N.S.W. DEPARTMENT OF EDUCATION
HIGHER SCHOOL CERTIFICATE EXAMINATION 1970
MATHEMATICS PAPER C (2S) (EQUIVALENT TO 2 UNIT)

Instructions: Time 3 hours. All questions may be attempted. In every question, all necessary working should be shown. Marks will be deducted for careless or badly arranged work.

QUESTION 1 (12 Marks)

(i) Express the following numbers as decimals with two figure accuracy:

(a) 3^{-3} (b) $27^{1/3}$ (c) $256^{1/4}$

(ii) Find the equation of the normal to the parabola $y = 1 + 3x - 3x^2$ at the point (1, 1).

(iii) Find the equation of the line through (-1, -1) parallel to the line joining (-2, -3) to the origin.

(iv) Write down the values of (a) $|7| + |-9|$ (b) $|-7| - |-9|$

QUESTION 2 (9 Marks)

(i) Simplify $\frac{\cos(180° - A)}{\sin(90° - A)}$.

(ii) Differentiate $\sin(\frac{\pi}{4} - x)$.

(iii) Given that $\cos 30° = \frac{\sqrt{3}}{2}$, $\cos 45° = \frac{1}{\sqrt{2}}$, $\sin 30° = \frac{1}{2}$, $\sin 45° = \frac{1}{\sqrt{2}}$, deduce the value of $\cos 15°$.

QUESTION 3 (9 Marks)

(i) What is the area of the region above the x-axis and below the curve $y = x(2 - x)$?

(ii) Write down primitives (indefinite integrals) of (a) $\frac{2}{x^3}$ (b) $\frac{1}{x+2}$

(iii) A circular sector AOB, centre O, of radius 10 cm, contains an angle of $\frac{\pi}{4}$ radians. The straight edges OA, OB are joined to form a right circular cone. What is the base radius of the cone in cm?

QUESTION 4 (10 Marks)

Consider the function defined for all x by $f(x) = 2\sin(\pi x)$

(i) Sketch the graph of $y = f(x)$ for $-2 \le x \le +2$

(ii) What is the range of the function?

(iii) What is its period?

(iv) Evaluate $\int_{-0.25}^{1.75} 2\sin(\pi x)\,dx$

QUESTION 5 (10 Marks)

For the curve $y = \frac{1}{2}x^2(x^2 - 1)$

(i) find the equation of the tangent at (1, 0);

(ii) what is the angle between this tangent and the tangent at (-1, 0)?

(iii) find the point of intersection of the tangents to the curve at (-1, 0) and (1, 0).

QUESTION 6 (10 Marks)

(i) P_1, P_2 are points on the curve $y = x^2$ whose x coordinates are respectively $2 - h$ and $2 + h$. Show that the equation of the line through P_1, P_2 is $y = 4x + h^2 - 4$.

Find the limiting position of this line as $h \to 0$.

(ii) Find the derivatives of (a) $x\sin x$ (b) $\log_e(\frac{1}{x})$

QUESTION 7 (10 Marks)

The position of a particle moving along the x-axis is given by $x = 8e^{-2t} - 8 + 16t$, where t is the time in seconds and x is measured in cm.

(i) Show that the particle is at rest when $t = 0$.

(ii) What is the velocity after 1 second? (Give two significant figures.)

(iii) What is the limiting velocity which the particle approaches as t increases?

(iv) Prove that the acceleration is $32 - 2v$, where v is the velocity.

QUESTION 8 (10 Marks)

(i) Find all real roots of the equation $(x^2 + 2x)^2 - (x^2 + 2x) - 6 = 0$

(ii) What is the natural (i.e. largest possible) domain of the function $\log_e x$? For what values of x in the interval $-\pi \le x \le \pi$ has the expression $\log_e(\cos x)$ a real value?

QUESTION 9 (10 Marks)

(i) Differentiate (a) $\frac{1}{\sin x + \cos x}$ (b) $(x^3 + 3x)^{1/3}$

(ii) Use Simpson's Rule to calculate approximately $\int_0^4 f(t)\,dt$ from the following table:

t	0	1	2	3	4
$f(t)$	0	30	50	40	10

If t represents time in seconds and $f(t)$ velocity in cm per second, what is a possible physical interpretation of $\int_0^4 f(t)\,dt$?

QUESTION 10 (10 Marks)

Two dice with their faces numbered 1 to 6 are tossed.

(i) By listing the possible outcomes find the probability of

(a) one die showing a 6 with the other showing at least 4

(b) one die showing a 6 with the other showing 3 or less

(ii) Deduce from the results in part (i) the probability that neither die shows a 6.