N.S.W. DEPARTMENT OF EDUCATION
HIGHER SCHOOL CERTIFICATE EXAMINATION 1969
MATHEMATICS PAPER C (2S) (EQUIVALENT TO 2 UNIT)

Instructions: Time 3 hours. All questions may be attempted. In every question, all necessary working should be shown. Marks will be deducted for careless or badly arranged work.

QUESTION 1 (12 Marks)

(i) Determine the axis of symmetry and the coordinates of the vertex of the parabola $y = 3x^2 - 6x + 1$.

(ii) Use shading to indicate on a diagram the region in which the following inequalities hold simultaneously: $y + 1 < 0,\quad x + y - 3 < 0,\quad x > 4.$

(iii) Express $\dfrac{1}{\sqrt{50} - 7}$ with a rational denominator, and hence give its value as a decimal with four significant figures $\{\sqrt{50} = 7.071\}$.

(iv) What is the probability of a year selected at random but which is not a leap year having fifty-three Sundays?

QUESTION 2 (9 Marks)

(i) State the periods of the following functions(a) $\tan x$ (b) $\sin(3x - \pi)$

(ii) Find (a) $\dfrac{d}{dx}(2\cos\frac{1}{2}x)$ (b) $\dfrac{d^2(\sin x + \cos x)}{dx^2}$

¶(iii) Find the angle of intersection of the lines $2x + y = 1, 3x = 6y + 1$.

QUESTION 3 (9 Marks)

(i) Find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$ of the curve $y = e^{-x^2/2}$. Hence state for what values of x the curve is concave down.

(ii) Write down a primitive function of each of (a) $\dfrac{1}{2x+3}$ (b) $-\sin\pi x$ (c) $\dfrac{x^4}{5}$

(iii) Two bags contain respectively 2 red, 3 black balls and 4 red, 7 black balls. A bag is chosen at random, then a ball is drawn at random. What is the probability this ball is red?

QUESTION 4 (10 Marks)

(i) The gradients of the lines joining P(x, y) to $(-2, 0)$ and to $(0, 0)$ are m and m' respectively, and it is given that $m = \dfrac{m' - m}{1 + mm'}$. Show that the locus of P is a circle and state its radius.

(ii) What well known geometrical result about a circle is illustrated by part (i)?

QUESTION 5 (10 Marks)

(i) Differentiate $\log_e(\log_e x)$. Hence evaluate $\displaystyle\int_e^{e^2} \frac{dx}{x\log_e x}$

(ii) Show that the line $y - x + 1 = 0$ is a tangent to the parabola $4y = x^2$ and give the coordinates of its point of contact.

QUESTION 6 (10 Marks)

(i) For n an arbitrary positive integer, find an expression for the sum:

$$3^n - 3^{n+1} + 3^{n+2} - 3^{n+3} + \ldots + 3^{3n}$$

(ii) Illustrate on a real number line the set x such that $|x - 1| + |x + 1| > 2$

QUESTION 7 (10 Marks)

Sketch, not on squared paper, the graphs of $y = 2\sin x$, $y = \sin 2x$, $y = 2\sin x + \sin 2x$ for $0 \le x \le 2\pi$ on the same set of axes.

Find the areas under the curve $y = 2\sin x + \sin 2x$ between

(a) $x = 0$ and $x = \dfrac{\pi}{2}$ (b) $x = \dfrac{\pi}{4}$ and $x = \dfrac{3\pi}{4}$

QUESTION 8 (10 Marks)

(i) State the natural (i.e. the largest possible) domain of the function given by $y = \sqrt{1+x} - \sqrt{1-x}$.

(ii) Show that the function is monotonically increasing throughout its domain.

(iii) What are its greatest and least values?

QUESTION 9 (10 Marks)

(i) The mass m of bacteria in a medium with ample food increases at a rate proportional to m. In one hour this mass increases from 1000 micrograms to 1200 micrograms. What will be the mass of the bacteria after 10 hours?

(ii) Find a constant k such that $y = e^x\cos x$ satisfies $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + ky = 0$.

QUESTION 10 (10 Marks)

(i) A particle moves along a straight line, its distance from the origin at time t being given by $x = t^3 - 4t^2 + 5t$. When does the particle come to rest?

(ii) A particle moves in a straight line with acceleration at time t being given by $\dfrac{d^2x}{dt^2} = \cos t - 2\cos 2t$.

When $t = 0, x = 0$ and $\dfrac{dx}{dt} = 0$.

Determine x as a function of t and find the maximum distance from the origin reached by the particle.