?

QUESTION 4 (10 Marks)

- (i) A particle is moving in a straight line and its velocity at time t is given by $v = 3 \sin 2t$.
- Calculate (to 3 significant figures) its velocity when $t = 0.1$.
 - Write down an expression for its acceleration at time t .
- (ii) For another particle moving in a straight line its acceleration at time t seconds ($t > 0$) is given by $a = 4 - \frac{8}{3} \text{ m/s}^2$. If the velocity is 10 m/s when $t = 2$, what is its velocity when $t = 3$?

QUESTION 5 (10 Marks)

- (i) For the graph of $y = x^3 + 3x^2 - 9x$ find
- the coordinates of the stationary points
 - the set of values of x for which the graph is concave upwards.
- (ii) The parabola $y = ax^2 - c$ and the circle $x^2 + y^2 = 16$ meet on both the x and y axes. If a and c are both positive, what are their values?

QUESTION 6 (10 Marks)

- (i) A field is triangular, with two sides of 80 metres and 40 metres enclosing an angle of 130° .
- Calculate the area of the field.
 - Use the cosine rule to calculate the length of the third side to the nearest metre.
- (ii) An astronaut observes that the moon, which has a diameter of 3500 km, subtends an angle of 1° at his eye. Calculate the approximate distance of the astronaut from the moon.

QUESTION 7 (10 Marks)

- (i) Find and test all real solutions of the equation $|x + 1| = 2x + 7$
- (ii) (a) Sketch, using the same axes, but not on graph paper, $y = x^2 - 2x - 3$ and $xy = -1$
- (b) Write down an equation whose roots are given by the abscissae of the points of intersection of these two graphs.

QUESTION 8 (10 Marks)

- (i) It is known from experience that 90% of the eggs from a certain farm are fresh. If a cook chooses 3 of these eggs, what is the probability:
- that all 3 are fresh?
 - that at least one is stale?

- (ii) Find the equation of the locus of the point which moves so that its distance from the x -axis is always equal to its distance from the point $(1, 3)$. Name the locus.

QUESTION 9 (10 Marks)

- (i) What is the equation of the tangent to the curve $y = \log_e x$ at the point $(1, 0)$?
- (ii) The area under the curve $y = \frac{1}{x}$ between $x = 1$ and $x = b$ is equal to 1 square unit. What is the value of b ?

(ii) Use the mid-ordinate rule with one strip to estimate $\int_{3.7}^{4.3} \frac{1}{1 + \sqrt{x}} dx$

QUESTION 10 (10 Marks)

- (i) Find the volume swept out when the region under the curve $y = e^x$, for $1 \leq x \leq 1.5$ is rotated about the x -axis, giving your answer to 3 significant figures.
- (ii) The number of bacteria (P) in a colony, after t minutes is given by $P = 1000e^{0.007t}$. Find:
- the number of bacteria when $t = 10$
 - the rate at which the colony is increasing when $t = 10$.