N.S.W. DEPARTMENT OF EDUCATION
HIGHER SCHOOL CERTIFICATE EXAMINATION 1981
MATHEMATICS - 2 UNIT COURSE

TIME 3 HOURS

Instructions All questions may be attempted. All questions are of equal value. In every question, all necessary working should be shown. Marks will be deducted for careless or badly arranged work.

QUESTION 1 (10 Marks)

(i) Express $\dfrac{1}{4-\sqrt{13}}$ in the form $a+b\sqrt{13}$, where a, b are rational numbers.

(ii) Evaluate $\int_0^1 e^{3x}\,dx$.

(iii) Solve the equation $\frac{1}{3}(x+7)-\frac{1}{6}(x-2)=2$.

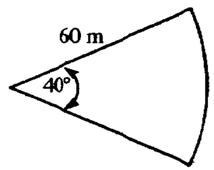
(iv) Find the exact value of $49^{-1/2}\times 27^{2/3}$.

(v) For what values of x is $2x^2 \geq x$?

QUESTION 2

(i) Express as a single fraction in its simplest terms $\dfrac{2x-3}{2}-\dfrac{x-1}{5}$.

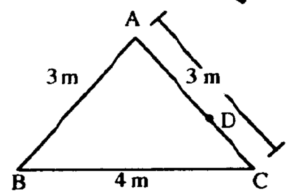
(ii) A playing field is in the form of a sector of a circle, of radius 60 m, as in the diagram (which is not to scale). Find, correct to the nearest square metre, the area of the field. (You may use the approximate value of $\frac{22}{7}$ for π.)



(iii) In the diagram (not drawn to scale), AB = 3 m, AC = 3 m and BC = 4 m. The point D on AC is such that AD = 2 m.

(a) Find the exact value of cos A.

(b) Calculate the length of BD, correct to 3 significant figures.



QUESTION 3

(i) Differentiate with respect to x: (a) $(5-3x)^{10}$ (b) $x^3\log_e 2x$.

(ii) Find the gradient of the tangent to the curve $y = x^{1/2}$ at the point (9, 3).

(iii) The function $f(x)$ is defined by the rule $f(x)=9x(x-2)^2$ in the domain $-1 \leq x \leq 3$. Draw a sketch of the graph of $y=f(x)$, showing clearly the turning points, the intercepts with the x and y axes, and the values at the end-points of the domain. What is the range of $f(x)$?

QUESTION 4 (10 Marks)

Without using squared paper, plot on the Cartesian plane the three points A, B, C, whose co-ordinates are (-5, 3), (1, -5), (2, 2), respectively.

(i) Calculate the length AB.

(ii) Find the equation of the line AB.

(iii) The line through C, perpendicular to AB, meets AB at N. Find the coordinates of N.

(iv) Hence, or otherwise, find the area of the triangle ABC.

QUESTION 5 (10 Marks)

(i) Solve the quadratic equation $x(2x-3)=5$.

(ii) Find all values of θ such that $\cos\theta = \frac{1}{2}$, and $0° \le \theta \le 360°$. Also find the exact value of $\sin\theta$ for each of these values of θ.

(iii) The first term of an arithmetic series is 2, and the fourth term is three times the third term. Find the sum of the first ten terms of the series.

QUESTION 6 (10 Marks)

(i) The roots of the quadratic equation $px^2 - x + q = 0$ are -1, 3. Find p and q.

(ii) Use Simpson's Rule with three function values to give an estimate of $\int_{-\pi/3}^{\pi/3} \cos^2 x\, dx$.

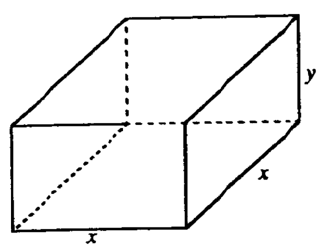
(iii) Draw a sketch of the curve $y = \frac{1}{\sqrt{1+x}}$, from $x=0$ to $x=3$.

The region bounded by this curve, the x-axis and the ordinates $x=0$ and $x=3$ is rotated about the x-axis. Find the volume of the solid so formed.

QUESTION 7 (10 Marks)

(i) The third term of a geometric series is 54, and the sixth term is 2. Find the common ratio, and the sum of the first 6 terms.

(ii) A rectangular box, open at the top, is to be constructed out of thin sheet metal on a square base of side x units, as in the figure.



(a) If the box holds a volume of 500 cubic units, and its height is y units, find a formula for y in terms of x.

(b) Show that the area A square units of sheet metal required is given by $A = x^2 + \frac{2000}{x}$.

(c) Hence find the least area of sheet metal required to make the box.

QUESTION 8 (10 Marks)

(i) (a) Find the points of intersection of the line $y = x + 1$ with the parabola $y = 2x^2$.

(b) Also find the area enclosed between this line and the given parabola.

(ii) For what values of m does the line $y = m(x+1)$ have no intersection with the parabola $y = 2x^2$?

(iii) Find the equations of the two tangents to the parabola $y = 2x^2$ which pass through the point (-1, 0).

QUESTION 9 (10 Marks)

(i) "In a certain State, of population 4.8 million, about 1200 persons are killed on the roads each year. Therefore, there is an approximately 1 in 4000 chance of a particular person being killed in a road accident each year."

Is this statement valid? Give brief reasons for your answer.

(ii) In a game of chess between two players X, Y, of about equal ability, the player with the White pieces, having the first move, has a probability of 0.5 of winning, and the probability that the player with the Black pieces wins is 0.3. What is the probability that the game ends in a draw (i.e. neither player wins)?

The two players X, Y, play each other twice in a chess competition, each player having the White pieces once. In the competition a player who wins a game scores 1 point, a player who loses a game scores 0 points, and a draw scores as $\frac{1}{2}$ point to each player. What is the probability that, as a result of these two games,

(a) X scores 2 points? (b) X scores less than $1\frac{1}{2}$ points?

QUESTION 10 (10 Marks)

(i) A particle is projected vertically upwards at time $t=0$ from a fixed point B, above the ground. After t seconds the height h metres that the particle is above the ground is given by $h = \frac{49}{5}(3 + 2t - t^2)$. Find

(a) the height of B above the ground;

(b) the maximum height of the particle above the ground;

(c) the time of flight of the particle from the instant of projection until it strikes the ground.

(ii) S is the point (1, 0), l is the line $x=4$, and P is the point (x, y).

(a) Find the distance SP.

(b) Find the perpendicular distance of P from l.

(c) Hence show that the equation of the locus of points P, such that the perpendicular distance of P from l is twice the distance SP, is $3x^2 + 4y^2 = 12$.