N.S.W. DEPARTMENT OF EDUCATION
HIGHER SCHOOL CERTIFICATE EXAMINATION 1985
MATHEMATICS - 2/3 UNIT COURSE

TIME 3 HOURS

Instructions All questions may be attempted. All questions are of equal value. In every question, all necessary working should be shown. Marks will be deducted for careless or badly arranged work.

Standard integrals are provided; approved calculators may be used.

QUESTION 1

(i) Given that $x^3 = 100\,000$, find x, rounded off correct to the nearest whole number.

(ii) If $S = \pi r(r + 2h)$, find S, rounded off correct to one decimal place, when $r = 1.200$ and $h = 4.005$

(iii) Use the formula $a^2 = b^2 + c^2 - 2bc \cos A$ to find A correct to the nearest degree, when $a = 22$, $b = 12$, $c = 13$, and A lies between 0° and 180°.

(iv) Solve the equation $5(x - 1) - 1 = 29$

(v) A manufacturer increases the price of a car by 20% to a new selling price of \$9000. What was the selling price of the car before this increase?

QUESTION 2

(i) Differentiate $(x + 2)^{21}$.

(ii) Given that $y = \dfrac{e^{2x}}{2x + 1}$, show that $\dfrac{dy}{dx} = \dfrac{4xe^{2x}}{(2x + 1)^2}$

(iii) Find the equation of the tangent to the curve $y = x \sin x$ at the point $(\pi, 0)$.

(iv) The curve $y = 3x^2 + \dfrac{a}{x^2}$ has a turning point at $x = 3$. Find the constant a.

QUESTION 3

(i) Evaluate (a) $\displaystyle\int_1^9 t^{1/2}\,dt$ (b) $\displaystyle\int_0^{\pi/2} \sec^2 \frac{1}{2}\theta \, d\theta$

(ii) Show that if $f(x) = 1 + \sin 2x$, then $f'(x) = 2\cos 2x$. Hence show that $\displaystyle\int_0^{\pi/12} \frac{2\cos 2x}{1 + \sin 2x}\,dx = 0.405$, rounded off correct to 3 decimal places.

(iii) Find the area between the curve $y = \dfrac{1}{(1 + 3x)^2}$, the x-axis and the ordinates $x = 0$ and $x = 1\frac{1}{3}$.

QUESTION 4

(i) Solve the quadratic equation $3x^2 - 4x - 4 = 0$.

(ii) The vertices of the triangle OAB are the points O(0, 0), A(0, 2) and B(3, -1).

(a) Draw a sketch diagram of the triangle.

(b) The point K on AB is such that OK is perpendicular to AB. Find the coordinates of K, and show the point K on your diagram.

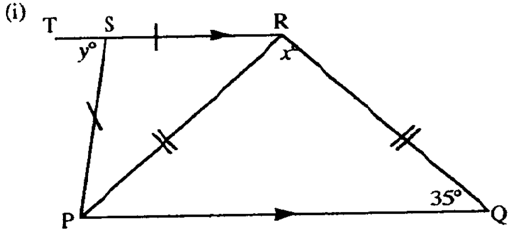
(c) Find the area of the triangle OAB.

(d) The line through the point B, perpendicular to OA, meets KO produced at S. Find the coordinates of S.

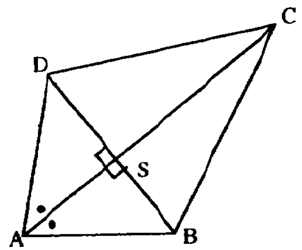
(e) Verify that AS is perpendicular to OB.

QUESTION 5

(i)



(ii)



(i) The diagram (not to scale) shows a quadrilateral PQRS, in which PQ ∥ SR, PS = SR, and PR = RQ. Also T is a point on RS produced.

(a) Draw a neat sketch of this diagram in your answer book.

(b) Given that $\angle RQP = 35°$, and $\angle PRQ = x°$, find x, giving reasons.

(c) If also $\angle TSP = y°$, find y, giving reasons.

(ii) In the diagram (not to scale), ABCD is a quadrilateral. The diagonals AC, BD intersect right angles, and $\angle DAS = \angle BAS$.

(a) Draw a neat sketch of the above diagram in your book.

(b) Explaining the reasons for each step, use congruent triangles to prove that DA = AB.

(c) Hence prove that DC = CB.

QUESTION 6

(i) A solid of revolution is formed by rotating about the x-axis the portion of the curve $y = 2(1 + e^{2x})$ between the ordinates $x = 0$ and $x = \frac{1}{2}$.

Show that the volume of this solid is $\pi(e^2 + 4e - 3)$ cubic units.

(ii) The numbers 1, 2, 3, 4 are painted on to four otherwise similar balls. The balls are placed in a bag, and one ball is drawn out at random. The number on this ball is written down and the ball is then replaced in the bag. A second ball is then drawn at random and the number on it written down. Using a tree diagram, or otherwise, what is the probability that the sum of the two written numbers is 6?

(iii) 100 green tickets and 100 red tickets are sold in a raffle in which there are three prizes. One ticket is drawn for first prize, discarded, and then a second ticket is drawn for the second prize. This is also discarded before the third prize is drawn.

(a) What is the probability that all three prizes are won by green tickets?

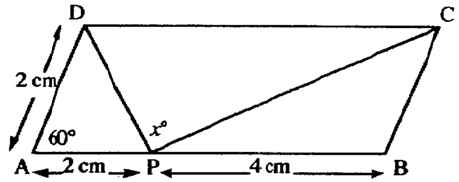
(b) What is the probability that at least one green ticket wins a prize?

QUESTION 7

(i) (a) For an arithmetic series of first terms a and common difference d, write down the formula for the nth term. Use this formula to prove that the sum of the first n terms of this series is $\frac{n}{2}\{2a + (n-1)d\}$.

(b) The sum of the first seven terms of an arithmetic series is five times the seventh term. Also the sum of the sixth and seventh term is 40. Find the sum of the first ten terms of the series.

(ii) In the figure (not to scale), ABCD is a parallelogram in which AB = 6 cm, AD = 2 cm, and $\angle DAB = 60°$. The point P on AB is such that AP = 2 cm, and $\angle DPC = x°$.



(a) Write down the length of DP.

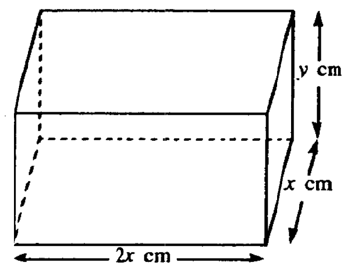
(b) Use the cosine rule for each of the triangles PBC, PCD to show that $\cos x° = -\frac{\sqrt{7}}{14}$.

QUESTION 8

(i) (a) Draw on a sketch diagram the lines $y = x$ and $y = x + 1$.

(b) Indicate on your diagram, by shading, the region of the (x, y) plane determined by those points which satisfy all the inequalities $|x| \le 1$ and $y \ge x$ and $y \le x + 1$.

(ii) An open rectangular box has four sides and a base, but no lid, as in the figure.



The dimensions of the base are x cm, $2x$ cm, and the height is y cm.

(a) Write down the formulae for the area A cm^2 of the outer surface of the box, and the volume V cm^3 contained by the box.

(b) Given that A = 150, eliminate y to obtain a formula $V(x)$ for the volume as a function of x. Hence

(α) show that $x \le 5\sqrt{3}$.

(β) find the value of x for which V is maximum and verify that the maximum value of V is $\frac{500}{3}$.

QUESTION 9

(i) t minutes after a jet engine starts operation, the rate of fuel burn, R kg per minute, is given by the relation $R = 10 + \frac{10}{1+2t}$.

(a) Draw a sketch of R as a function of t.

(b) What is the rate of burn, R, after 7 minutes?

(c) What value does R approach as t becomes very large?

(d) Calculate the total amount of fuel burned in the first 7 minutes.

(ii) The acceleration a metres per second per second of a moving object is given at time t seconds ($t \geq 0$) by $a = 4\pi^2 \cos \pi t$.

At time $t = 0$, the object is at the point $x = 0$, and travelling with velocity $V = 2\pi$ metres per second.

(a) Find the velocity V and the displacement x as a function of t, for $t \geq 0$.

(b) Find, for t in the range $0 \leq t \leq 4$, the values of t for which the object is stationary.

(c) Show that, for t in the range $0 \leq t \leq 4$, the largest value of x is $2(2 + \sqrt{3}) + \frac{19\pi}{3}$.

QUESTION 10

(i) A parabola whose equation is of the form $y = Bx^2$ (where B is a constant), has the line $20x - y + 20 = 0$ as a tangent.

(a) Find the value of B.

(b) Sketch the parabola and the tangent line, showing the coordinates of the point of contact.

(c) Find the coordinates of the focus and the equation of the directrix of the parabola.

(iii) A population $N(t)$ varies with time according to the law $N(t) = Ce^{kt}$, where C, k are positive constants and $t \geq 0$.

(a) Show that if a, b are two positive numbers such that $a + b = 1$, then $N(at + bu) = (N(t))^a (N(u))^b$ for any $t \geq 0$, $u \geq 0$.

(b) Hence, or otherwise, find N(13), given that N(3) = 10 and N(18) = 100.