N.S.W. DEPARTMENT OF EDUCATION
HIGHER SCHOOL CERTIFICATE EXAMINATION 1982
MATHEMATICS - 2 UNIT COURSE

TIME 3 HOURS

Instructions All questions may be attempted. All questions are of equal value. In every question, all necessary working should be shown. Marks will be deducted for careless or badly arranged work.

QUESTION 1

(i) Express as integers (a) $\sqrt{7} \times \sqrt{63}$ (b) $8^{-2/3} \times 4^{3/2}$ (c) $\log_2 8$

(ii) Solve the simultaneous equation $a + 3d = 1$, $a + 53d = 101$.

(iii) Solve the quadratic equation $3x^2 = 5x - 2$

(iv) Find the exact value of (a) $\int_1^2 \frac{dx}{x}$ (b) $\int_{\pi/3}^{\pi/2} \cos x \, dx$.

QUESTION 2

(i) Solve the equation $2(x+3) = 3(x-1)$

(ii) In the triangle ABC, the angle B is 90°, AB = 6 m and AC = 7 m. Find

(a) the size of angle A, correct to the nearest degree;

(b) the length of BC, in metres, correct to two decimal places.

(iii) In the formula $v^2 = u^2 - 2fs$, find s, given that $v = 3$, $u = 4$ and $f = -3$.

(iv) (a) A rectangular sheet of plate steel has length 20 m, width 10 m and thickness 10 mm. Find the volume of the sheet in cubic metres.

(b) Six equal circular holes, each of radius 2 m, are cut in the sheet so that none of the holes intersect.

Assuming the approximate value of 3.142 for π, obtain, in cubic metres correct to two decimal places, the volume of steel remaining in the sheet. (The volume of a right circular cylinder of radius r and height h is $\pi r^2 h$).

QUESTION 3

(i) (a) Find the equation of the straight line PQ, given that P has coordinates (3, 4) and that PQ is parallel to the line whose equation is $y = 2x + 9$.

(b) The line PQ intersects the x-axis at Q. Find the coordinates of the midpoint of PQ.

(ii) Use Simpson's Rule with 3 function values to give an estimate of $\int_0^1 4^x dx$.

(iii) A particle moves from rest at a point O in a straight line, so that t seconds after leaving O its velocity, v metres/s, is given by $v = 4(t - t^3)$. Find

(a) the time taken for the particle to come to rest again;

(b) the distance it travels during this time.

QUESTION 4

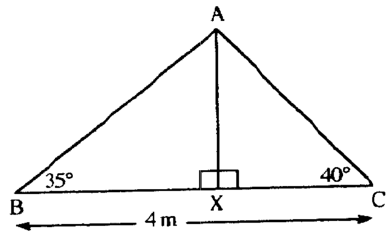
(i) Two cards are drawn successively at random (without replacement) from a pack of playing cards. You may assume that the pack has 52 cards, with four suits (spades, hearts, diamonds and clubs) of 13 cards each. What is the probability that:

(a) the first card drawn is a spade?

(b) both cards are spades?

(c) both cards are the same suit (not necessarily spades)?

(ii) In the diagram (not to scale), the angle B = 35°, the angle C = 40°, and BC = 4 m.



(a) Apply the sine rule to the triangle ABC to write down an expression for the length of AC.

(b) Hence find the length of AX, in metres, correct to two decimal places.

{X is the foot of the perpendicular from A to BC.}

QUESTION 5

(i) Differentiate $\cos(1 + x^3)$ with respect to x.

(ii) Find the area between the curve $y = x^4$, the x-axis, and the ordinates $x = 1$ and $x = 2$.

(iii) The function $f(x)$ is defined by the rule $f(x) = 4x e^{-2x}$, in the domain $-\frac{1}{2} \le x \le 2$.

(a) Draw up a table of values of $f(x)$, correct to one decimal place, for each of the values $x = -\frac{1}{2}, 0, \frac{1}{2}, 1, 1\frac{1}{2}, 2$.

(b) Use the derivative of $f(x)$ to find the coordinates of the turning point of $f(x)$, and determine whether it is a maximum or minimum.

(c) Hence draw a sketch of $f(x)$, showing clearly the turning point and the values at the end points of the domain.

QUESTION 6

(i) For what values of x is $x^2 \ge (x+1)(x+2)$?

(ii) Write down a single expression for the sum of the first n terms of the geometric series $1 + x^2 + x^4 + x^6 + \ldots$

What is the sum to infinity when $x = \frac{1}{3}$?

(iii) The first term of an arithmetic series is 4, and the fifth term is four times the third term. Find the common difference.

(iv) The parabola $y = ax^2 + bx + c$ has its vertex at (2, 1) and passes through the point (0, 0). Find a, b, and c.

QUESTION 7

(i) Given that $\cos x = \frac{4}{5}$, and $0 < x < \frac{\pi}{2}$, find the exact values for (a) $\sin x$ (b) $\cos 2x$.

(ii) (a) The coordinates of P are (2, 1). Show that P lies on both the parabolas $4y = x^2$ and $4y = (x - 4)^2$. Show also that P is the only point of intersection of the two curves.

(b) Find the equation of the tangent at P to the parabola $4y = (x - 4)^2$. Also find the coordinates of the other point Q at which this tangent intersects the parabola $4y = x^2$.

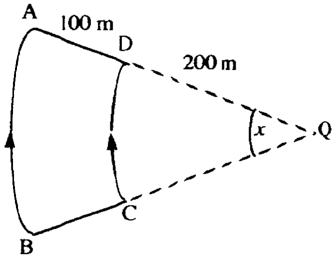
QUESTION 8

(i) The mass M in grams of a radioactive substance may be expressed as $M = Ae^{-kt}$, where t is the time in years, and k is a constant.

(a) At time $t = 0$, $M = 10$. Find A.

(b) After 5 years the mass is 9 grams. Find the mass after 20 years.

(ii) In the figure, AB and CD are circular arcs which subtend an angle x radians at the centre Q, where $0 < x < \pi$, and AQ, BQ are radii. The length AD is 100 metres, and DQ = 200 metres.



(a) What is the length of each of the arcs AB and DC?

(b) A man lives at A, and there is a bus stop at B, with the paths AB, BC, CD and DA in the figure forming a road system. For what values of x is it shorter for the man to walk along the route ADCB rather than along the arc AB?

(c) Given $x = \frac{\pi}{5}$, find the area enclosed by the roads linking A, B, C, D.

QUESTION 9

(i) (a) Plot (not on squared paper) the points A(1, 0), B(4, 0) and C(0, 2) on a sketch diagram.

(b) Show that the length CB is twice that of CA.

(c) A point P(X, Y) moves such that the length of PB is twice that of PA. Show that the locus of P is a circle, and determine its centre and radius. Draw a sketch of the circle on your diagram.

(ii) Five similar discs are taken and each marked with one and only one letter. Two are marked with A, one with H, one with L and the final one with O. They are then placed in a box. The five discs are drawn at random one at a time from the box without replacement. What is the probability that

(a) the first disc drawn is marked A?

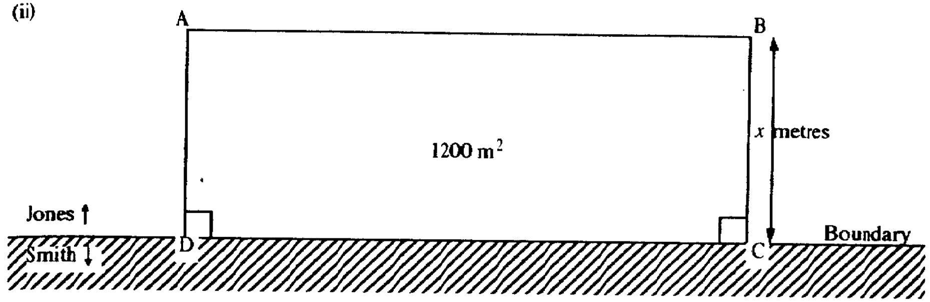
(b) the order of selection of the five discs spells the word ALOHA?

QUESTION 10

(i) The quadratic equation $x^2 + Lx + M = 0$ has one root twice the other. Prove that $2L^2 = 9M$.

Prove also that the roots are rational whenever L is rational.

(ii)



Farmer Jones wishes to fence off a rectangular yard ABCD of area 1200 m^2 from a paddock, as in the figure, with side CD against the property of Farmer Smith. Fencing cost \$3 per metre, and Smith has agreed to pay half the cost of fencing the side CD.

Let \$$y$ be the cost to Jones of fencing the yard, and let x metres be the length of BC. Obtain a formula for y as a function of x, and hence find the minimum cost to Jones of fencing the yard, assuming Smith meets his share of the expense.