N.S.W. DEPARTMENT OF EDUCATION
HIGHER SCHOOL CERTIFICATE EXAMINATION 1980
MATHEMATICS - 2 UNIT COURSE

TIME 3 HOURS

Instructions All questions may be attempted. All questions are of equal value. In every question, all necessary working should be shown. Marks will be deducted for careless or badly arranged work.

QUESTION 1 (10 Marks)

(i) Differentiate with respect to x: (a) $x^{2.7}$ (b) $\cos(\log x)$ (c) $x\sqrt{1+x}$

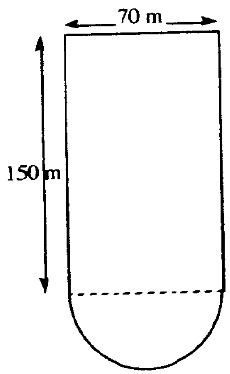
(ii) (a) Find an indefinite integral (primitive function) of $\dfrac{2x}{x^2+1}$

(b) Find the exact value of $\int_{\pi/3}^{\pi} \cos(\frac{1}{2}x)\,dx$.

(iii) Given that $f(x) = x^2 + x$, find the values of b for which $f''(b) = f(b)$.

QUESTION 2 (10 Marks)

(i) A sporting field is in the shape of a rectangle with a semicircle at one end as shown in the diagram.



Using the approximate value of $\frac{22}{7}$ for π, find:

(a) the area of the entire field;

(b) the total cost of fencing the boundary of the field at a cost of $30 per metre.

(ii) Solve the equation $\frac{1}{2}(x+2) - \frac{1}{5}(x-3) = 1$

(iii) Factorize completely $3x^2 - 12y^2$.

QUESTION 3 (10 Marks)

(i) The line $x = 1$ meets the curve $y = x^3 + 5$ at P. Write down the coordinates of P.

(ii) Find the equation of the tangent line l to the curve $y = x^3 + 5$ at P.

(iii) Verify that Q(-5, -12) lies on the line l.

(iv) The curve $y = x^3 + 5$ meets the y-axis at R. Find the equation of the line QR.

(v) If θ is the acute angle between PQ and QR, find the exact value of $\tan\theta$

QUESTION 4 (10 Marks)

(i) Given that $\sin\theta = \frac{3}{4}$ and $0° < \theta < 90°$, find, as a single expression with rational denominator, the exact

values of: (a) $\cos\theta$; (b) $\cos\theta + \tan\theta$.

(ii) Find all values of θ such that $\sin 2\theta = 1$ and $0 \le \theta \le 2\pi$.

(iii) Solve the inequality $x^2 - 4x > 0$.

(iv) The first term of an arithmetic series is 1, and the 26th term is 2. Find the sum of the first 50 terms of the series.

QUESTION 5 (10 Marks)

(i) Sketch the parabola $16y = x^2$, and write down the coordinates of the focus S.

(ii) Find the equation of the tangent to this parabola at the point P with co-ordinates (4, 1).

(iii) The straight line joining P to the focus S intersects the parabola again at R. Find the coordinates of R, and the equation of the tangent at R.

(iv) Show that the tangents at P and R intersect on the directrix.

QUESTION 6 (10 Marks)

(i) Six cards labelled A, B, C, D, E and F, are drawn, one at a time, from a hat. What is the probability that card A or card E will be the third card drawn?

(ii) A box contains 5 good and 3 defective light bulbs. Two are drawn at random.

(a) What is the probability that the first one drawn is defective?

By drawing a tree diagram, or otherwise, calculate the probability that the two light bulbs drawn are:

(b) both defective; (c) both good; (d) one defective and one good.

QUESTION 7 (10 Marks)

(i) (a) Write down an expression for the sum of n terms of a geometric series with first term a and common ratio r, where $r \neq 1$.

For what values of r does this series have a limiting sum as n increases indefinitely?

(b) By writing the recurring decimal 0.3030 3030... as an infinite geometric series, express it as a rational number in its lowest terms.

(ii) Show that the volume obtained by rotating about the x-axis the area beneath the curve $y = e^x$, from $x = 0$ to $x = 2$, has magnitude $\frac{\pi}{2}(e^4 - 1)$.

(iii) The points P, Q have coordinates (-1, 0) and (3, 3) respectively, and a point R has coordinates (x,y).

Find a condition that PR is perpendicular to QR, and hence show that the equation of the locus of points R satisfying this condition is $x^2 + y^2 - 2x - 3y - 3 = 0$.

QUESTION 8 (10 Marks)

(i) The function $f(x)$ is defined by the rule $f(x) = \begin{cases} 0 \text{ if } x \le 0 \\ 2x \text{ if } x > 0 \end{cases}$

(a) Sketch the function $f(x)$, from $x = -2$ to $x = 2$ (b) Evaluate $\int_{-2}^{2} f(x)\,dx$

(ii) The function $f(x)$ is defined by rule $f(x) = x^3 - 3x^2$ in the domain $-1 \le x \le 3$.

(a) Draw a sketch of the graph of $y = f(x)$, showing clearly the turning points, the intercepts with the x- and y-axes, and the values at the extremes of the domain.

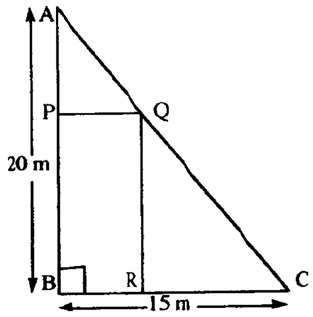
(b) Indicate on your sketch the region bounded entirely by parts of the graph of $y = f(x)$ and the x-axis. Find the area of this region.

QUESTION 9 (10 Marks)

(i) (a) Given that $x = 3$ is the root of the quadratic equation $mx^2 - 20x + m = 0$, find the exact value of the other root.

(b) Find all values for k for which the quadratic equation $kx^2 - 8x + k = 0$ has real roots.

(ii) In the triangle ABC, AB = 20 m, BC = 15 m, and $\angle ABC = 90°$. BPQR is a rectangle inscribed in ABC, as in the figure, with $PQ = x$ metres.

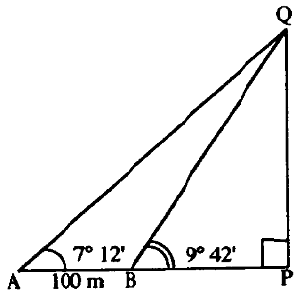


(a) Find the length of AP in terms of x, and hence show that the area of the rectangle BPQR is $x\,(20 - \frac{4}{3}x)$ square metres.

(b) Hence find the maximum possible area of the rectangle BPQR.

QUESTION 10 (10 Marks)

(i) The diagram given was sketched by a surveyor, who measured the angle of elevation of a tree top on the other side of a river to be 7° 12' at the point A. At the point B, 100 metres directly towards the tree from A, the angle of elevation was 9° 42'.



(a) Derive an expression for the height of the tree.

(b) Calculate the height of the tree correct to three significant figures.

(ii) Two particles P and Q move along a line l, their displacements at time $t \ge 0$ with respect to a fixed point 0 on l being $x(t)$ and $X(t)$ respectively.

(a) Given that $\frac{d^2x}{dt^2} = 6 + e^{-t}$, and that $\frac{dx}{dt} = -1$ at $t = 0$, and $x = 0$ at $t = 0$, find an expression for $x(t)$.

(b) If $X(t) = 2\sin 5t + 3t^2 + 2$, prove that $X(t) > x(t)$ for all $t \ge 0$.

Explain this result in terms of the motions of the particles P and Q.