HIGHER SCHOOL CERTIFICATE EXAMINATION 1992
MATHEMATICS - 2/3 UNIT

Direction to Candidates

Time allowed - Three hours (includes reading time)

All questions may be attempted. All questions are of equal value. All necessary working should be shown in every question. Marks may not be awarded for careless or badly arranged work.

Standard integrals are provided; approved calculators may be used.

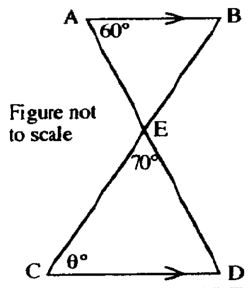
QUESTION 1

(a) Find the average of the numbers 75, 29, 80, 42, 65, 59, 38. Give your answer correct to one decimal place.

(b) Factorize $x^2 + 5x - 6$.

(c) Solve $4x + 7 = 3(x - 2)$.

(d) In the diagram AB is parallel to CD. Find the value of θ. Give reasons.



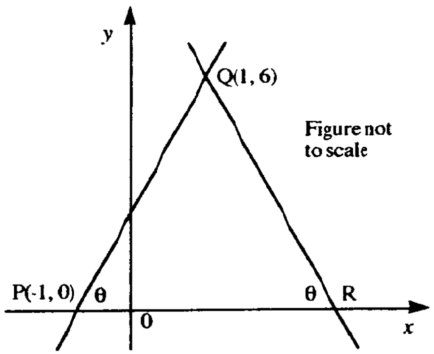
(e) Solve the pair of simultaneous equations $x + y = 2, 2x - y = 7$.

(f) A sum of \$10 000 is placed in a bank account and earns 12% interest per annum, compounded annually.

How much money is in the account at the end of 6 years, just after the final interest has been paid.

QUESTION 2

In the diagram P and Q have coordinates (-1, 0) and (1, 6) and $\angle QPR = \angle QRP = \theta$. Copy the diagram.



(a) Find the coordinates of the midpoint of PQ.

(b) Show that PQ has equation $y = 3x + 3$.

(c) Show that $\tan\theta = 3$.

(d) Show that the gradient of QR is -3.

(e) Show that the equation of QR is $3x + y - 9 = 0$.

(f) Find the coordinates of R.

(g) Find the perpendicular distance from P to QR.

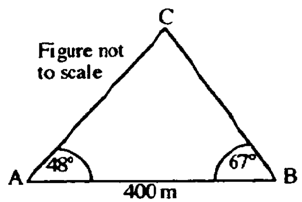
(h) On your diagram, shade in the region satisfying both the inequalities: $y \le 3x + 3$ and $3x + y - 9 \ge 0$.

QUESTION 3

(a) Differentiate: (i) e^{3x} (ii) $\ln(5x - 1)$ (iii) $x\tan x$.

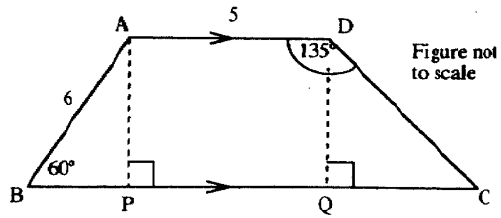
(b) Find: (i) $\int (2x + 3)^{10} dx$ (ii) $\int \sin\frac{x}{2} dx$

(c) Use the sine rule to calculate the length of the side BC to the nearest metre.



QUESTION 4

The diagram shows a trapezium ABCD in which AD is parallel to BC. AB = 6, AD = 5, $\angle ABC = 60°$, and $\angle ADC = 135°$. Perpendiculars are drawn from A and D to meet BC at P and Q.



(i) Show that BP = 3

(ii) Show that AP = $3\sqrt{3}$

(iii) Find the exact value of BC.

(iv) Find the area of the trapezium ABCD. Leave your answer in surd form.

(b) Pat and Chris each throw a die.

(i) Find the probability that they throw the same number.

(ii) Find the probability that the number thrown by Chris is greater than the number thrown by Pat.

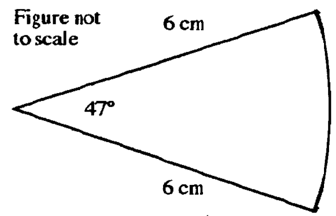
(c) Ten kilograms of sugar is placed in a container of water and begins to dissolve. After t hours the amount A kg of undissolved sugar is given by $A = 10e^{-kt}$.

(i) Calculate k, given that $A = 3.2$ when $t = 4$.

(ii) After how many hours does 1 kg of sugar remain undissolved?

QUESTION 5

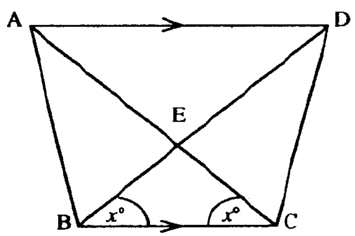
(a)



(a) The diagram shows the sector of a circle.

Find the area of this sector. Give your answer to the nearest square centimetre.

(b)



(b) In the diagram AD is parallel to BC and $\angle DBC = \angle ACB = x°$.

(i) Show that AE = DE

(ii) Prove that the triangles ABC and DCB are congruent.

(ii) Deduce that $\angle ABD = \angle DCA$.

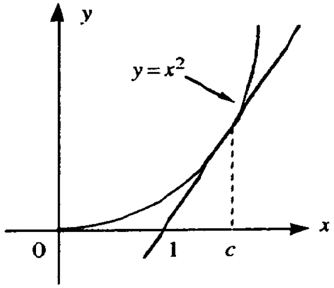
(c) A timber worker stacks log. The logs are stacked in layers, where each layer contains one log less than the layer below. There are five logs in the top layer, six logs in the next layer, and so on. There are n layers altogether.

(i) Write down the number of logs in the bottom layer.

(ii) Show that there are $\frac{1}{2}n(n + 9)$ logs in the stack.

QUESTION 6

(a)



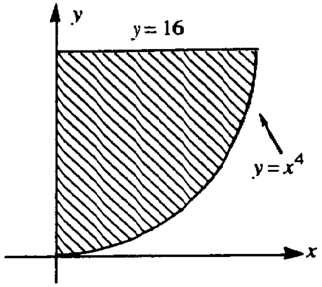
(a) The diagram shows a graph of the parabola $y = x^2$ and the tangent to the parabola at $x = c$.

(i) Find the gradient of the tangent at $x = c$.

(ii) Find the equation of the tangent at $x = c$.

(iii) Find the value of c if the tangent intersects the x-axis at $x = 1$.

(b)



(b) The shaded region in the diagram is bounded by the curve $y = x^4$, the y-axis and the line $y = 16$.

Calculate the volume of the solid of revolution formed when this region is rotated about the y-axis.

(c) The speed of a train was recorded at intervals of one minute. The times, in minutes, and the corresponding speeds v, in kilometres per hour, are listed in the table below.

time (min)	0	1	2	3	4
v (km/h)	0	25	34	30	40

(i) Explain why the distance x, in km, travelled by the train in these four minutes is given by $x = \int_0^{1/15} v\,dt$.

(ii) Estimate x by using Simpson's Rule with five function values.

QUESTION 7

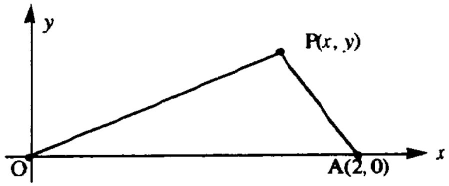
(a) Solve the equation $3^{2x} + 2.3^x - 15 = 0$.

(b) (i) Write down the gradient of AP in terms of x and y.

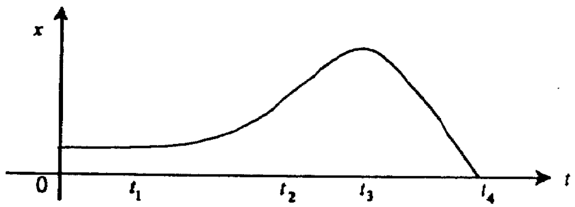
(ii) Show that the equation of the locus of all points P such that OP is perpendicular to AP is $x^2 - 2x + y^2 = 0$.

(iii) Deduce that the locus of all points such that OP is perpendicular to AP is a circle.

Write down the centre and radius of this circle.



(c)



A particle moves in a straight line and the above graph shows the distance x of the particle from a fixed point at time t.

(i) Is the particle moving faster at time t_1 or at time t_2 ? Why?

(ii) What is the velocity at time $t = 0$? Why?

(iii) Sketch the graph of the velocity v as a function of t.

QUESTION 8

(a) Twenty-five kangaroos were released on an island. The population of P kangaroos on the island t years later is given by $P = -t^3 + 6t^2 + 25$, for $0 \le t \le 6$.

(i) After how many years was the population a maximum?

(ii) What was the maximum population?

(iii) Sketch the curve $P = -t^3 + 6t^2 + 25$, for $0 \le t \le 6$.

(iv) When was the population increasing most rapidly?

(b) A box contains twelve chocolates all exactly the same appearance. Four of the chocolates are hard and eight are soft. Kim eats three chocolates chosen randomly from the box. Using a tree diagram, or otherwise, find the probability that:

(i) the first chocolate Kim eats is hard;

(ii) Kim eats three hard chocolates;

(iii) Kim eats exactly one hard chocolate.

QUESTION 9

(a) The diagram shows part of the curves $y = \sin x$ and $y = \cos x$..

(i) Find the x coordinates of the two marked points A and B.

(ii) Calculate the area of the shaded region.

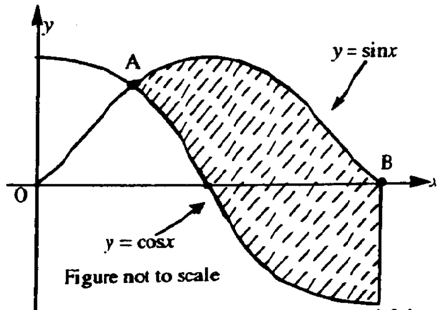


Figure not to scale

(b) (i) Sketch the parabola P which has focus (2, 3) and directrix $y = -1$.

(ii) Find the equation of P.

(c) Let m be a negative number. Show that the equation $\sin x = mx$ has $x = 0$ as its only solution satisfying $-\pi \le x \le \pi$.

QUESTION 10.

(a) (i) For what values of r does the geometric series $a + ar + ar^2 + \ldots +$ have a limiting sum?

For these values of r write down the limiting sum.

(ii) Find a geometric series with common ratio $\frac{1}{w}$ that has limiting sum $\frac{1}{1-w}$.

(b) A truck is to travel 1000 kilometres at a constant speed of v km/h. When travelling at v km/h, the truck consumes fuel at the rate of $(6 + \frac{v^2}{50})$ litres per hour.

The truck company pays 50 cents/litre for fuel and pays each of the two drivers \$20 per hour whilst the truck is travelling.

(i) Let the total cost of fuel and the drivers' wages for the trip be C dollars. Show that $C = 10v + 43\,000\frac{1}{v}$.

(ii) The truck must take no longer than 12 hours to complete the trip, and speed limits require that $v \leq 100$.

At what speed v should the truck travel to minimize the cost C?