HIGHER SCHOOL CERTIFICATE EXAMINATION 1990
MATHEMATICS - 2/3 UNIT

Direction to Candidates

Time allowed - Three hours (includes reading time)

All questions may be attempted. All questions are of equal value. All necessary working should be shown in every question. Marks may not be awarded for careless or badly arranged work.

Standard integrals are provided; approved calculators may be used.

QUESTION 1

(a) Calculate $\frac{171.8}{14.3 \times 5.7}$ correct to two decimal places.

(b) The speed of light is 299 725 kilometres per second. Write this number correct to four significant figures.

(c) Factorize $2x^2 - 7x - 15$.

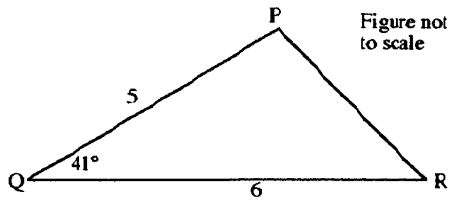
(d) The volume V of a sphere is given by $V = \frac{4}{3}\pi r^3$. If a sphere has volume 5 cm^3, find the radius correct to two decimal places.

(e) Solve $3 - (4 - x) = 5x$.

(f) Mark on a number line the values of x for which $|x - 2| < 3$.

QUESTION 2

(a) Find the length of the side PR using the cosine rule and give your answer correct to two decimal places.



(b) (i) Plot on a number plane the point A(4, 3). Draw the interval AO where O is the origin.

(ii) Plot the points B(0, -3) and C(4, 0) on your diagram.

(iii) Show that the line BC has equation $4y = 3x - 12$.

(iv) Show that OACB is a parallelogram. Give reasons.

(v) Find the area of the parallelogram OACB.

(vi) Calculate the length of the diagonal AB.

QUESTION 3

(a) (i) Rationalize the denominator of $\frac{2}{2-\sqrt{3}}$.

(ii) Find integers a and b such that $\frac{2}{2-\sqrt{3}} = a + \sqrt{b}$.

(b) Differentiate: (i) $7x^5 + 3$; (ii) $2\sin x + \sqrt{x}$; (iii) $x \ln x$.

(c) Find: (i) $\int (\cos x + 5x^2)\, dx$ (ii) $\int_0^1 (1 + e^{-x})\, dx$.

QUESTION 4

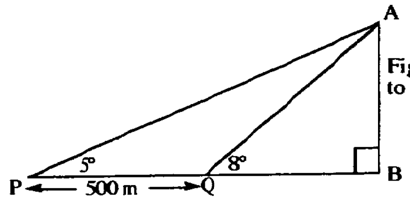
(a) The sum of the first n terms of a certain arithmetic series is given by $S_n = \frac{n(3n+1)}{2}$.

(i) Calculate S_1 and S_2.

(ii) Find the first three terms of this series.

(iii) Find the expression for the n-th term.

(b) Kaye observes a cliff AB from her yacht at position P. She then sails 500 metres closer to the cliff to position Q. The angle of elevation of the cliff top from P is $\angle APB = 5°$ and from Q is $\angle AQB = 8°$.



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(i) Copy the diagram and find $\angle PAQ$.

(ii) Use the sine rule to calculate AQ to the nearest metre.

(iii) Hence, or otherwise, find QB to the nearest 10 metres.

QUESTION 5

(a) Consider the curve given by $y = 1 + 3x - x^3$, for $-2 \le x \le 3$.

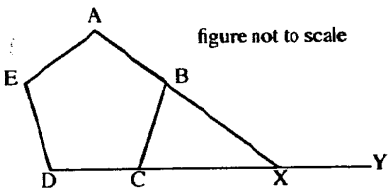
(i) Find the stationary points and determine their nature.

(ii) Find the point of inflexion.

(iii) Sketch the curve for $-2 \le x \le 3$.

(iv) What is the minimum value of y for $-2 \le x \le 3$?

(b) In the diagram, ABCDE is a regular pentagon and AB and DC produced meet at X. The point Y lies on DCX produced.



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(i) Copy the diagram and find the size of $\angle ABC$.

(ii) Find the size of $\angle BXY$ giving reasons.

QUESTION 6

(a) Find all values of x with $0° \le x \le 360°$ for which $\tan x = \frac{1}{\sqrt{3}}$.

(b) A particle P is at the origin at time $t = 0$ and moves so that its velocity for $t \ge 0$ is given by $v = \frac{1}{t+1}$.

(i) What is the acceleration of P when $t = 1$?

(ii) What is the displacement x of P from the origin when $t = 1$?

(c) Bob tosses two dice with faces numbered 1 to 6. He records the maximum of the two uppermost faces as a score.

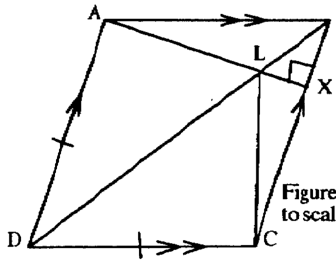
(ii) Find the probability that he records the score 1 in a single throw of the two dice.

(ii) Find the probability that he records the scores 1, 1, 1 in three tosses of the two dice.

(iii) Find the probability that he records the score 6 in a single throw of the two dice.

QUESTION 7

(a) ABCD is a rhombus, AX is perpendicular to BC and intersects BD at L.



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(i) Copy the diagram and state why $\angle ADB = \angle CDB$.

(ii) Prove that the triangles ALD and CLD are congruent.

(iii) Show that $\angle DAL$ is a right angle.

(iv) Hence or otherwise find the size of $\angle LCD$.

(b) A box contains 8 red and 11 green marbles. Sue-Mei randomly selects three marbles one at a time and without replacement. What is the probability that she selects green, red, green in that order?

(c) A quantity Q of radium at time t in years is given $Q = Q_0 e^{-kt}$ where k is a constant and Q_0 is the initial amount of radium at time $t = 0$.

(i) Given that $Q = \frac{1}{2}Q_0$, when $t = 1690$ years, calculate k.

(ii) After how many years does only 20% of the initial amount of radium remain?

QUESTION 8

(a) Find the equation of the normal to the curve $y = x^2 + x$ at the point where $x = 1$.

(b) On a number plane shade in the region given by the two conditions $x^2 + y^2 \le 4$ and $x + y \ge 1$.

(c) It is assumed that the number $N(t)$ of termites in a certain mound at time $t \ge 0$ is given by $N(t) = \frac{A}{2 + e^{-t}}$, where A is a constant, and t is measured in months.

(i) At time $t = 0$, $N(t)$ is estimated at 3×10^5 termites. What is the value of A?

(ii) What is the value of $N(t)$ after one month?

(iii) How many termites would you expect to find in the mound when t is very large?

(iv) Find an expression for the rate at which the number of termites increases at any time t.

QUESTION 9

(a) Consider the function given by $y = \sin^2 x$

(i) Copy and complete the following table. (Note that x is measured in radians.)

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
y	0				

(ii) Apply Simpson's rule with five function values to find an approximation to $\int_0^{\pi} \sin^2 x \, dx$.

(b) A cylinder of radius r cm and height h cm is inscribed in a cone with base radius 6 cm and height 20 cm as in the diagram.

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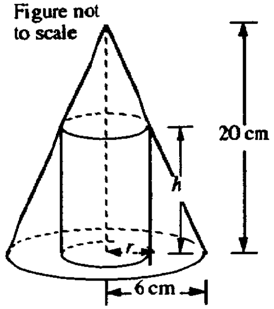

(i) Show that the volume V of the cylinder is given by $V = \dfrac{10\pi r^2(6-r)}{3}$.

(ii) Hence find the values of r and h for the cylinder which has maximum volume.

(iii) What is the maximum volume?

QUESTION 10

(a) The shaded region lying between the curve $y = 1 - x^2$ and the x-axis is rotated about the x-axis.

Find the volume of the solid so formed.

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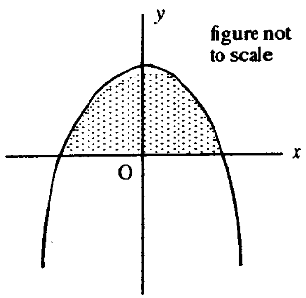

(b) A farmer borrows \$80 000 to purchase new machinery. The interest is calculated monthly at the rate of 2% per month, and is compounded each month.

The farmer intends to repay the loan with interest in two equal annual instalments of \$M at the end of the first and second years.

(i) How much does the farmer owe at the end of the first month?

(ii) Write an expression involving M for the total amount owed by the farmer after 12 months, just after the first instalment of \$M has been paid.

(iii) Find the expression for the amount owed at the end of the second year and deduce that

$$M = \frac{80\,000 \times (1.02)^{24}}{(1.02)^{12} + 1}.$$

(iv) What is the total interest over the two year period?