N.S.W. DEPARTMENT OF EDUCATION
HIGHER SCHOOL CERTIFICATE EXAMINATION 1983
MATHEMATICS - 2/3 UNIT COURSE

TIME 3 HOURS

Instructions All questions may be attempted. All questions are of equal value. In every question, all necessary working should be shown. Marks will be deducted for careless or badly arranged work.

QUESTION 1

(i) Solve the quadratic equation $5x^2 - 2x - 3 = 0$.

(ii) Express as a rational number: (a) $\dfrac{\sqrt{32}-\sqrt{8}}{3\sqrt{2}}$ (b) $25^{-1/2}$ (c) $\log_4 16$ (d) $\int_0^{\pi/4} \cos 2x\,dx$

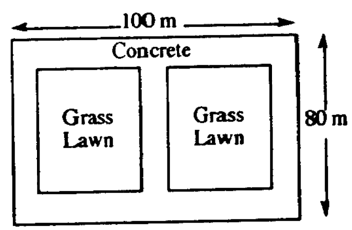
(iii) Differentiate y with respect to x, where $y = \log_e(1+x^3)$

(iv) Given that $f(x) = \cos 3x$, find a value of b such that $f'(b) = 3$

QUESTION 2

(i) Solve the equation: $x = 2(x+2) - 3(x-1)$

(ii) The figure (not to scale) represents a rectangular school yard, of length 100 m and width 80 m, containing two rectangular grassed lawns and concrete paths, each path being of width 2 m. What percentage of the yard area has grass on it?



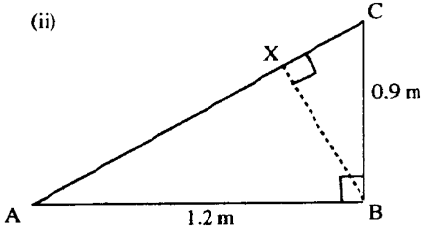
(iii) There are two prizes in a raffle in which 53 tickets are sold. The first prize is obtained by drawing a ticket at random, and this ticket is not replaced for the draw for the second prize. A man buys two tickets in the raffle. What is the probability that:

(a) he wins first prize; (b) he wins both prizes; (c) he does not win a prize?

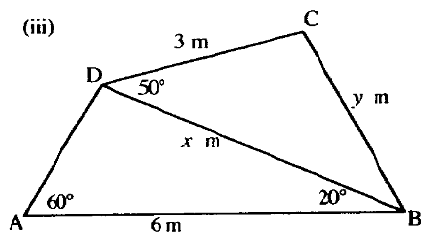
QUESTION 3

(i) Given that $a^2 = b^2 + c^2 - 2bc\cos A$ and that $a = 1.5$ cm, $b = 2$ cm, $c = 3$ cm, find the angle A, correct to the nearest degree.

(ii)



(iii)



(ii) In the figure, the angle B is 90°, and AB = 1.2 m, BC = 0.9 m. Find the angle A, correct to the nearest degree.

In the same triangle, the point X on AC is such that BX is perpendicular to AC. Find the lengths of AX and XC.

(iii) In the figure (not drawn to scale), AB = 6 m, CD = 3 m, ∠DAB = 60°, ∠DBA = 20°, ∠CDB = 50°, DB = x m and BC = y m. Use the sine rule to find x and the cosine rule to find y.

QUESTION 4

(i) The points A, B and O, have coordinates (3, 4), (-4, 3), and (0, 0) respectively, and M is the midpoint of AB. Sketch these points showing the coordinates of M.

(a) Find the equation of the line l_1 which passes through A, and is parallel to OB.

(b) Find the equation of the line l_2 which passes through O and M.

(c) Let C be the intersection of l_1 and l_2. Show that BC is parallel to OA.

(ii) Find the coordinates of the points of intersection, A, B, of the line $y = x + 3$ with the parabola $y = 5x - x^2$.

Find also the equations of the tangents to this parabola at A and B.

The tangent at A meets the y-axis at E, and the tangent at B meets the y-axis at F. Show that the length of EF is 8 units.

QUESTION 5

The function $f(x)$ is defined by the rule $f(x) = 5x\sqrt{x+1}$, in the domain $-1 \le x \le 1$.

(a) Draw up a table of values of $f(x)$, correct to one decimal place, for each of the values $x = -1, -\frac{1}{2}, 0, \frac{1}{2}, 1$.

(b) Use the derivative of $f(x)$ to find the turning point of $f(x)$.

(c) Hence draw a sketch of $f(x)$ showing clearly the turning point, and the values at the end-points of the domain.

(d) The area between the x-axis and the portion of the curve $y = f(x)$ from $x = -1$ to $x = 0$, is rotated about the x-axis. Find the volume of the solid of revolution so formed.

(e) Use Simpson's Rule with three function values, obtained from the table drawn up in (a), to give an estimate of $\int_0^1 5x\sqrt{x+1}\,dx$.

QUESTION 6

(i) (a) Write down the formulae for the nth term and the sum of the first n terms of an arithmetic series with first term a and common difference d.

(b) The first term of an arithmetic series is 10, and the eighth term is 8. What is the sum of the first 21 terms of this series?

(c) The fifth term of an arithmetic series is 14, and the sum of the first ten terms is 165. Find the first term of this series.

(ii) A pair of dice are thrown together at random, and the numbers 1 to 6 on each die are equally likely to appear. Find the probability that:

(a) a 3 or a 5 is thrown, (b) the sum of the two numbers thrown is at least 10.

QUESTION 7

(i) Sketch the function $y = 2\sin 2x$, for x in the domain $0 \le x \le 2\pi$.

Determine the values of x for which $2\sin 2x \ge 1$.

(ii) A point P moves in a straight line so that, t seconds after starting from the origin O, its velocity v metres per second is given by $v = 2 - 3e^{-t}$.

(a) Obtain formulae for the acceleration of P, and the displacement x metres of P from O, at time t.

(b) Find the time at which P is instantaneously at rest, and show that at this time $x = 2\log_e(\frac{3}{2}) - 1$.

(c) Discuss the motion of P for very large values of t.

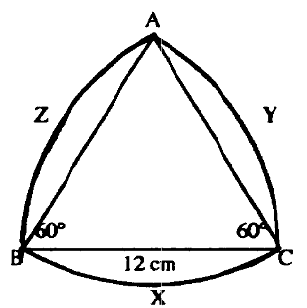
QUESTION 8

(i) In the figure (not drawn to scale), ABC is an equilateral triangle of side 12 cm. The circular arcs AZB, BXC, CYA are constructed with centres C, A, B, respectively. Find:

(a) the length of the arc AZB, and the area of the sector CAZBC;

(b) the length of the perimeter AZBXCYA;

(c) the area enclosed by this perimeter.



(ii) The quadratic expression $Q(x)$ is given by $Q(x) = (1+m)x^2 + 4x + m - 1$.

(a) Find the range of values of m for which the equation $Q(x) = 0$ has no real roots.

(b) Find the range of values of m for which $Q(x) > 0$.

QUESTION 9

(i) The intensity I amps of an alternating current in an electrical circuit is given by the formula $I = Ae^{-at} - 2\cos \pi t$, where A, a are constants and t seconds is the time elapsed after the circuit is switched on.

(a) Given that $I = 0$ at $t = 0$, find A.

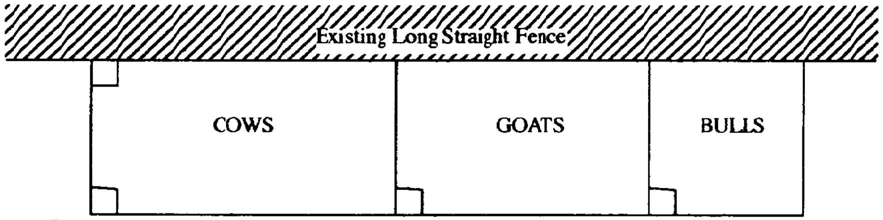
(b) Given also that $I = 1.2$ when $t = 2.5$, find I when $t = 5$.

(ii) Sketch the points A, B whose coordinates are (-1, 0) and (1, 1) respectively.

The point P has coordinates (X, Y). Given that $\angle APB = 90°$, show that the locus of P is the curve whose equation is $X^2 + Y^2 - Y = 1$.

QUESTION 10

(i)



This year Farmer Jones needs to construct three rectangular holding paddocks for cows, goats and bulls, as in the figure, against an existing long straight fence.

If he has 1 km of fencing available, what is the maximum total area of holding paddocks that he can construct?

(ii) Draw a sketch of the curve $y = \log_e(1+x)$, from $x = 0$ to $x = 1$.

The portion of the curve from $x = 0$ to $x = 1$ is rotated about the y-axis. Find the volume of the solid of revolution so formed.