# Mathematics

## Essential Mathematics

## Rationale and aims

### Rationale

Mathematics is the study of function and pattern in number, geometry and data. It provides both a framework for thinking and a means of communication that is powerful, logical, concise and precise. Essential mathematics focuses on using mathematics to make sense of the world. The emphasis is on providing students with the mathematical skills and understanding to solve problems and undertake investigations in a range of workplace, personal, training and community settings. There is an emphasis on the use and application of information and communication technologies in the course. The course includes investigation of the application of mathematical understanding and skills in workplaces or community settings.

### Aims

- Students will apply understanding and skills in practical computation, displaying and describing data, analysing and critically interpreting statistical information reported in the media, and applying geometric and network techniques in design and planning.
- Students will choose and use a wide range of appropriate technologies to achieve the first aim.
- Students will evaluate the reasonableness of their solutions.

## Organisation

### Organisation of the course

The senior secondary mathematics curriculum consists of four courses in mathematics. The courses are differentiated, each focusing on a pathway that will meet the learning needs of a particular group of senior secondary students. Each course is organised into four units typically for completion over four semesters.

Essential mathematics (Course A) has been designed as a standalone subject. However there has been consideration given to students who may wish to pick up the course at unit 3, having previously not studied mathematics in the senior years or studied units 1 and 2 of General mathematics (Course B).

### Coherence and continuity K-12

The *Shape of the Australian Curriculum – Mathematics* describes three content strands: number and algebra; statistics and probability; and Measurement and geometry. In the senior secondary years these strands have been continued but no longer used as major organisers. This is because the ideas in each of the strands both converge and diverge.

The proficiency strands of understanding, fluency, reasoning and problem solving have been integrated into the content descriptions, as in the K -10 curriculum.

The senior secondary mathematics curriculum builds on the K-10 curriculum by offering courses to cater for the needs and requirements of every student wishing to undertake further study of mathematics in the senior years.

### Course Content

Course A is organised around key focus areas of:

· Measurement – quantification, estimation and calculation

· Finance – understanding and making of informed decisions about financial situations

· Location / Direction – development and application of problem solving skills

· Statistics – data management in public, business and community situations

· Algebra – workplace and community situations

· Design – geometry building on measurement concepts and skills

· Probability – simple probabilities and applications

## Unit Content

| UNIT 1 | | UNIT 3 | |
|---|---|---|---|
| Topic 1: | Measurement | Topic 1: | Finance 2 |
| Topic 2: | Finance 1 | Topic 2: | Data Analysis 2 |
| Topic 3: | Investigation | Topic 3: | Investigation |
| UNIT 2 | | UNIT 4 | |
| Topic 1: | Time and Place 1 | Topic 1: | Design |
| Topic 2: | Data Analysis 1 | Topic 2: | Probability |
| Topic 3: | Algebra | Topic 3: | Time and Place 2 |
| Topic 4: | Investigation | | |

## General Capabilities

Good teaching in each of the learning areas will always contribute to a students' development of general capabilities and understanding of the crosscurriculum dimensions. The Australian Curriculum reinforces this expectation by incorporating the general capabilities and cross–curriculum dimensions into the content descriptions, in ways appropriate to each learning area. eg mathematics provides a framework for thinking and a means of communication that is powerful, logical, concise and precise.

In writing the Australian curriculum, considerable attention has been paid to the development of literacy and numeracy as the foundations on which much further learning depends. ICT has been incorporated into all topic areas. Thinking skills and the scope for creativity is inherent in all mathematical problem solving and so is an integral part of the Australian Curriculum: Mathematics.

Other general capabilities of self management, teamwork, intercultural understanding, ethical behaviour and social competence are able to be addressed in a classroom environment where the content can be delivered within a context suitable for a particular student group.

## Cross Curriculum Dimensions

Three cross-curriculum dimensions are dealt with explicitly in the Australian Curriculum: Indigenous history and culture, a commitment to sustainable living and Asia and Australia's engagement with Asia.

The Australian Senior Secondary Mathematics Curriculum provides opportunities for the study of Mathematics within a relevant context, giving students the means to explore, evaluate and comment with on information presented to them.

## Use of technology

The *Shape of the Australian Curriculum – Mathematics* states that available technology should be used for teaching and learning situations. Technology can include computer algebra systems, graphing packages, financial and statistical packages and dynamic geometry. These can be implemented through either a computer or calculator.

Technology can aid in developing skills and allay the tedium of repeated calculations. For example a technology can be used to complete recursive calculations.

There are many resources available on the internet and in state and territory portals that also have application for learning in the senior mathematics courses.

The decision about using technology in assessment programs is not within the province of the curriculum, jurisdictional assessment agencies will make that decision.

## Unit 1

### Measurement

Measurement is fundamental to mathematics. The ability to quantify, estimate and calculate is essential in many practical applications based on work, community or life experiences. Students will be taught to understand concepts, acquire skills and solve problems related to:

Linear measure. This includes:

• metric units of length, their abbreviations, conversions between them, level of accuracy needed and appropriate choices of units

• estimation of lengths

• relationships between metric units of length and units of length in other measurement systems (e.g. centimetres to inches)

• calculating perimeters of familiar shapes (triangles, squares, rectangles, polygons, circles, sectors of circles, composites of these)

• a variety of practical applications of linear measure such as student heights, sports playing fields line marking costs and car dimensions.

Area measure. This includes:

• metric units of area, their abbreviations, conversions between them, and appropriate choices of units

• estimation of areas

• relationships between metric units of area and units of area in other measurement systems (e.g. hectares to acres)

• using formulas to calculate areas of common shapes (triangles, squares, rectangles, parallelograms, trapeziums, circles, sectors)

• finding the area of irregular figures by decomposition into common shapes

• finding the surface area of familiar solids (cubes, rectangular and triangular prisms and cylinders) and pyramids (rectangular- and triangular-based) and spheres

• using addition of areas of faces to find the surface area of irregular solids

• a variety of practical applications of area measure such as costs of turfing sports playing fields, costs of painting buildings and packaging needs of a factory.

Mass. This includes:

• metric units of mass (and weight), their abbreviations, conversions between them, and appropriate choices of units

• estimation of mass

• a variety of practical applications of mass such as packaging, food and workplace examples (e.g. cranes, forklifts).

Volume and capacity. This includes:

• metric units of volume, their abbreviations, conversions between them, and appropriate choices of units

• relationship between volume and capacity recognising that $1\ cm^3 = 1\ ml$ and $1\ m^3 = 1\ kL$

• estimation of volume and capacity

• using formulas to find the volume and capacity of common objects (cubes, rectangular and triangular prisms and cylinders)

• a variety of practical applications of volume such as drink containers, bags and alcoholic content.

## Finance 1

The study of aspects of finance involving earning and purchasing in practical contexts allows the development of a working knowledge of and a critical approach to these financial situations. It assists students to make informed decisions about financial situations they are likely to encounter in their lives. Students will be taught to understand concepts, acquire skills and solve problems related to:

Wages and income. This includes:

• comparing various ways of earning an income from various types of work such as full time, part time, casual, self employment, commission, contract and piecework

• research on which occupations have different ways of earning, using job guides, employment sections in newspapers or online career software packages

• interpreting tables of award rates for different ages, different industries and different categories

• wages through timesheets, overtime, penalty rates

• income earned through commission only, retainers and commissions, and piecework

• conversion of yearly incomes to and from weekly, fortnightly and monthly incomes

• additional income sources such as leave loadings, special allowances (tools, uniforms, hazards, responsibility, etc) and special bonuses (shares, cash)

• entitlements of holiday and sick leave

• percentage increase and decrease of income.

Income support and benefits. This includes:

• government benefits and allowances such as student allowances, living-away-from-home allowances and Centrelink benefits

• means-tested incomes and associated benefits and the effects on benefits due to changes of income.

Taxes and deductions. This includes:

• personal tax deductions from wages and salaries using tables, formulas, online calculators and spreadsheets

• tax thresholds and the effect of taxes on net incomes

• Medicare levy

• other deductions such as superannuation, union fees and private health cover.

Buying and paying for goods and services. This includes:

• GST, such as calculating the amount of GST on a product, calculating GST as a percentage, with consideration of exemptions and other countries' taxes on purchases

• discounts and mark-ups such as finding the discounted amount and finding the mark-up as a percentage

• finding best buys from unit pricing comparisons

• labelling such as content information, preservatives and their codes, and comparison between similar products

• buying on terms and no interest such as total costs, interest calculations, benefits, hidden traps and fees and charges

• comparisons of methods of payment such as cash, cheques, direct debits, credit cards, BPAY, loans and time payments

• consumer rights and information access

• budgeting plans for specific purchases using appropriate technology.

## Investigation

Students investigate realistic problems of interest to themselves which involve the application of mathematical relationships, concepts and skills that have been addressed in this unit. The subject of the investigation may be derived from one or more topic(s) covered in this unit. It may also draw on previous mathematical ideas or develop new understanding and skills. An investigation may be initiated by a student, a group of students or the teacher. In some instances, teachers may give students a clear, detailed and sequential set of instructions for part of the investigation or to initiate the investigation. In other situations, the teacher may provide broad guidelines allowing the student or group of students sufficient scope to develop themes or aspects of their own choice. As this is the first investigation of this course it is envisaged that the investigation would be briefer and teacher guidance and scaffolding would be greater than in Units 2 and 3. Students could present their findings to the class to give them an opportunity to learn from each other. It is expected that this would take approximately one fifth of the available time, but it may be completed throughout the semester or in a block. This could be an opportunity for students to pursue particular interests of their own or to engage with the community (local, national, global) or the workplace. It is assumed that appropriate technology will be used in the investigation.

An investigation should include:

• a description of the problem and its relationship to the topics of the unit

• realistic time frames to plan, conduct, analyse and report on the investigation

• specific conditions that focus the investigation

• the method for conducting the investigation

the appropriate application of the mathematical model or strategy, including:

• the generation or collection of relevant data and/or information, with details of the process of collection

• mathematical calculations and results

• appropriate representations

• the outcome of the investigation which could include the analysis and interpretation of results.

The mode of presentation of the investigation may include:

• a written or oral report

• multimedia formats

• visual presentations

• a physical model with accompanying notes

• other appropriate formats.

• Note: these are suggestions only. There are many other opportunities for investigations that are relevant to particular students' interests, future pathways and local issues.

Possible investigations relating to the topics of Unit 1 are:

• evaporation rates and surface area, e.g. dams, tanks

• error in measurements, e.g. packaging

• relationship between body parts, e.g. arms span versus height, heart rate versus height

• strategies for estimation in particular contexts, e.g. trades estimates, hospitality estimates

• comparison of volumes and containers, e.g. optimising carton size, packing and stacking

• sports field line marking, e.g. maximising different sports on the one playing area, lining athletics tracks

• historical investigation, e.g. comparison of different methods of measure, standards or systems of measure across the world

• unit cost comparison, e.g. comparing different brands of the same product, comparing sizes of a type of product

• earning in specific occupations, e.g. income over a year with taxes, deductions and job-related expenditure from weekly payslips

• retail strategies, e.g. discounts and volume of sales to make profits, profit and loss analysis in terms of mark-ups and discounts

• small business, e.g. pricing of tradespersons' fees, comparisons of fees and charges for different occupations.

## Unit 2

## Time and place 1

Students' interaction with the community through their everyday life requires them to develop skills and interpret relevant information in a variety of situations involving time, distance, speed and direction. The development of problem-solving skills in these areas will be explored in practical contexts. Local, regional and Australia-wide maps will be used. Applications of time and rates will be considered to enhance skills in this area. Emphasis will be on the use of maps and technology in everyday contexts (e.g. timetables). Students will be taught to understand concepts, acquire skills and solve problems related to:

Time. This includes:

• units of time, conversions between units, fractional, digital and decimal representations

• 12-hour and 24-hour clocks

• calculating time intervals such as time between events

• interpreting timetables such as bus, train and ferry timetables

• planning the most time-efficient routes using several timetables which may involve electronic technologies

• interpreting complex timetables such as tide charts, sunrise charts and moon phases

• comparing travel time between different modes of transport.

Distance. This includes:

• using scales to find distances on maps such as road maps, street maps, bushwalking maps, online maps and cadastral maps

• optimising distances through trial and error and systematic methods such as shortest path problems, routes to visit all towns and routes to use all roads

• finding stopping distances for vehicles with different speeds, of different sizes and under different road conditions.

Speed. This includes:

• typical units of speed associated with different activities such as walking, running, swimming, flying

• finding speed, distance or time using the formula speed = distance/time

• time or cost for journeys from estimated distances using maps

• conversion between units of speed such as km/h to m/s

• estimation of speed

• interpreting distance versus time graphs

• calculating and interpreting average speed.

Direction. This includes:

• interpreting map keys and coordinates

• using a variety of maps to find places using compass directions and bearings. Maps could include street directories, site maps (e.g. hospitals, schools, shopping centres), road maps and cadastral maps

• giving directions using compass directions and distances to develop routes.

Applications.

• This includes detailed descriptions and directions for travel routes including speed, distance, estimated time and direction and fuel consumption, where relevant.

## Data analysis 1

Data in the form of numbers, tables, lists, graphs, images and charts are met in more and more aspects of daily life. Data is put before us by corporations, governments and the media for a variety of purposes: to inform us, to persuade us, or simply to justify a conclusion. Statistical methods can be used to extract information from data. They can also help us to make judgments about the reliability of that information. In this topic students look at ways of exploring data collected by others, and also data they collect themselves from direct observation, experiments and surveys. These explorations include presenting and summarising the data, describing the summarised data, and discovering information in the form of patterns in the summarised data. Students will work with data that is meaningful to them, including data that relates to their community. This community data can be compared with data from other communities —locally, nationally and globally. Various media sources should be used to ensure a variety of real-life contexts. Students will be taught to understand concepts, acquire skills and solve problems related to:

Classifying data. This includes:

• categorical data

• numerical data.

Presenting and interpreting data. This includes:

• presenting categorical data – tables, bar charts

• presenting numerical data – frequency distributions, graphs, histograms, recognising outliers

• features such as axes, labels, legends and scales

• other visual forms of data seen in context such as maps and pictograms

• comparing the suitability of different presentation methods in practical contexts

• practical examples illustrating distortions of data

• presenting real-world data informatively using technology.

Summarising data and interpreting the data summary. This includes:

• mode – for categorical data and count data

• measures of central tendency: the arithmetic mean, the median

• suitability of measures of central tendency in various contexts

• the effect of outliers on the mean and the median

• quartiles, deciles and percentiles and their interpretation

• informal ways of describing spread such as spread out, dispersed, tightly packed, clusters, gaps, more or less dense regions and outliers

• statistical measures of spread such as the range, interquartile range, standard deviation and their interpretation

• examples illustrating inappropriate uses, or misuses, of measures of central tendency and spread.

Comparing data sets. This includes:

• back-to-back stem plots

• five-number summaries and box plots

• comparing shape characteristics of histograms such as symmetry and skewness.

## Algebra

Formulas are used in many aspects of life from the workplace to the community. Being able to evaluate a formula given numerical values of variables is important in many contexts. Evaluating variables, not just the subject, in these formulas is useful and this can be achieved by solving simple equations. All formulas used in this topic must be relevant to the students today, in their future workplaces or within their community. Students will be taught to understand concepts, acquire skills and solve problems related to:

Formula. This includes:

• introduction to algebra and the development and use of formulas through everyday contexts such as Australian rules football, medicine dosages and service charges

• the use of practical formulas such as workplace, community-based, health and sporting formulas

• evaluation of the subject of the formula through numerical substitution of values and assessing the reasonableness of the result

• determining the appropriate degree of accuracy

• formulas involving variables expressed as words such as Vital Lung Capacity = 0.041 x Height – 0.018 x Age – 2.69.

Equations. This includes:

• solving simple equations with a single variable

• solving equations using informal techniques such as guess, check and improve

• solving equations using other techniques such as backtracking and balance methods.

Solving equations derived from formulas to find a variable other than the subject. This includes:

• the use of practical formulas such as workplace, community-based, health and sporting formulas

• substitution then solving

• formulas involving variables expressed as words.

## Investigation

Students investigate realistic problems of interest to themselves which involve the application of mathematical relationships, concepts and skills that have been addressed in this unit. The subject of the investigation may be derived from one or more topic(s) covered in this unit. It may also draw on previous mathematical ideas or develop new understanding and skills. An investigation may be initiated by a student, a group of students or the teacher. In some instances, teachers may give students a clear, detailed and sequential set of instructions for part of the investigation or to initiate the investigation. In other situations, the teacher may provide broad guidelines allowing the student or group of students sufficient scope to develop themes or aspects of their own choice. It is expected that this would take approximately one fifth of the available time but it may be completed throughout the semester or in a block. This could be an opportunity for students to pursue particular interests of their own or to engage with the community (local, national, global) or the workplace.

An investigation should include:

• a description of the problem and its relationship to the topics of the unit

• realistic time frames to plan, conduct, analyse and report on the investigation

• specific conditions that focus the investigation

• the method for conducting the investigation

An investigation should include the appropriate application of the mathematical model or strategy, including:

• the generation or collection of relevant data and/or information, with details of the process of collection

• mathematical calculations and results

• appropriate representations

• the outcome of the investigation which could include the analysis and interpretation of results.

The mode of presentation of the investigation may include:

• a written or oral report

• multimedia formats

• visual presentations

• a physical model with accompanying notes

• other appropriate formats.

• Note: these are suggestions only. There are many other opportunities for investigations that are relevant to particular students' interests, future pathways and local issues.

Possible investigations relating to the topics of Unit 2 are:

• planning a car trip, e.g. around the state or territory, minimising fuel consumption

• planning a trip around the city, e.g. use of multiple modes of transport and timetables

• planning a timetable, e.g. new routes or different transport types

• comparing public and private transport, e.g. time taken, costs, cost to environment

• optimising routes, e.g. garbage truck run, visiting all tourist sites with minimum backtracking

• interpreting maps, e.g. orienteering, bushwalking routes

• statistical investigations, e.g. quality control such as how many smarties in a box, length of stride in relation to sport, Olympic statistics

• critiquing arguments based on statistics, e.g. environmental issues, health issues

• statistical misinterpretation, e.g. creating a biased interpretation from data, analysis of media representations

• use of formulas, e.g. dosages for different ages, sport applications.

## Unit 3

### Finance 2

The study of aspects of finance involving credit, investments and budgeting in practical contexts allows the development of a working knowledge and a critical approach to these financial situations. It assists students to make informed decisions about financial situations they are likely to encounter in their lives. Students will be taught to understand concepts, acquire skills and solve problems related to:

Consumer credit. This includes:

• credit cards and store cards — reading and interpreting statements, interest calculations, minimum payments, interest-free periods, fees and charges

• personal loans — conditions of availability, interest calculations, repayments, fees and charges and comparisons with credit cards

• debit cards such as fees and charges, benefits and disadvantages.

Budgeting. This includes:

• personal living costs such as transport, entertainment, food and clothing

• household costs such as rent, bonds, food, utilities costs, insurance and other bills

• costs of services such as car registration, insurance, maintenance and repair

• developing a budget based on investigating personal and household expenses against estimated income.

Payment plans. This includes:

• mobile phones — comparison of types of plans, costs, fees and charges

• internet — comparison of types of plans, costs, fees and charges

• consumer rights and information access.

Investments. This includes:

• term deposits with simple interest considering the effects of changing the principal, interest rate and time periods using formulas, graphs and technology

• investment accounts with compound interest considering the effects of changing the principal, interest rate and time periods using graphs and technology

• savings accounts such as comparison with other investments, maintaining balances with various deposits and withdrawals, fees and charges using appropriate technology

• long-term investments such as superannuation and real estate.

## Data analysis 2

People are exposed to statistical information almost every day. This information comes in the form of descriptions of real-world situations and arguments designed to convince people of some particular point of view. It is important to consider such statistical descriptions and statistical arguments critically. A good place to start is how the data that underpins the descriptions and arguments is collected. Data can be acquired simply by observation or by conducting an experiment or a census. The media frequently reports on relationships between two variable quantities. Sometimes these relationships are surprising because they are unexpected (for example, taller people earn more). Sometimes they are not related to each other in any causal sense, but merely show some sort of correlation because of another factor affecting each (increase in the number of houses in Australia, and increase in the world high-jump record). They can provide evidence in a well-studied area (for example, the relationship between fast food and obesity) with a view to educating the public and perhaps changing behaviour. By studying two sets of numerical data jointly it may be possible to establish whether a relationship exists, and possibly to use one variable to make predictions about the other. Students will be taught to understand concepts, acquire skills and solve problems related to:

Censuses. This includes:

• population and samples

• procedure for conducting a census

• advantages and disadvantages.

Sample Surveys. This includes:

• understanding the purpose of sampling – to provide an estimate of population values when a census is not used

• kinds of samples – quota samples, convenience samples, random samples and their advantages and disadvantages.

Simple survey planning and procedure.  This includes:

• identification of target population

• consideration of how to ensure representativeness of sample

• questionnaire design principles such as simple language, unambiguous questions, possible use of multiple-choice questions, privacy and ethics, and absence of bias.

Sources of errors in surveys. This includes:

• sampling errors

• non-sampling errors: interviewer bias, measurement errors, data processing error and non-responses.

Possible misrepresentation of the results of a survey. This includes:

• misunderstanding the procedure

• misunderstanding the reliability of generalising the survey findings to the entire population.

Assessing the relationship between two variables. This includes:

• informal interpretation of patterns and features in bivariate scatterplots

• describing patterns in the scatterplots between two numerical variables in terms of direction (positive or negative), form (linear or non-linear) and strength (strong, moderate or weak).

• identifying the dependent and independent variable

• finding a line of best fit using your eye and technology

• using technology to find the correlation coefficient as an indicator of the strength of linear association

• using the line of best fit to make predictions recognising the dangers of extrapolation

• distinguishing between causality and correlation through examples.

• strength of linear association

• extrapolation

## Investigation

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The mode of presentation of the investigation may include:

• a written or oral report

• multimedia formats

• visual presentations

• a physical model with accompanying notes

• other appropriate formats.

• Note: these are suggestions only. There are many other opportunities for investigations that are relevant to particular students’ interests, future pathways and local issues.

Possible investigations relating to the topics of Unit 3 are:

• payment plans, e.g. comparing ‘deals’, car loans

• credit cards, e.g. effect of paying off the minimum, comparing cards for fees, hidden charges

• cost benefit analysis, e.g. frequent flyer points and the costs, paying on terms

• living away from home, e.g. renting and its expenses

• expenses, e.g. buying and running a car

• investments, e.g. various investments with differing interest rates, frequency of contributions

• surveys using sampling methods on topics of interest to the student, comparison of own community with results from other places

• investigating relationships between numerical variables on topics of interest to students

• looking on the Australian Bureau of Statistics website for Australian statistical data in one particular area of interest, e.g. population, average weekly earnings, crime and safety. Explain the procedure the ABS used for collecting the data, describe the degree of accuracy of the data, and report on any interesting information the data reveals

• report on the procedures used for minimising all kinds of errors in the results of the most recent five-yearly Australian Population Census

• cluster and strata sampling: explain how these methods differ from simple random sampling and why they may be preferred to simple random sampling

• is there a systematic relationship between hand span and forearm length?

• investigate the relationship between the percentage of sugar in a breakfast cereal and the price of that breakfast cereal.

## Unit 4

### Design

The ability to interpret scale drawings is a skill that has many practical applications in work, community or life experiences. This unit consolidates the previous measurement unit in terms of specific applications of those skills. The ability to interpret design and scale drawings and develop costing for construction gives students skills for the future in terms of work or recreational activities. Throughout this topic the use of calculators and other appropriate technologies is vital. Many software packages have geometric tools to assist the drawing and construction of plans and elevations. Students will be taught to understand concepts, acquire skills and solve problems related to:

Interpreting scale drawings. This includes:

• interpreting commonly used symbols and abbreviations in scale drawings

• finding actual measurements from scale drawings such as lengths, perimeters and areas

• estimating and comparing quantities, materials and costs using actual measurements from scale drawing such as packaging, clothes, painting, bricklaying and landscaping.

Creating scale drawings. This includes:

• drawing conventions of scale drawings such as scales in ratio, clear indications of dimensions and clear labelling

• manual construction and using software packages to create scale drawings

• planning and design of drawings such as placing furniture in a room, packaging on shelves, plants and objects in landscaping.

Three-dimensional objects. This includes:

• interpreting plans and elevation views

• sketching elevation views using technology

• interpreting diagrams of three-dimensional objects

• using geometric software tools to construct three-dimensional scale drawings.

Applications to plan, design and cost the development of a three-dimensional item.

## Probability

Uncertainty is an inseparable part of life. We devote a lot of attention to coping with the chance elements in life. For example, we take out insurance against ill health, car accidents, theft of our belongings and fire in our home. People look up the weather forecast before they go hiking, factory managers constantly check the quality of items coming off the production line so as to have early warning of machinery slipping out of alignment, supermarkets plan their reordering of shelf products so that they minimise the chance of being out of stock at times of peak customer demand. In all of these contexts the mathematical analysis of chance processes is central to decision-making. Students will be taught to understand concepts, acquire skills and solve problems related to:

Probability expressions. This includes:

• commonly used probability terms such as possible, probable, likely and certain

• ways of expressing probabilities formally as fractions, decimals, ratios and percentages

• odds.

Simulations. This includes:

• performing simulations with the use of technology

• recognising that repetitions of chance events are likely to produce different results

• the notion of relative frequency as probability

• recognising the law of large numbers which is that outcomes for successive trials of a chance experiment follow no discernible pattern but the relative frequency of occurrence of any particular outcome remains reasonably constant for a large number of trials

• identifying factors that could complicate the simulation of real-world events.

Simple probabilities. This includes:

• estimating probabilities for familiar events and processes

• calculating probabilities which could use sample space, arrays, tree diagrams and formulas.

Applications. This include probabilities associated with:

• games

• traffic lights

• simple queuing problems.

## Time and place 2

As communication technologies allow us to engage with the world easily an understanding of the issues involved with time and distance for Australia is vital. This unit builds on the skills acquired in the topic Time and place 1 in Unit 2. It develops a world perspective of knowledge and understanding of places, position, time zones, distances and speed which are all part of the connected world of today. It uses more sophisticated maps incorporating some aspects of navigation both in the air and on the sea. Emphasis will be on the use of maps and electronic technologies in practical contexts. Students will be taught to understand concepts, acquire skills and solve problems related to:

Location. This includes:

• locating positions on the earth surface given latitude and longitude using GPS, globes, atlases and electronic technologies

• finding distances between two places on earth with the same longitude using appropriate formulas and technology.

Time. This includes:

• the relationship between longitude and time

• time zones in Australia and its neighbours

• Greenwich Mean Time (GMT or UTC) and the International Dateline

• finding differences in times between two places on the earth

• solving problems associated with time zones such as interaction on the internet or phone

• travelling east and west and incorporating time zone changes.

Direction. This includes:

• interpreting and using grid references on topographical maps

• using compass bearings both forward and reverse to plot and determine points, places and routes on maps such as topographical maps and land-use maps

• interpreting contour maps including gradients, scales and distances

• interpreting simple navigation charts

• plotting courses and determining location using maps, charts, pairs of compasses, dividers and parallel rulers.

Speed. This includes:

• nautical miles and their conversion to and from kilometres (1 NM = 1.852 km)

• knots , 1 knot = 1 NM per hour, and its conversion to and from km/h

• simple examples of the effects of currents and wind on the direction travelled by boats, planes and windsurfers.

Applications requiring the use of more than one of the concepts of:

• direction and speed

• distance or time such as travelling to different countries considering distance travelled

• time departed and local time of arrival or aeroplane routes considering latitude and longitude

• distance travelled, speed travelled and fuel used.

## General Mathematics

## Rationale and aims

### Rationale

Mathematics provides both a framework for thinking and a means of communication that is powerful, logical, concise and precise. General mathematics is designed to equip students with the confidence, understanding, skills and strategies to apply mathematical techniques to the analysis and solution of problems. The course provides an introduction to some areas of discrete mathematics, including non-calculus methods of optimisation. Statistics and financial mathematics and their applications are important parts of this course. General mathematics is designed for students who wish to undertake further studies in areas such as agricultural, health and social sciences, business and education where mathematical knowledge facilitates problem solving and decision-making.

### Aims

- Students will apply understanding and skills in data analysis, growth and decay, linear models**,** proportional reasoning, decision mathematics, matrices and geometry and trigonometry.
- Students will choose and use appropriate technology to assist in understanding mathematical concepts, complete investigations and to solve problems in a range of contexts.
- Students will evaluate their methods of investigations, the reasonableness of their solutions and communicate their findings in a concise and systematic manner using appropriate mathematical vocabulary.

## Organisation

## Organisation of the course

The senior secondary mathematics curriculum consists of four courses in mathematics. The courses are differentiated, each focusing on a pathway that will meet the learning needs of a particular group of senior secondary students. Each course is organised into four units typically for completion over four semesters.

General mathematics (Course B) has been designed as a standalone course or studied in conjunction with Mathematical methods (Course C). Students may choose to move from General mathematics to Essential mathematics (Course A) at the end of Unit 1 or 2.

## Coherence and continuity K-12

The *Shape of the Australian Curriculum – Mathematics* describes three content strands: number and algebra; statistics and probability; and Measurement and geometry. In the senior secondary years these strands have been continued but no longer used as major organisers. This is because the ideas in each of the strands both converge and diverge.

The proficiency strands of understanding, fluency, reasoning and problem solving have been integrated into the content descriptions, as in the K -10 curriculum.

The senior secondary mathematics curriculum builds on the K-10 curriculum by offering courses to cater for the needs and requirements of every student wishing to undertake further study of mathematics in the senior years.

## Course Content

Course B is organised around key focus areas of:

· Rates and ratios – with an emphasis on financial literacy and numeracy

· Matrices – their nature and use in practical contexts

· Measurement and geometry

· Graphs and networks – the representation of relationships and the properties of networks

· Data analysis – patterns of data in relation to variables

· Linear modelling and linear programming

· Growth and decay in sequences – arithmetic and geometric sequences

· Time series analysis – the analysis and interpretation of data relating to workplace and wider applications

· Financial modelling

## Unit Content

| | | | |
|---|---|---|---|
| **UNIT 1** | | **UNIT 3** | |
| **Topic 1:** | Rates and Ratios | **Topic 1:** | Data Analysis 2 |
| **Topic 2:** | Matrices | **Topic 2:** | Graphs and Networks 2 |
| **Topic 3:** | Measurement and Geometry | **Topic 3:** | Growth and Decay in Sequences |
| **Topic 4:** | Graphs and Networks 1 | | |
| **UNIT 2** | | **UNIT 4** | |
| **Topic 1:** | Data Analysis 1 | **Topic 1:** | Data Analysis 3 |
| **Topic 2:** | Linear Modelling | **Topic 2:** | Time Series Analysis |
| **Topic 3:** | Linear Programming | **Topic 3:** | Financial Modelling |
| **Topic 4:** | Price Index Numbers | | |

## General Capabilities

Good teaching in each of the learning areas will always contribute to a students' development of general capabilities and understanding of the crosscurriculum dimensions. The Australian Curriculum reinforces this expectation by incorporating the general capabilities and cross-curriculum dimensions into the content descriptions, in ways appropriate to each learning area. eg mathematics provides a framework for thinking and a means of communication that is powerful, logical, concise and precise.

In writing the Australian curriculum, considerable attention has been paid to the development of literacy and numeracy as the foundations on which much further learning depends. ICT has been incorporated into all topic areas. Thinking skills and the scope for creativity is inherent in all mathematical problem solving and so is an integral part of the Australian Curriculum: Mathematics.

Other general capabilities of self management, teamwork, intercultural understanding, ethical behaviour and social competence are able to be addresses in a classroom environment where the content can be delivered within a context suitable for a particular student group.

## Cross Curriculum Dimensions

Three cross-curriculum dimensions are dealt with explicitly in the Australian Curriculum: Indigenous history and culture, a commitment to sustainable living and Asia and Australia’s engagement with Asia.

The Australian Senior Secondary Mathematics Curriculum provides opportunities for the study of Mathematics within a relevant context, giving students the means to explore, evaluate and comment with on information presented to them.

## Use of technology

The *Shape of the Australian Curriculum – Mathematics* states that available technology should be used for teaching and learning situations. Technology can include computer algebra systems, graphing packages, financial and statistical packages and dynamic geometry. These can be implemented through either a computer or calculator.

Technology can aid in developing skills and allay the tedium of repeated calculations. For example a technology can be used to complete recursive calculations.

There are many resources available on the internet and in state and territory portals that also have application for learning in the senior mathematics courses.

The decision about using technology in assessment programs is not within the province of the curriculum, jurisdictional assessment agencies will make that decision.

## Unit 1

### Rates and Ratios

This topic extends students’ prior experience and understanding of rates, ratio and proportion to applications in consumer mathematics, the workplace and leisure. Study of this topic assists students to become financially literate and numerate members of society. The use of technology to support the computational aspects of this topic is assumed. Students will be taught to understand concepts, acquire skills and solve problems related to:

Rates, ratio and proportion. This includes:

• reviewing the concepts of rates, ratio and proportion and the expression of ratios and proportions as percentages

• reviewing the determination of percentage increase and decrease in contexts such as the impact of inflation on costs and wages, unit costs, mark-ups and discounts, including an understanding of GST

• calculating profit or loss in absolute and percentage terms

• converting annual interest rates to, for example, daily, monthly or quarterly interest rates, depending on the payment or repayment frequency

• calculating future values and simple and/or compound interest paid or earned, in contexts such as bank accounts, investments and loans without payments or repayments

• comparing prices and values using the unitary method, such as amount per 100 g of an item or amount per dollar

• applying the unitary method for solving ratio and proportion problems to workplace and leisure contexts such as determining drug doses, scaling up recipes to commercial production, currency exchange rates, map scaling, etc.

• logarithms and their application to logarithmic scales such as decibels in acoustics, Richter scale for earthquake magnitude, octaves in music, pH in chemistry.

## Matrices

Matrices provide a convenient and compact way of storing sets of numbers where two parameters are involved. They can be used in a wide variety of contexts, including the representation and processing of tabulated information, the representation and analysis of social networks, and the representation and modelling of economies. In this topic, students will be introduced to matrices and their arithmetic, and will apply them in a range of practical contexts. The use of technology with matrix arithmetic capabilities to support the computational aspects of this topic is assumed. Students will be taught to understand concepts, acquire skills and solve problems related to:

### Matrices and matrix arithmetic. This includes:

• recognising a matrix as a rectangular arrangement of numbers or symbols, types of matrices (row, column, square, zero, identity) and determining the size (order) of a matrix

• determining the sum or difference of two matrices and the process of multiplying a matrix by a scalar

• understanding the process of matrix multiplication and that, for matrices A and B, AB does not necessarily equal BA

• determining the product of two 2 x 2 matrices and the product of a row and a column matrix with and without the aid of technology; evaluating products involving larger matrices with the aid of technology

• recognising, formulating and solving practical problems that can be represented and processed using matrices and matrix products, e.g. costing problems, analysing social networks.

### The inverse of a matrix. This includes:

• determining whether two matrices are multiplicative inverses of each other

• calculating the determinant of a 2 x 2 matrix with and without the aid of technology

• determining if a 2 x 2 matrix has an inverse and determining that inverse with and without the aid of technology; evaluating the inverses of larger square matrices (3 x 3 and larger) with the aid of technology

• recognising, formulating and solving practical problems that can be represented and processed using matrices and their inverses, e.g. coding and decoding, modelling simple economies.

## Measurement

Students will already be fluent in calculations with simple geometric shapes and objects from previous study. The emphasis here is to enhance students' understanding using practical applications and investigations from everyday situations. The use of technology to support the computational aspects of this topic is assumed. Students must use appropriate units and levels of accuracy. Students will be taught to understand concepts, acquire skills and solve problems related to:

Reviewing the calculation of lengths and angles in right-angled triangles in two and three dimensions, including using Pythagoras' theorem and the three trigonometric ratios sine, cosine and tangent.

Reviewing the calculation of perimeter and area of circles, triangles, rectangles and composites.

Finding the area of a triangle given two sides and the included angle, or three sides, using Heron's formula.

Similar figures. This includes:

- finding and applying the scale factor of similar figures to solve scaling problems
- interpreting and constructing scale drawings
- obtaining measurements from scale drawings to solve problems
- comparing the ratios of the areas of similar figures.

Volume and surface area of three-dimensional objects. This includes:

- calculating the volumes and surface areas of spheres, rectangular prisms, cylinders, cones, pyramids and composite three-dimensional objects
- finding and applying the scale factor of similar three-dimensional objects to solve scaling problems involving surface area and volume.

## Graphs and networks 1

Graphs are mathematical tools for representing relationships between discrete objects. There are many different types of relationships that can be represented in this way, making graphs and networks, and the theory relating to them, widely applicable. In this topic, students will be introduced to undirected graphs and their use in understanding and analysing the properties of networks. The use of technology to support the graphical and computational aspects of this topic is assumed. Students will be taught to understand concepts, acquire skills and solve problems related to:

The language of graph theory. This includes:

- understanding the meanings of the terms vertex, edge, loop and isomorphic graph
- determining the degree of a vertex
- identifying connected graphs, simple graphs, complete graphs, planar graphs, subgraphs, edge-weighted graphs and trees
- understanding that a graph can be represented as an adjacency matrix.

The basic properties of graphs. This includes:

- applying Euler's rule relating to the number of vertices, edges and faces of a plane graph
- the concept of traversability in connected graphs and understanding its relationship to the degree of the vertices in a graph
- identifying paths and circuits in connected graphs
- applying the concepts of Euler paths and circuits and understanding the conditions for their existence
- the concepts of Hamilton paths and circuits
- understanding the concept of a minimum spanning tree and finding it in a connected graph by inspection or by using Prim's algorithm.

The applications of non-directed graphs. This includes:

• using minimum spanning trees to solve minimum connector problems, e.g. providing power from a single power station to several towns

• solving practical problems using Hamilton paths and circuits, e.g. planning a tourist route around a city

• solving practical problems using Euler circuits and paths, e.g. planning a garbage bin collection route.

## Unit 2

### Data analysis 1

The purpose of data analysis is to obtain information from data. This topic aims at consolidating and extending the statistics taught in K–10 with its emphasis on the data investigation cycle. In particular, this topic is concerned with identifying, describing, summarising and interpreting patterns in data relating to a single variable that might arise in such an investigation, using an increasingly sophisticated range of statistical tools. The use of technology to support the graphical and computational aspects of this topic is assumed. Students will be taught to understand concepts, acquire skills and solve problems related to:

Making sense of data relating to a single categorical variable. This includes:

• classifying categorical data as ordered (e.g. placing in a race – first, second, third) or unordered (e.g. eye colour)

• using tables and bar charts to organise and display categorical data

• describing and interpreting any patterns observed in bar charts, including the use of the modal category.

Making sense of data relating to a single numerical variable. This includes:

• reviewing the use of dot plots, stem plots, box plots, histograms and line graphs to organise and display numerical data

• using 'Q1 – 1.5 x IQR' and 'Q3+ 1.5 x IQR' as cutoff points for identifying possible outliers and incorporating these when using box plots to display data

• using a logarithmic scale to display the distributions of variables whose values range over several orders of magnitude (e.g. the distribution of salaries in a large company, gross domestic product for developing and developed countries)

• describing features such as shape in histograms, stem plots or dot plots (uni or multimodal, symmetric or skewed, bell-shaped), centre and spread, outliers (unusual values), and interpreting any pattern in the context of the data

• summarising a data distribution by choosing and calculating an appropriate measure of centre (median or arithmetic mean) and spread (range, interquartile range, or standard deviation).

Comparing a numerical data set across two or more groups. This includes:

• using parallel box plots to visually compare groups in terms of centre, spread and outliers

• comparing groups using medians, means, IQRs, ranges or standard deviations, as appropriate

• reporting comparisons of a numerical data set across two or more groups in a systematic and concise manner.

### Linear modelling

Linear equations are the simplest equations we can use to model relationships in the real world and are widely applicable. In this topic, the basic ideas of linear equations are reviewed and extended in the context of modelling real-world phenomena. The use of technology to support the graphical and computational aspects of this topic is assumed. Students will be taught to understand concepts, acquire skills and solve problems related to:

Plotting, sketching and interpreting straight line graphs. This includes:

• sketching the graph of a linear equation both with and without the aid of technology

• determining the slope and intercepts of a straight line graph from both its equation and its plot

• interpreting in context the slope and intercept of a linear graph used to model a familiar linear relationship

• constructing and analysing a linear graph to model a given linear relationship, e.g. modelling the change in the level of water in a cylindrical tank over time when water is added to the tank at a constant rate.

Finding the point of intersection of two linear graphs. This includes:

• solving analytically (in simple cases), numerically and graphically

• applications, e.g. break-even analysis.

Using matrices to represent and solve a system of two or more simultaneous linear equations.

Plotting, sketching and interpreting piecewise linear and step graphs. This includes:

• sketching a piecewise linear graph or a step graph, both with and without the aid of technology

• setting up and analysing piecewise linear and step graphs to solve realistic problems such as modelling the level of water in a tank over time when water is drawn off at different rates and for different periods of time, and modelling the charging scheme for sending parcels of different weights through the post.

## Linear programming

Linear programming is a mathematical technique used to solve optimisation problems of a kind that arise naturally in business planning, industrial engineering, transportation, scheduling and resource allocation. In this topic the study of linear programming is restricted to those problems that can be solved graphically. The technique relies on an understanding of linear inequalities. This understanding is developed from students' prior experience and understanding of linear equations. The use of technology to support the graphical and computational aspects of this topic is assumed. Students will be taught to understand concepts, acquire skills and solve problems related to:

Plotting linear inequalities and systems of linear inequalities in one and two dimensions.

Linear programming. This includes:

• defining decision variables (limited to two)

• determining equations or inequalities defined by given constraints (with one or two variables) and graphing them

• determining the feasible region and its vertices graphically

• defining the objective function

• using the 'sliding objective function method' to determine the values of the decision variables which maximise or minimise the objective function, including cases where more than one optimal answer exists.

### Price index numbers

It's a familiar saying: 'my money just doesn't buy as much as it used to'. This is evidence that our society (like most others) experiences a certain amount of price inflation over time. In recent years in Australia, price inflation, averaged over all the goods and services we consume, has mostly been less than 4% per year. To measure price inflation over time we use a statistical measure called a price index that finds the change in the average of each of two sets of values collected at two different points in time. In this topic we investigate alternative averages for defining a price index and explore some of the statistical methods involved in its construction. The Australian Consumer Price Index illustrates the way practical problems are handled in this context. The CPI is very widely used by government, business and industry for many different purposes. We explore the use of the CPI for 'deflating' specific prices or incomes over time to discount the effect of general price inflation, that is, to enable financial comparisons over time as if the general purchasing power of money remained unchanged. The use of technology to support the graphical and computational aspects of this topic is assumed. Students will be taught to understand concepts, acquire skills and solve problems related to:

Price index foundations. This includes:

• terminology: base period, current period, regimen of the index, simple arithmetic mean (AM); weighted AM; geometric mean (GM)

• two logical alternatives for defining a price index: the ratio of mean prices of a set of goods between two points of time, the mean of ratios of prices of each of a set of goods between two points of time

• the Laspeyres price index – the ratio of weighted AMs with base period weights

• the Paasche price index – the ratio of weighted AMs with current period weights

• the relative merits of these price indices in practice

• argument for averaging the Laspeyres and Paasche price index values

• why the GM is mathematically preferable to the AM for defining a price index number.

A price index in practice. This includes:

• the Australian Consumer Price Index (CPI): how the regimen is determined and why its composition is altered slightly over time; major components of the regimen; which price index approach is used

• valid uses of the CPI

• inappropriate uses of the CPI.

Use of the CPI to discount the effect of general price inflation over time. This includes:

• misleading comparisons of money amounts over time, through neglecting the impact of general price inflation, with examples (e.g. from the media)

• how to express a time series of money amounts in terms of constant purchasing power by deflating the money amounts using a price index

• exercises in using the CPI for deflating money values over time: collecting the time series values, selecting the appropriate price index values, deflating the time series, reporting the results and highlighting striking features. Examples: the annual cost of registering a six-cylinder passenger car over time, the value of the full single adult pension over time, the cover price of a particular magazine over time.

## Unit 3

### Data analysis 2

The purpose of data analysis is to obtain information from data. A key question when working with two variables observed on the same subjects is 'Are the variables associated and, if so, how?' In this topic a range of techniques for identifying and describing associations will be introduced. The use of technology to support the graphical and computational aspects of this topic is assumed. Students will be taught to understand concepts, acquire skills and solve problems related to:

Identifying and describing associations between two categorical variables, e.g. attitude to capital punishment (agree with, no opinion, disagree with) and sex (male, female). This includes:

• constructing two-way frequency tables, marginal distributions, row and column percentages

• using row or column percentages, as appropriate, to identify patterns that suggest the presence of an association

• describing an association in terms of differences observed in percentages across categories.

Identifying and describing associations between a numerical and categorical variable, e.g. height and sex (male, female). This includes:

• using back-to-back stem plots, parallel stem plots, dot plots or box plots to display the data as appropriate

• identifying patterns in the graphical displays that suggest the presence of an association, e.g. changes in centre, spread, shape across categories

• describing the association in terms of any changes observed in one or more of centre, spread or shape in the distribution of the numerical variable across the categories.

Identifying and describing associations between two numerical variables, e.g. height and foot length. This includes:

• constructing a scatterplot to identify patterns in the data suggesting association

• describing an association between two numerical variables in terms of direction (positive/negative), form (linear/non-linear) and strength (strong/moderate/weak)

• calculating and interpreting the correlation coefficient (r).

Association and causation. This includes:

• recognising that the presence of an association between two variables does not necessarily mean that the relationship between the variables is causal

• understanding that possible explanations for an association could include causation, a common response or confounding.

## Graphs and networks 2

Graphs are mathematical tools for representing relationships between discrete objects. There are many different types of relationships that can be represented in this way making graphs and the theory relating to them widely applicable in everyday situations. In this topic, students will extend their knowledge of networks and graphs to include the properties of digraphs. They will use this knowledge of both directed and undirected graphs to understand and analyse the properties and applications of directed networks. The use of technology with matrix capabilities to support the computational aspects of this topic is assumed. Students will be taught to understand concepts, acquire skills and solve problems related to:

The language and properties of digraphs (directed graphs). This includes:

• reviewing the language, properties and applications of non-directed graphs

• extending the concept of a graph to include graphs with directed edges (digraphs), including bipartite graphs

• representing a digraph by an adjacency matrix.

Solving small-scale connectivity problems in transport and communication networks. This includes:

• the identification of n-step paths by inspection or using matrix methods

• determining the shortest path between two points in a network by inspection (e.g. the shortest time to travel between two international cities with multiple routes).

The application of directed graphs to project planning using critical path analysis. This includes:

• constructing a network to represent the durations and interdependencies of activities that must be completed during the project

• locating the critical path using forward and backward scanning

• using the ‘crashing technique’ to reduce project completion time.

Solving small scale maximum flow problems by inspection in combination with the ‘minimum cut – maximum flow’ theorem.

Allocation/assignment problems of optimal resource management. This includes:

• representing an allocation/assignment problem using a graphical (bipartite graph) and/or a tabular or matrix representation

• applying the Hungarian algorithm to the tabular representation to find the optimal solution or solutions.

## Growth and decay in sequences

Sequences can be used to model a vast array of physical phenomena that involve growth or decay. The study begins with arithmetic and geometric sequences and their use to model linear and exponential growth and decay respectively. The study is then extended to include sequences that are generated by first-order linear recurrence relations in general, allowing more complex growth and decay situations to be modelled, for example the growth of a trout population in a lake where limited recreational fishing is permitted. Finally, the extension to first-order linear matrix recurrence relations enables growth and decay to be modelled in systems with multiple states, for example growth and decay in an insect population comprising eggs, juveniles and adults. The use of technology to support the graphical and computational aspects of this topic is assumed. Students will be taught to understand concepts, acquire skills and solve problems related to:

Arithmetic sequences. This includes:

• understanding the defining properties

• using a recurrence relation to generate terms

• the rule for determining the *n*th term of an arithmetic sequence

• the graphical representation and interpretation of arithmetic sequences

• summing the terms of a finite arithmetic sequence

• solving linear growth and decay problems in areas such as simple interest and flat rate depreciation.

Geometric sequences. This includes:

• understanding the defining properties

• using a recurrence relation to generate terms

• the rule for determining the *n*th term of a geometric sequence

• the graphical representation and interpretation of geometric sequences

• summing the terms of a finite or an infinite geometric sequence

• solving exponential growth and decay problems in areas such as simple population modelling, compound interest and depreciation by declining balance.

Sequences generated by first-order linear recurrence relations. This includes:

• using a first-order linear recurrence relation to generate terms of a sequence

• recognising arithmetic and geometric sequences as special cases

• the graphical representation and interpretation of sequences generated by first-order linear recurrence relations, including the condition for a steady-state solution

• using first-order linear recurrence relations to model growth and decay, for example the growth of a trout population in a lake where limited recreational fishing is permitted, a reducing balance loan.

Sequences generated by first-order linear matrix recurrence relations. This includes:

• using the recurrence relation to generate the terms of a matrix sequence

• transition matrices (including Leslie matrices) and their application to modelling growth and decay in systems with multiple states, for example growth and decay in an insect population comprising eggs, juveniles and adults, changing consumer preferences for a range of competing products.

## Unit 4

### Data analysis 3

Scatterplots are useful for identifying and describing, in general terms, associations between two numerical variables, but it would also be useful to have a more precise mathematical description. In the case of linear associations, this is achieved by fitting a straight line to the scatterplot using the least squares technique. By using data transformation, the linear modelling process can also be extended to analysing many non-linear associations. The emphasis throughout this topic is on graphical analysis. The use of technology to support the graphical and computational aspects of this topic is assumed. Students will be taught to understand concepts, acquire skills and solve problems related to:

Fitting a linear model to data. This includes:

• identifying the response variable and the explanatory variable

• using a scatterplot to identify the nature of the relationship between the variables

• modelling a linear relationship by fitting a least squares line of best fit

• interpreting the intercept and slope of the fitted line

• evaluating the quality of the linear fit by using a residual plot

• using the coefficient of determination in combination with a residual plot to assess the goodness-of-fit of the linear model

• using the equation of a fitted line to make predictions

• distinguishing between interpolation and extrapolation when using the fitted line to make predictions, and being aware of the dangers of extrapolation

• writing up the results of the analysis in a systematic and concise manner.

Modelling non-linear relationships. This includes:

• linearising the scatterplot using an appropriate data transformation (power or logarithmic)

• fitting a linear model to the transformed data and evaluating the quality of the fit by using a residual plot

- writing up the results of the analysis in a systematic and concise manner.

## Time series analysis

Governments and other organisations often collect data relating to the economy or social issues in which information is collected at regular intervals of time. When collated, this information then appears in the form of a time series. The ability to interpret and analyse such data is an increasingly important skill in both the workplace and the world at large where decision-making is increasingly influenced by information contained in time series data. The use of technology to support the graphical and computational aspects of this topic is assumed. Students will be taught to understand concepts, acquire skills and solve problems related to:

Describing and interpreting patterns in time series data. This includes:

- constructing time series plots
- describing time series plots by identifying features such as trend (long-term direction), seasonality (systematic, calendar-related movements) and irregular fluctuations (unsystematic, short-term fluctuations) and recognising when there are outliers (e.g. one-off real-world events) or signs of structural change (e.g. a discontinuity in the time series).

Analysing time series data. This includes:

- smoothing time series data numerically using moving means to help identify trends in time series with large fluctuations
- smoothing time series plots graphically using moving medians to help identify long-term trends in time series with large fluctuations
- calculating seasonal indices using yearly means
- deseasonalising a time series using a seasonal index
- fitting a least squares line, as an alternative to moving means or medians, to model long-term trends in time series data, adjusting for seasonality beforehand if necessary
- being aware of the dangers of extending forecasts too far into the future.

## Financial modelling

Financial modelling uses mathematical models to analyse the change in the value of loans, investments and assets over time. An understanding of financial modelling is a critical element in being a financially aware citizen in today’s world. The use of technology with financial mathematics capabilities to support the computational aspects of this topic is assumed. Students will be taught to understand concepts, acquire skills and solve problems related to:

Reviewing and extending simple and compound interest computations. This includes:

- using a recurrence relationship to model the growth of simple interest and compound interest investments
- solving problems involving simple interest, including determining any one of the principal, the annual interest rate, the time period in years and the amount accrued after that time period, given the other three
- solving problems involving compound interest, including determining any one of the principal, the annual nominal interest rate, the number of times that interest is compounded per year, the time period in years and the amount accrued after that time, given the other four
- determining the simple interest rate that must be paid to match the return of a compound interest investment over the same period of time
- determining for a compound interest investment or loan, the total interest earned or paid between any two compounding periods
- determining the effective interest rate for a compound interest loan or investment given the nominal annual interest rate and the number of

compounding periods per year.

Valuing assets over time. This includes:

• converting a percentage decrease into growth or decay rate

• adjusting book value of an asset to reflect its decreasing value over time using the flat rate (straight line), the declining (reducing) balance, and the unit cost methods

• comparing the three methods of depreciation to determine which produces the lowest book value after a given period of time.

Reducing balance loans (compound interest loans with regular repayments). This includes:

• using a recurrence relationship to model the reducing balance of a compound interest loan with regular repayments made immediately after each compounding period, including investigating the impact of the interest rate on the time to pay out the loan, the compounding and repayment period, and the repayment amount

• understanding the terms present value, future value, annual interest rate, repayment amount, and the number of repayments/compounding periods per year as they relate to reducing balance loans

• using technology to determine, for a reducing balance loan, any of one of the present value, future value, annual interest rate, repayment amount, and the number of repayments/compounding periods per year, given the other four

• using technology to investigate the effect of changing the repayment amount or interest rate on the time taken to repay the loan.

Simple annuities and perpetuities (compound interest investments with regular payments). This includes:

• using a recurrence relationship to model the changing value of an annuity, including investigating the impact of the interest rate on the lifetime of the annuity, the compounding and payment period, and the payment amount

• using technology to determine the future value of an annuity after a given number of payments

• using a recurrence relationship to model the reducing balance of a perpetuity

• determining the regular payment that can be expected from a perpetuity given the amount to be invested, the interest rate and the payment rate.

Adding to an investment. This includes:

• using a recurrence relationship to model the value of an investment over time when fixed amounts are added to the investment at regular intervals. This includes investigating the impact on the final amount accrued of the interest rate, the compounding and payment period, and the payment amount

• using technology to determine any one of the present value, future value, annual interest rate, repayment amount, and the number of repayments/compounding periods per year, given the other four

• using technology to investigate the effect of changing the payment amount or interest rate on the final value of the investment.

## Mathematical Methods

## Rationale and aims

### Rationale

Mathematics is the study of function and pattern in number, geometry and data. It provides both a framework for thinking and a means of communication that is powerful, logical, concise and precise. Mathematical methods is designed for students with an interest in mathematics and whose future pathways may involve mathematics at university. The focus is on function, calculus and statistics and the course provides a strong foundation for further studies in disciplines in which mathematics has an important role, including economics, political and social sciences and all branches of physical and biological sciences.

### Aims

- Students will deepen their algebraic understanding and skills with linear, quadratic and polynomial functions, extend their knowledge to include absolute value, exponential, logarithmic and trigonometric functions and will apply this to an understanding of the concepts of functions and graphing.
- Students will appreciate the significance of differential and integral calculus, and understand and apply its concepts in a range of situations and with a range of functions.
- Students will further extend understanding and skills in probability and statistics and will use this as a basis for developing knowledge of elementary statistical inference, an ability to apply this in simple situations and an appreciation of its importance in modern statistics.
- Students will choose and use appropriate technology to assist in the development of mathematical concepts.

## Organisation

### Organisation of the course

The senior secondary mathematics curriculum consists of four courses in mathematics. The courses are differentiated, each focusing on a pathway that will meet the learning needs of a particular group of senior secondary students. Each course is organised into four units typically for completion over four semesters.

Mathematical methods (Course C) has been designed as a standalone course, or to be taken in conjunction with General mathematics (Course B) or with Specialist mathematics (Course D).

### Coherence and continuity K-12

The *Shape of the Australian Curriculum – Mathematics* describes three content strands: number and algebra; statistics and probability; and Measurement and geometry. In the senior secondary years these strands have been continued but no longer used as major organisers. This is because the ideas in each of the strands both converge and diverge.

The proficiency strands of understanding, fluency, reasoning and problem solving have been integrated into the content descriptions, as in the K -10 curriculum.

The senior secondary mathematics curriculum builds on the K-10 curriculum by offering courses to cater for the needs and requirements of every student wishing to undertake further study of mathematics in the senior years

### Course Content

Course C is organised around key focus areas of:

· Algebra – concepts and skills as a foundation for functions and calculus

· Functions and graphs – formalised relationships between variable quantities

· Calculus

· Trigonometry

· Discrete and continuous random variables and their importance for understanding statistical inference

· Linear equations – systematic methods for solution, and introduction to matrices

## Unit Content

| | | | |
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| **UNIT 1** | | **UNIT 3** | |
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## General Capabilities

Good teaching in each of the learning areas will always contribute to a students' development of general capabilities and understanding of the crosscurriculum dimensions. The Australian Curriculum reinforces this expectation by incorporating the general capabilities and cross-curriculum dimensions into the content descriptions, in ways appropriate to each learning area. eg mathematics provides a framework for thinking and a means of communication that is powerful, logical, concise and precise.

In writing the Australian curriculum, considerable attention has been paid to the development of literacy and numeracy as the foundations on which much further learning depends. ICT has been incorporated into all topic areas. Thinking skills and the scope for creativity is inherent in all mathematical problem solving and so is an integral part of the Australian Curriculum: Mathematics.

Other general capabilities of self management, teamwork, intercultural understanding, ethical behaviour and social competence are able to be addresses in a classroom environment where the content can be delivered within a context suitable for a particular student group.

## Cross Curriculum Dimensions

Three cross-curriculum dimensions are dealt with explicitly in the Australian Curriculum: Indigenous history and culture, a commitment to sustainable living and Asia and Australia's engagement with Asia.

The Australian Senior Secondary Mathematics Curriculum provides opportunities for the study of Mathematics within a relevant context, giving students the means to explore, evaluate and comment with on information presented to them.

## Use of technology

The *Shape of the Australian Curriculum – Mathematics* states that available technology should be used for teaching and learning situations. Technology can include computer algebra systems, graphing packages, financial and statistical packages and dynamic geometry. These can be implemented through either a computer or calculator.

Technology can aid in developing skills and allay the tedium of repeated calculations. For example a technology can be used to complete recursive calculations.

There are many resources available on the internet and in state and territory portals that also have application for learning in the senior mathematics courses.

The decision about using technology in assessment programs is not within the province of the curriculum, jurisdictional assessment agencies will make that decision.

## Unit 1

### Algebra

Algebra is the basis of much of this course so students entering the course will need basic algebra skills. In this topic some algebraic techniques and skills are revised and new ones are introduced. They are all important for a proper understanding of functions, for manipulating formulas and for solving equations, as they arise throughout the course. In order to help understanding, the new algebraic concepts should be taught in conjunction with the corresponding parts of the Functions and graphs and the Calculus 1 topics. They should also be illustrated by practical applications. Assumed knowledge: the concepts, understandings and skills contained in the Number and Algebra strand at Year 10 and 10A levels of the National Curriculum. Graphics calculators and computers can be used to illustrate practically every aspect of this topic. Students will be taught to understand concepts, acquire skills and solve problems related to:

Logarithms. This includes:

- review of indices (including fractional indices) and index laws: $a^x a^y = a^{x+y}$, $a^{-x} = \frac{1}{a^x}$, $\left(a^x\right)^y = a^{xy}$, $a^0 = 1$, and $\left(ab\right)^x = a^x b^x$
- logarithms defined as indices: $a^x = b \, is \, equivalent \, to \, x = \log_a b,$ i.e. $a^{\log_a b} = b$
- logarithmic scales such as decibels in acoustics, Richter scale for earthquake magnitude, octaves in music, pH in chemistry
- solving equations involving logarithms by converting them to indicial equations, e.g. $\log_2 x = 5 \ implies \, 2^5 = x \ implies \ x = 32$
- solving equations involving indices using logarithms, e.g $2^x = 100 \ implies \ x = \log_2 100 \approx 6.644$
- using index laws to establish laws for the logarithm of products, quotients and powers: $\log_a\left(xy\right) = \log_a x + \log_a y$, $\log_a\left(\frac{1}{x}\right) = -\log_a x$, $\log_a x^n = n\log_a x$ and $\log_a 1 = 0$
- converting from one base to another: $\log_b x = (\log_b a)(\log_a x)$.

Numerical surds. This includes:

• radicals and conversions to and from fractional indices, e.g. $\sqrt[3]{5} = 5^{1/3}$ and $\frac{1}{\sqrt{7}} = 7^{-1/2}$

• simplification and manipulation, e.g. $\sqrt[3]{80} = 80^{1/3} = \left(2^4 \times 5\right)^{1/3} = 2^{4/3}5^{1/3}$

• rationalising denominators, e.g. $\frac{2}{3-\sqrt{5}} = \frac{2}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}} = \frac{6+2\sqrt{5}}{9-5} = \frac{6+2\sqrt{5}}{4} = \frac{3}{2} + \frac{\sqrt{5}}{2}$.

Rational algebraic expressions. This includes:

• simplification and manipulation of expressions such as $\frac{a}{x+c} + \frac{b}{y+d}$ and $\frac{x}{x+1} + \frac{x^2-1}{3x}$.

Polynomials. This includes:

• remainder and factor theorems

• applications to polynomials of low degree.

Binomial theorem. This includes:

• factorials and the notation: $n! = n \times (n-1) \times (n-2) \times \cdots \times 3 \times 2 \times 1$

• the notation $\binom{n}{r}$ *and* ${}^nC_r$ for the number of r-element subsets of an n-element set and the number of combinations of $n$ different objects taken $r$ at a time, and the formula $\binom{n}{r} = \frac{n!}{r!(n-r)!}$

• the expansions of $(1+x)^2$, $(1+x)^3$ and $(1+x)^4$, evaluated by hand

• the numbers $\binom{n}{r}$ as binomial coefficients, i.e. as coefficients in the expansion of $(1+x)^n$

• properties of Pascal's triangle, such as the entries are binomial coefficients, each interior entry is the sum of the two entries above it, and the row sums are powers of 2

• the binomial theorem: $(x+y)^n = x^n + \binom{n}{1}x^{n-1}y + \cdots + \binom{n}{r}x^{n-r}y^r + \cdots + y^n$.

## Functions and graphs

Students will already be familiar with the use of equations and formulas to describe relationships between variable quantities. These ideas are formalised via the function concepts – a vital starting point for the development of differential and integral calculus. In this topic the simplest types of functions are studied systematically. The ideas are illustrated throughout with graphs which are a powerful and effective way of conveying information. Assumed knowledge: the concepts, understandings and skills contained in the Number and Algebra strand at Year 10 and 10A levels of the National Curriculum. Graphics calculators and computers with graphics capabilities can be used to illustrate practically every aspect of this topic. Students will be taught to understand concepts, acquire skills and solve problems related to:

The function concept. This includes:

• functions as mappings between sets, and as rules or formulas that define one variable quantity in terms of another

• domain and range, independent and dependent variables.

Linear functions. This includes:

- the defining rule $y = ax + b$
- straight line graphs, slope $a$, and $y$ – intercept $b$
- fitting a linear function given value and/or slope information.

Quadratic functions. This includes:

- the squaring function $y = x^2$
- the general quadratic $y = ax^2 + bx + c$
- completing the square: $y = ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a}$
- fitting a quadratic function given value and/or turning point information
- parabolic graphs, turning points, zeros and the role of the discriminant
- graphs of $y = ax^2 + bx + c$ and $y = a(x-b)^2 + c$.

Cubic functions. This includes:

- the cubing function $y = x^3$
- graphs of $y = a(x-b)^3 + c$ and $y = k(x-a)(x-b)(x-c)$
- qualitative behaviour of graphs of simple cubics: limits at ±¥, maximum number of zeros and turning points.

Transformations of graphs (linear changes of scale and origin). This includes:

- the relationships between the graphs of $y = f(x)$, $y = af(x)$, $y = f(bx)$, $y = f(x-c)$ and $y = f(x) + d$, and the roles of $a, b, c \ and \ d$.

Graphs of relations. This includes:

- graphs of equations of the form $(x-a)^2 + (y-b)^2 = r^2$ and $x = f(y)$, where $f$ is a linear or quadratic function
- vertical line property of the graphs of functions.

## Calculus 1

This is the first of four topics devoted to calculus, a major component of the course because of its utility and beauty, and its accessibility to students at this particular level of intellectual development. This topic includes the basic elements of differential calculus. A ‘first principles’ approach to differentiation is taken and applied to simple power functions and polynomials. It is illustrated throughout using graphs and Cartesian geometry, and with simple applications in practical situations. Assumed knowledge: the concepts, understandings and skills contained in the Number and Algebra strand at Year 10 and 10A levels of the National Curriculum. Calculators and computers with graphics capabilities can be used to illustrate practically every aspect of this topic. Students will be taught to understand concepts, acquire skills and solve problems related to:

The concept of the derivative. This includes:

- the notations $\delta x$ and $\delta y$ for changes in the independent and dependent variables $x$ and $y$ respectively

- the ratio $\frac{\delta y}{\delta x}$ as the slope of a chord of the graph of $y = f(x)$
- the ratio $\frac{\delta y}{\delta x}$ as the average rate of change of $y$ with respect to $x$
- intuitive, informal introduction to limits, restricted to the behaviour of $\frac{\delta y}{\delta x}$ as $\delta x \to 0$
- the derivative $\frac{dy}{dx}$ defined as $\lim_{\delta x \to 0} \frac{\delta y}{\delta x}$
- the derivative as the slope or gradient of the tangent line of the graph of $y = f(x)$
- the derivative as the instantaneous rate of change
- function notation: the derivative $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \frac{dy}{dx}$, where $y = f(x)$.

Computation of derivatives. This includes:

- graphical and numerical approaches to estimating the values of the derivative, for simple power and polynomial functions
- the formula $\frac{d}{dx}\left(x^n\right) = nx^{n-1}$ established algebraically for positive integers $n$ using the binomial theorem
- linearity of the derivative: $\frac{d}{dx}(ky) = k\frac{dy}{dx}$ and $\frac{d}{dx}(y_1 + y_2) = \frac{dy_1}{dx} + \frac{dy_2}{dx}$
- derivatives of polynomials of low degree.

Applications of the derivative. This includes:

- finding instantaneous rates of change
- finding the slope of a tangent and the equation of the tangent
- velocity as instantaneous rate of change of position.

## Unit 2

### Trigonometry

Trigonometry begins as a branch of geometry studying the ratios of the sides of right-angled triangles. However when we begin to treat the trigonometric ratios as functions, the applications become very much broader. For example, trigonometric functions are commonly used to model periodic phenomena as diverse as light and sound waves, tidal motion, sunlight intensity and molecular vibrations. In this topic we see how the trigonometric functions are defined. However the first task is to choose the right scale. This involves a new angle measure. Assumed knowledge: the concepts, understandings and skills contained in the Number and Algebra and the Measurement and Geometry strands at Year 10 and 10A levels of the National Curriculum. Graphics calculators and computers with graphics capabilities can be used to illustrate practically every aspect of this topic. Students will be taught to understand concepts, acquire skills and solve problems related to:

Circular measure. This includes:

- the concept of angle as the amount of turning
- radian measure defined by the formula $\theta = \ell / r$, where $\theta$ is the radian measure of the angle in a sector of a circle where the radius is $r$ and the arc length is $\ell$.

• the relationships $360° = 2\pi$ and $180° = \pi$ (so $30°$ = $30 \times \pi / 180 = \pi / 6$ and so on)

• extended angle measures: angles greater than $2\pi \ (360°)$ corresponding to anticlockwise turning (rotation) through more than one full circle, and negative angles corresponding to clockwise turning (rotation)

• lengths of arcs and areas of sectors in circles.

Trigonometric ratios. this includes:

• revision of sine, cosine and tangent as ratios of side lengths in right-angled triangles

• the reciprocal trigonometric ratios: . $\sec\theta, \ \csc\theta \ and \ \cot\theta$ .

Extended domains for trigonometric functions. This includes:

• the unit circle definition of $\cos\theta \ and \ \sin\theta$ as the $x \, and \, y$ coordinates of the point on the unit circle corresponding to the angle $\theta$

• exact values of $\sin\theta, \ \cos\theta \, and \, \tan\theta$ , at special values of $\theta$, i.e. multiples of $\pi / 6 \ and \, \pi / 4$

• extended definition of $\sin\theta, \ \cos\theta \, and \ \tan\theta$ , for all real $\theta$

• periodicity of the trigonometric functions

• the graphs of $y = \sin x, \ \ y = \cos x \ \ and \ y = \tan x$ on extended domains

• the graphs of $y = af\left(bx\right),$ where $f(x) is \ \sin x, \cos x \, or \ \tan x$ .

## Algebra, functions and graphs 1

The study of algebra functions and graphs continues with the introduction of new classes of functions and an examination of special properties and features of graphs. The new functions are fractional and negative powers, exponentials and logarithms. These have a wealth of applications in mathematical modelling. The emphasis here is on the algebraic aspects, graphs and simple applications. This should be seen as preparation for the calculus of these types of functions, the subject of another topic in this unit. Graphics calculators and computers with graphics capabilities can be used to illustrate practically every aspect of this topic. Students will be taught to understand concepts, acquire skills and solve problems related to:

Power functions. This includes:

• qualitative features of the graphs of $y = x^n$ for integral values of $n$

• qualitative features of the graphs of $y = x^{1/2} \ \ and \ y = x^{1/3}$ .

Exponential functions. This includes:

• the defining formula $f\left(x\right) = a^x, \left(a > 0\right)$

• algebraic properties of exponential functions: index laws and change of base

• algebraic and graphical solution of simple equations involving exponential functions

• properties of their graphs, limits at ±¥, graphical effects of changes of base

• applications in exponential growth and decay models such as populations, compound interest, radioactive decay.

Logarithmic functions. This includes:

• the defining formula $y = \log_a x$ $(a > 1)$

- algebraic properties of logarithmic functions: index laws and change of base
- algebraic and graphical solution of simple equations involving logarithmic functions
- properties of their graphs, graphical effects of changes of base
- applications in geometric (exponential) growth and decay models.

Features of graphs. This includes:

- intercepts and asymptotes
- turning points, local and global maxima and minima
- reflections and inverses: the relationships between the graphs of $y = x^2$ and $y = \sqrt{x}$, and between the graphs of $y = a^x$ and $y = \log_a x$.

Transformations of graphs (linear changes of scale and origin). This includes:

- the relationship between the graphs of $y = f(x)$ and $y = af\left(b\left(x-c\right)\right)+d$ and the roles of $a,b,c$ and $d$.

## Calculus 2

The study of calculus continues in a natural way. The derivatives of fractional and negative powers, exponential and logarithmic functions are determined, and some fundamental differentiation rules are examined. The process of antidifferentiation is introduced, and antiderivatives of many of the functions examined so far are obtained. Further applications of differentiation such as optimisation and curve sketching are studied in practical contexts. There are also some basic applications of antidifferentiation. Graphics calculators and computers with graphics capabilities can be used to illustrate practically every aspect of this topic. Students will be taught to understand concepts, acquire skills and solve problems related to:

Differentiation. This includes:

- the notion of the derivative as a function
- the formula $\frac{d}{dx}x^n = nx^{n-1}$, for all rational numbers $n$, but with $n$ restricted in applications to small integral values and simple fractions
- the chain rule $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$, and its use for obtaining derivatives of functions such as $\left(2x-1\right)^3$ and $\sqrt{x^3+1}$
- curve sketching for simple polynomials and rational functions, stationary points, local maxima and minima, asymptotes and behaviour near $\pm\infty$
- displacement time graphs, with velocity as the slope of the tangent.

Antidifferentiation. This includes:

- antidifferentiation as the reverse of differentiation
- the notation $\int f\left(x\right)dx$ for the antiderivative, primitive or indefinite integral
- the formula $\int x^n dx = \frac{1}{n+1}x^{n+1}+c$ for $n \neq -1$
- linearity: $\int kf\left(x\right)dx = k\int f\left(x\right)dx$, $\int\left(f\left(x\right)+g(x)\right)dx = \int f\left(x\right)dx + \int g\left(x\right)dx$
- identification of families of curves with the same derivative function
- determining $f\left(x\right)$, given $f'\left(x\right)$ and an initial condition $f\left(a\right)=b$
- determining displacement given velocity in linear motion problems.

## Discrete random variables

This is the first of two topics devoted to the study of random variables and their associated probabilities. Together they set the framework for the topic on the statistical inference. Assumed knowledge: the concepts, understandings and skills contained in the Number and Algebra and the Statistics and Probability strands at Year 10 and 10A levels of the National Curriculum. Technology should be very widely used in this topic. Simulations can be used to introduce and explore many of these concepts. In particular, technology can be used to explore the effects of changing the parameters n and p in the graphical representation of a binomial distribution. Students will be taught to understand concepts, acquire skills and solve problems related to:

### Discrete random variables. This includes:

• reviewing the formulas $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, and $P(A \cup B) = P(A) + P(B)$ for the special case of mutually exclusive events

• conditional probability defined by $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$

• independence defined by $P(A \cap B) = P(A) \times P(B)$

• the concept of discrete random variable, illustrated with examples

• the probability function of a discrete random variable and its properties

• the cumulative distribution function of a discrete random variable and its relation to the probability function

• the mean (expected value), variance and standard deviation of a discrete random variable, evaluated in simple cases, and the notations $\mu,\ \sigma^2\ and\ \sigma,\ E(X)$ and $Var(X)$

• the effects of linear changes of scale and origin on the mean and the variance.

• the Bernoulli random variable with parameter $p$, with examples in real, everyday situations with two possible outcomes

### The Bernoulli distribution. This includes:

• the mean $\mu = p$ and variance $\sigma^2 = p(1-p)$ of a Bernoulli random variable.

• the binomial distribution as a commonly used model for the probability of random events

### The binomial distribution. This includes:

• independent Bernoulli trials, and the binomial random variable as a sum of independent Bernoulli random variables

• the probability function of the binomial distribution with parameters $n$ and $p$ and its graphical representation

• the mean $\mu = np$ and variance $\sigma^2 = np(1-p)$ of the binomial distribution

• applications of the binomial distribution, including conditions for its applicability.

# Unit 3

## Calculus 3

The study of differential calculus continues with the calculus of exponential and logarithmic functions, and with a deeper examination of differentiation and integration. We develop the basic properties of definite integrals, and study the fundamental theorem of calculus – the link computer algebra capabilities can be used to illustrate practically every aspect of this topic. Students will be taught to understand concepts, acquire skills and solve problems related to:

Exponential functions. This includes:

- estimating the limit of $\frac{a^h - 1}{h}$ as $h \to 0$ using technology, for various values of $a > 0$
- recognising $e$ as the unique number $a$ for which the above limit is $1$.
- the formulas $\frac{d}{dx}\left(e^x\right) = e^x$ and $\int e^x dx = e^x + c$.
- the natural logarithm $\ln x = \log_e x$

Logarithmic functions. This includes:

- the inverse relationship of the functions $y = e^x$ and $y = \ln x$
- the formulas $\frac{d}{dx}\left(\ln x\right) = \frac{1}{x}$ and $\int \frac{1}{x} dx = \ln x + c$
- the formulas $\frac{d}{dx}\left(\ln f(x)\right) = \frac{f'(x)}{f(x)}$ and $\int \frac{f'(x)}{f(x)} dx = \ln f(x)$.
- the product and quotient rules

Differentiation rules. This includes:

- application of these rules for differentiating functions such as $xe^x$, $\frac{1}{x^n}$ and $\frac{lnx}{x}$.

The second derivative. This includes:

- the second derivative as the rate of change of the first derivative function
- acceleration as the second derivative of displacement
- concavity and points of inflection, and applications in curve sketching
- the second derivative test for a local maximum or minimum.

Definite integrals. This includes:

- $\int_a^b f\left(x\right) dx$ as a limit of sums of the form $\sum_i f\left(x_i\right)\delta x_i$
- $\int_a^b f\left(x\right) dx$ as area under the curve $y = f\left(x\right)$ if $f\left(x\right) > 0$
- $\int_a^b f\left(x\right) dx$ as a sum of signed areas
- properties of definite integrals: $\int_a^b f\left(x\right) dx + \int_b^c f\left(x\right) dx = \int_a^c f\left(x\right) dx$ (additivity) and $\int_a^b \left(cf\left(x\right) + dg(x)\right) dx = c\int_a^b f\left(x\right) dx + d\int_a^b g\left(x\right) dx$ (linearity).

Fundamental theorem. This includes:

- the area function $F(x)=\int_a^x f(t)\,dt$
- the theorem: $F'(x)=\frac{d}{dx}\left(\int_a^x f(t)\,dt\right)=f(x)$
- the formula $\int_a^b f'(x)\,dx=f(b)-f(a)$ and its use for calculating definite integrals.

Applications of integration. This includes:

- areas under a curve and between curves
- total change from instantaneous or marginal rates of change
- the change in position of an object moving in a straight line given its velocity
- the average value $\frac{1}{b-a}\int_a^b f(x)\,dx$ of a function over an interval.

## Linear equations

Students will already have some experience in solving simultaneous equations by eliminating variables. In this topic they will learn a systematic method of solving systems of linear equations, commonly known as Gaussian elimination. They will use matrix notation for this method. As a result students will learn a useful algebraic technique. They will also be given a gentle introduction to matrices and one of their more common uses. Students will be taught to understand concepts, acquire skills and solve problems related to:

Systems of linear equations. This includes:

- the general form of a system of linear equations in several variables
- the notion of a solution of a system of linear equations in several variables
- solving systems of linear equations by systematic (Gaussian) elimination of variables
- three types of solutions: a unique solution, no solution and infinitely many solutions.

Row reduction methods. This includes:

- the augmented matrix as an efficient way of representing a system of linear equations
- row reduction of the augmented matrix as the equivalent of systematic elimination of variables in equations
- row echelon form
- obtaining the solution from the row echelon form of the augmented matrix in the unique case
- using technology to solve large systems (more than three equations or three variables).

## Continuous random variables

In this topic students will be introduced to the notion of a continuous random variable with emphasis on the normal probability distribution. Normal distributions and other continuous distributions can be applied in a wide range of areas. Technology should be very widely used in this topic.

Simulations can be used to introduce and explore many of these concepts, to calculate probabilities in more complex cases and to explore the effects of changing the parameters *n* and *p* in a normal distribution graph. Students will be taught to understand concepts, acquire skills and solve problems related to:

Continuous random variables. This includes:

- the concept of a continuous random variable, illustrated with examples such as uniform, exponential and normal random variables
- the difference between discrete and continuous random variables
- the probability density function of a continuous random variable and its properties
- calculating probabilities by evaluating integrals
- the cumulative distribution function of a continuous random variable and its relation to the probability function
- the mean, variance and standard deviation of a continuous random variable $X$, to be evaluated in simple cases.
- the effects of linear changes of scale and origin on the mean and the variance.

The normal distribution. This includes:

- the normal distribution as a commonly used continuous distribution
- the probability density function of the normal distribution with parameters $\mu$ and $\sigma$ and its graphical representation
- the standard normal distribution
- cutoff points $Z_\alpha$ for the standard normal distribution, defined by $P(Z > Z_\alpha) = \alpha$, where $Z$ is the standard normal variable and $0 < \alpha < 1$
- calculating probabilities associated with a given normal distribution
- determining quantiles associated a given normal distribution.

## Unit 4

### Statistical inference

This topic unites past work on statistical description and probability and moves it in a new direction, namely statistical inference. The techniques of statistical inference enable us to make statements about characteristics of a population on the basis of the information in a random sample from that population. A sample, by definition, contains only partial information about the population so the statements we make about the population cannot be made with certainty. They are, in other words, probabilistic statements. Here we use techniques of statistical inference to estimate a population proportion. There is scope in this topic for applying technology to simulate sampling to determine the behaviour of the sample proportion. Students will be taught to understand concepts, acquire skills and solve problems related to:

Sampling from ‘two-outcome’ populations. This includes:

- the concept of a ‘two-outcome’ population, with the outcomes typically labelled ‘success’ and ‘failure’, and with the proportion of successes as the population parameter $p$
- simple random sampling, and associated issues such as correct identification of the underlying population, survey procedures, sources of errors in surveys, deliberate and unintentional bias, selection bias, response and non-response bias.

The sample proportion. This includes:

- the sample proportion $f$ defined by $f = X / n$, where $X$ is the number of ‘successes’ in a sample of size $n$
- the concept of the sample proportion $f$ as a random variable

- the sampling distribution of $f$ and its relation to the binomial distribution
- using simulation to illustrate the sampling distribution of $f$
- the mean $p$ and variance $p(1-p)/n$ of the sample proportion $f$
- the standard error $e=\sqrt{f(1-f)/n}$ as an estimate of the standard deviation of the sample proportion $f$
- the approximate normality of the sampling distribution of $f$, if $np>5\ and\ n(1-p)>5$.

Confidence intervals for a proportion. This includes:

- the sample proportion $f$ as a point estimator of the population proportion $p$
- the $(100-\alpha)\%$ confidence interval $f-Z_{\alpha/2}e<p<f+Z_{\alpha/2}e$ for the population proportion $p$, and its interpretation
- the conservative 95% confidence interval $f-1/\sqrt{n}<p<f+1/\sqrt{n}$
- the margin for error $Z_{\alpha/2}e$ versus level of confidence: factors governing the width of the confidence interval for a population proportion
- determining the sample size $n$ required for a confidence interval with a specified margin of error

## Algebra, functions and graphs 2

In this topic the algebraic and graphical properties of absolute value functions and trigonometric functions are studied. This completes a suite of functions commonly used in mathematical models of real-world phenomena. This is followed by an introduction to non-linear curve fitting – a particularly useful aspect of mathematical modelling in applied science. Calculators and computers with graphics capabilities can be used to illustrate practically every aspect of this topic. Students will be taught to understand concepts, acquire skills and solve problems related to:

Absolute values. This includes:

- critiquing the reporting of polls, including self-selection ‘voting’ polls, in the media.
- the notation $|x|$
- interpretation of $|x|\ and\ |x-y|$ as distances between points on the real number line
- the graph of $y=|x|\ and\ variants\ such\ as\ y=|ax+b|$
- solving equations of the type $|ax+b|=c,$ algebraically and graphically.

Trigonometric expressions and functions. This includes:

- properties of trigonometric functions: periodicity, symmetry, complementarity and phase differences
- features of the graphs of $y=\sin x,\ y=\cos x\ and\ y=\tan x$: period, amplitude, phase shifts, asymptotes
- the relationship between the graphs of $y=f(x)\ and\ y=af(b(x+c))+d,$ where $f(x)$ is $\sin x,\ \cos x\ or\ \tan x,$ the roles of $a,b,c\ and\ d,$ with particular reference to changes in amplitude, period and phase
- solving equations of the type $af(bx+c)=d,$ where $f(x)$ is $\sin x,\ \cos x\ or\ \tan x,$ algebraically or graphically.

Curve fitting. This includes:

- review of properties of the least squares line of best fit
- fitting exponential functions $y=Ae^{bx}$ through two points

- log-linear transformations and log-linear plots
- the least squares line of best fit in the log-linear plane
- the correspondence between the exponential curve $y = Ae^{bx}$ and the straight line $\ln y = \ln A + bx$ in the log-linear plane.

## Calculus 4

The study of calculus is completed with the calculus of trigonometric functions, an integration technique and some useful applications of differentiation and integration. Students who successfully complete these calculus topics will gain an appreciation of the power of the subject and its wide scope of applications. They will also be well equipped to undertake studies in any of the many disciplines in which mathematical modelling is important. Calculators and computers with graphics capabilities can be used to illustrate practically every aspect of this topic. Students will be taught to understand concepts, acquire skills and solve problems related to:

Trigonometric functions. This includes:

- the formulas $\frac{d}{dx}(\sin x) = \cos x \ \ and \ \ \frac{d}{dx}(\cos x) = -\sin x,$ made plausible using numerical estimations of the limits, or using geometric constructions
- the formula $\frac{d}{dx}(\tan x) = \frac{1}{cos^2 x},$ established via the quotient rule
- derivatives of related functions such as $x\sin x,\ \cos(2x-3)\ and\ e^{\tan x}$ using the rules for differentiation
- the indefinite integrals $\int \sin x dx = -\cos x + c \ \ and \ \ \int \cos x dx = \sin x + c$.

Integration techniques. This includes:

- indefinite integrals of the form $\int f(ax+b)\,dx$, such as , $\int (3x+5)^3\,dx$ $\int \frac{dx}{2x+3}$ and $\int \sin(4x-1)\,dx$.

Applications of calculus. This includes:

- optimisation over finite intervals
- the increments formula: $\delta y \cong \frac{dy}{dx} \times \delta x$
- related rates as instances of the chain rule: $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
- exponential growth and decay: $y = Ae^{kx}$ as the solution of $\frac{dy}{dx} = ky$.

# Specialist Mathematics

## Rationale and aims

### Rationale

Mathematics is the study of function and pattern in number, geometry and data. It provides both a framework for thinking and a means of communication that is powerful, logical, concise and precise. Specialist mathematics is designed for students with a strong interest in mathematics including those intending to study mathematics, physical sciences or engineering at university. The course is intended to be taken in conjunction with Mathematical methods. The course contains topics in functions and calculus that build on and deepen the ideas presented in Mathematical methods. Vectors, complex numbers and recursive methods are introduced. The emphasis is on the application of mathematics.

**Aims**

• Students will apply understanding and skills in geometry and trigonometry, difference equations, differential and integral calculus, vectors and complex numbers and differential equations and use these applications to model real life situations.
•Students will choose and use appropriate technology to assist in understanding mathematical concepts and solve problems.
•Students will develop rigorous mathematical reasoning skills and understand the power of mathematics to model situations.

## Organisation

### Organisation of the course

The senior secondary mathematics curriculum consists of four courses in mathematics. The courses are differentiated, each focusing on a pathway that will meet the learning needs of a particular group of senior secondary students. Each course is organised into four units typically for completion over four semesters.

Specialist mathematics (Course D) is designed to be taken in conjunction with Mathematical methods (Course C). There has been consideration given to studnets who have studied units 1 and 2 in Course C (Mathematical Methods) to enter Course D (Specialist Mathematics) at unit 3.

### Coherence and continuity K-12

The *Shape of the Australian Curriculum – Mathematics* describes three content strands: number and algebra; statistics and probability; and Measurement and geometry. In the senior secondary years these strands have been continued but no longer used as major organisers. This is because the ideas in each of the strands both converge and diverge.

The proficiency strands of understanding, fluency, reasoning and problem solving have been integrated into the content descriptions, as in the K -10 curriculum.

The senior secondary mathematics curriculum builds on the K-10 curriculum by offering courses to cater for the needs and requirements of every student wishing to undertake further study of mathematics in the senior years.

### Course Content

Course D is organised around key focus areas of:

• Mathematical proofs and applications
•Real and complex numbers - properties, representations and uses
•Recurrence relations - the concept of iteration
•Matrices - applications to study of solutions to linear equations and transformations of two-dimensional figures
•Graph theory - the algebraic equation of a curve and parametric equations to describe a curve
•Trigonometry - more advanced functions and modelling periodic phenomena
•Kinematics - application of calculus to solving problems
•Mathematical induction
•Vectors in one-, two-, and three dimensional situations
•Differential calculus - further methods of differentiation

## Unit Content

| | |
|---|---|
| **Unit 1** | **Unit 3** |
| **Topic 1:** Mathematical proof | **Topic 1:** Mathematical induction |
| **Topic 2**: Real and complex numbers | **Topic 2:** Vectors |
| **Topic 3**: Recurrence relations | **Topic 3:** Differential calculus |
| **Topic 4**: Matrices | **Topic 4**: Complex numbers |
| **Unit 2** | **Unit 4** |
| **Topic 1:** Parametric equations | **Topic 1:** Integral calculus |
| **Topic 2**: Graph theory | **Topic 2**: **OPTIONS – 3 options – only one to be taken**<br><br>*Option 1: Statistical inference*<br><br>*Option 2: Vectors and Dynamics*<br><br>*Option 3: Further calculus techniques and inequalities* |
| **Topic 3**: Trigonometry | **Topic 4**: Kinematics |

## General Capabilities

Good teaching in each of the learning areas will always contribute to a students' development of general capabilities and understanding of the crosscurriculum dimensions. The Australian Curriculum reinforces this expectation by incorporating the general capabilities and cross-curriculum dimensions into the content descriptions, in ways appropriate to each learning area. eg mathematics provides a framework for thinking and a means of communication that is powerful, logical, concise and precise.

In writing the Australian curriculum, considerable attention has been paid to the development of literacy and numeracy as the foundations on which much further learning depends. ICT has been incorporated into all topic areas. Thinking skills and the scope for creativity is inherent in all mathematical problem solving and so is an integral part of the Australian Curriculum: Mathematics.

Other general capabilities of self management, teamwork, intercultural understanding, ethical behaviour and social competence are able to be addresses in a classroom environment where the content can be delivered within a context suitable for a particular student group.

## Cross–Curriculum Dimensions

Three cross-curriculum dimensions are dealt with explicitly in the Australian Curriculum: Indigenous history and culture, a commitment to sustainable living and Asia and Australia's engagement with Asia.

The Australian Senior Secondary Mathematics Curriculum provides opportunities for the study of Mathematics within a relevant context, giving students the means to explore, evaluate and comment with on information presented to them.

## Use of technology

The *Shape of the Australian Curriculum – Mathematics* states that available technology should be used for teaching and learning situations. Technology can include computer algebra systems, graphing packages, financial and statistical packages and dynamic geometry. These can be implemented through either a computer or calculator.

Technology can aid in developing skills and allay the tedium of repeated calculations. For example a technology can be used to complete recursive calculations.

There are many resources available on the internet and in state and territory portals that also have application for learning in the senior mathematics courses.

The decision about using technology in assessment programs is not within the province of the curriculum, jurisdictional assessment agencies will make that decision.

## Unit 1

### Mathematical proof

This topic enables students to apply the basic structure of mathematical proof. The proofs first suggested involve numbers. The techniques of proof developed in this context are then applied in Euclidean geometry. Students have studied geometry in previous years. They have worked with results concerning parallel lines, triangles, quadrilaterals, used congruence and similarity. Students will apply theorems and ideas from earlier work to develop further results in Euclidean geometry. They will apply them to the solution of both numerical and theoretical problems. Computers and calculators with geometry capabilities can be used for constructions and in the solution of numerical problems. Assumed knowledge: conditions for congruency of triangles, similarity of triangles, properties of triangles and quadrilaterals. These concepts and the presentation of geometric arguments were introduced in the curriculum for Years 7–10. Students will be taught to understand concepts, acquire skills and solve problems related to:

The nature of proof. This includes:

• implication, converse, equivalence, contrapositive

• proof by contradiction

• correct use of the symbols $\Rightarrow$ (implication), $\Leftrightarrow$ (equivalence) and $=$ (equality)

• informal use of the quantifiers 'for all' and 'there exists' (and their negations)

• counter-examples.

• these ideas will also be illustrated through deductive Euclidean geometry

These will be demonstrated through proving and discussing results such as:

• there are infinitely many prime numbers

• for $n$ a positive integer $n^2$ is odd if and only if $n$ is odd

• the sum of two consecutive odd numbers is divisible by 4

• for all positive integers $n$ and $m$, if $n$ and $m$ are even then $n+m$ is even

• for all real numbers $x$ and $y$, if $x$ is rational and $y$ is irrational, then $x+y$ is irrational

• for all non-zero real numbers $x$ and $y$, we have $\frac{1}{x+y} \neq \frac{1}{x} + \frac{1}{y}$.

Circle properties. These include the following theorems and their application to proving further results and solving problems:

• an angle in a semicircle is a right angle

• the angle at the centre subtended by an arc of a circle is twice an angle at the circumference subtended by the same arc

• angles at the circumference of a circle subtended by the same arc are equal

• the opposite angles of a cyclic quadrilateral are equal

• chords of equal length subtend equal angles at the centre and conversely chords subtending equal angles at the centre of a circle have the same length.

Results involving chords, tangents and secants of circles, and the application of these results. The results include:

• the alternate segment theorem

• when two chords of a circle intersect, the product of the lengths of the intervals on one chord equals the product of the lengths of the intervals on the other chord

• when a secant (meeting the circle at $B$ and $A$) and a tangent (meeting the circle at $T$) are drawn to a circle from an external point $(M)$, the square of the length of the tangent equals the products of the lengths to the circle on the secant $(AM \times BM = TM^2)$.

Concurrency properties in triangles and proofs of associated results. These include:

• concurrency of medians (centroid)

• concurrency of altitudes (orthocentre)

• concurrency of perpendicular bisectors of sides (circumcentre)

• concurrency of bisectors of the vertex angles (incentre).

## Real and complex numbers

This topic provides a satisfying completion to the study of numbers undertaken in earlier years at school. Students taking this unit will already be fluent in rational number arithmetic and familiar with the concept of the real number line. The primary aim in this topic is to enhance their understanding of real numbers in general, and irrational numbers in particular, by providing a concrete representation. The secondary purpose of this topic is to introduce students to complex numbers. This will involve a study of their basic arithmetic properties, their representations as points in the complex plane, and their uses as solutions of quadratic equations that have no real roots. Computers and calculators will be used to generate sequences, to perform complex arithmetic and to solve equations. Students will be taught to understand concepts, acquire skills and solve problems related to:

Infinite decimal expansions. This includes:

• rational numbers as terminating and recurring decimals

• irrational numbers as non-terminating, non-recurring decimals.

Existence of irrationals. This includes:

- direct construction of non-terminating, non-recurring decimals
- proofs of irrationality by contradiction for numbers such as $\sqrt{2}$ and $log_2 5$.

Approximation of irrationals by rationals. This includes:

- finite truncation of infinite decimal expansions
- $\sqrt{2}$ as the limit $x_{n+1} = \frac{1}{2}(x_n + \frac{2}{x_n})$.

Complex numbers. This includes:

- the imaginary number $i$ as a root of the equation $x^2 = -1$
- complex numbers in the form $a + ib$ with $a$ and $b$ real numbers
- real and imaginary parts of a complex number
- complex conjugates
- complex arithmetic: addition, subtraction, multiplication and division.

The complex plane. This includes:

- complex numbers as points in a plane with real and imaginary parts as Cartesian coordinates
- addition of complex numbers as vector addition in the complex plane
- location of complex conjugates in the complex plane.

Roots of equations. This includes:

- the general solution of quadratic equations with real coefficients
- complex conjugate solutions
- linear factors of real quadratic polynomial.

## Recurrence relations

Iteration is about applying the same procedure over and over again, where the output from one application becomes the input for the next. Its role in mathematics has become increasingly important in recent decades with the advent of modern computers, which often use programs heavily dependent on iteration. The aim in this topic is to introduce the concept of iteration by showing how it can be used to generate sequences, and by applying these sequences in a variety of commonly occurring situations. In another unit of this course students will study iterative methods of proof. Arithmetic and geometric sequences will be considered as special types of recurrence relations. Computers and calculators will be used to generate sequences through the use of a spreadsheet facility, built-in features or simple programs. Assumed knowledge: in 10A the emphasis was on the introduction of arithmetic and geometric sequences by establishing both iterative and explicit formulas. The study in this topic is of first-order linear recurrence relations, which builds on 10A. Students will be taught to understand concepts, acquire skills and solve problems related to:

The two parts of an iterative definition of a sequence: the initial values and the iterative step.

Examples of iterative definitions of sequences. This includes:

- the Fibonacci sequence

• the factorial sequence

• arithmetic sequences

• geometric sequences

• applications of first-order linear recurrence to problems in financial and population modelling and including long-term behaviour.

‘Direct’ formulas for sequences. This includes:

• finding direct formula for terms of arithmetic and geometric sequences

• finding formula for the partial sums of arithmetic and geometric sequences.

## Matrices

Matrices are rectangular arrays of numbers. They are a convenient way of displaying sets of numbers that depend on two parameters. As such, they have wide applications in computing, engineering, science, economics and statistics, as well as various branches of mathematics. Two of the most important applications will be studied in this topic. These are the use of matrices in the study of systems of linear equations and in the study of rotations and other transformations of two-dimensional figures. Matrices have their own algebraic properties. In many respects they behave like ordinary numbers but there are some important differences. Computers and calculators will be used to undertake some matrix calculations. Assumed knowledge: in this unit matrices are used in the solution of simultaneous linear equations in two variables and defining transformations of the plane. The study of simultaneous equations and transformations of the plane should have been undertaken in Years 7–10. Students will be taught to understand concepts, acquire skills and solve problems related to:

Matrix arithmetic. This includes:

• matrix definition and notation

• addition and subtraction of matrices, scalar multiplication, matrix multiplication, identity and inverse

• calculating determinant and inverse of $2 \times 2$ matrices

• solving matrix equations.

Simultaneous equations. This includes:

• solving simultaneous equations in two variables

• geometric interpretations of cases where there is a unique solution, no solution or infinitely many solutions.

Transformations in the plane. This includes:

• translations, dilations, rotations about the origin through angle q, reflections in a line which passes through the origin and shears

• composition of these transformations and matrix products

• relationship with the determinant: $|\det A|$ is the area magnification factor and $\det A < 0$ corresponds to orientation reversal

• geometric results by matrix multiplications, e.g. the combined effect of two reflections is a rotation.

# Unit 2

## Parametric equations

In earlier years students have used Cartesian equations to describe curves. The aims in this topic are to find the algebraic equation of a curve that has been geometrically specified and to use parametric equations to describe a curve. These techniques are used to investigate the geometric properties of parabolas in particular. Computers and calculators can be used to plot curves from their parametric equations. Assumed knowledge: quadratic functions and circles studied in 10A, including the solution of quadratic equations and sketching parabolas. Students will be taught to understand concepts, acquire skills and solve problems related to:

Geometric specification of circles, lines, parabolas, ellipses and hyperbolas.

Parametric equations and the 'corresponding' Cartesian equation. These include:

- circles: $x = a\cos t, y = a\sin t (x^2 + y^2 = a^2)$
- parabolas: $x = at^2, y = 2at \quad (y^2 = 4ax)$
- ellipses: $x = a\cos t, y = b\sin t \quad (\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1)$
- hyperboles: $x = a\sec t, y = b\tan t \quad (\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1)$.

The geometry of the parabola. This includes:

- the geometric definition of a parabola (focus-directrix)
- chords of parabolas
- tangents and normals of parabolas (Cartesian and parametric equations).

Proofs of geometric properties of parabolas. This includes:

- a chord joining two points on a parabola is a focal chord if and only if the tangents at the endpoints of the chords meet at the directrix
- the interval joining a point on the parabola to the focus, and the line through the point parallel to the x-axis are equally inclined to the tangent at the point.

## Graph theory

A graph is a way of showing connections between things. From a few simple ideas the study of graphs and their applications has grown into a lively branch of modern discrete mathematics. In this brief introduction to graph theory students will learn some basic concepts and simple algorithms. They will also learn some of the history of the subject. This will help convince students that mathematics is alive and growing, and is not, as is commonly believed, simply a collection of formulas and recipes. Students will be taught to understand concepts, acquire skills and solve problems related to:

Basic graph concepts. This includes:

- vertices, edges, edge weights
- examples of graphs
- subgraphs, connectedness and components
- simple graphs
- isomorphic graphs
- the complete graph $K_n$ on $n$ vertices

• bipartite graphs, and the complete bipartite graph $K_{m,n}$ on $m$ and $n$ vertices.

Circuits. This includes:

• the Konigsberg bridge problem

• Euler circuits and necessary and sufficient conditions

• Hamiltonian circuits and the travelling salesman problem.

Trees. This includes:

• definition of a tree

• the formula $E = V + 1$ relating the number of edges $E$ and the number of vertices $V$ in a tree

• spanning trees

• minimal spanning trees and Prim's algorithm.

Planarity. This includes:

• Euler's formula for simple connected plane graphs

• non-planarity of $K_5$ and $K_{3,3}$ and the proofs via Euler's formula

• Kuratowski's theorem: a graph is planar if and only if it contains no subdivision of $K_5$ or $K_{3,3}$

• Platonic solids and the proof of uniqueness via Euler's formula.

## Trigonometry

Students will already be familiar with the trigonometric functions and their basic properties. In this topic their understanding is extended by studying more advanced properties and by modelling various types of periodic phenomena, a task for which the trigonometric functions are ideally suited. Graphics or computer algebra system calculators and computers can be used for practical investigations of every aspect of this topic. Students will be taught to understand concepts, acquire skills and solve problems related to:

Compound angles. This includes:

• expansions of $\sin(A \pm B)$, $\cos(A \pm B)$ and $\tan(A \pm B)$ derived geometrically or via matrix multiplication

• double angle formulas for sine, cosine and tangent.

Trigonometric identities. This includes proofs, using established properties of the trigonometric functions, of simple identities such as:

• Pythagorean identities

• expansions of products of sines and cosines as sums and differences

• converting sums $a\cos x + b\sin x$ to $R\cos(A \pm \alpha)$ or $R\sin(x \pm \alpha)$

• triple angle formulas.

Trigonometric equations. This includes:

- graphing functions with rules of the form $y = f\left(a\left(x - b\right)\right)$ where $f$ is $\sin$, $\cos$ or $\tan$
- general solution of $f(a(x-b))=c$ where $f$ is $\sin$, $\cos$ or $\tan$
- general solution of $a\cos x + b\sin x = c$.

Periodic phenomena. This includes:

- using trigonometric functions to model periodic phenomena such as simple harmonic motion, and heights of tides and other forms of wave motion
- exploration of the graphs of trigonometric polynomials with applications in music
- illustrations of amplitude, period and phase shifts in practical contexts.

### Kinematics

Kinematics is the study of motion of a particle in a straight line. It provides examples of rates of change and average rates of change that are introduced in Course C (Mathematical Methods) Units 1 and 2. It is an application of calculus to practical situations by solving problems in straight line motion with both constant and non-constant acceleration determined by a polynomial rule. Through distance–time graphs the relationship between gradient and velocity is illustrated, as is the relationship between gradient and acceleration for velocity–time graphs. This topic also provides an opportunity for students to observe the relationship between area and displacement through geometric consideration of the velocity–time graph of a particle moving with constant acceleration. Computers and calculators can be used for plotting graphs and calculations. **Assumed knowledge**: calculus of Units 1 and 2 of Course C (Mathematical Methods). Students will be taught to understand concepts, acquire skills and solve problems related to:

Motion of a particle moving in a straight line. This includes:

- the formula for motion under constant acceleration derived by antidifferentiation
- distance-time graphs
- velocity-time graphs
- comparing motion of two particles travelling in a straight line such as particles meeting, comparative times of arrival
- velocity as the derivative of position (displacement) and acceleration as the derivative of velocity with respect to time
- determining position by antidifferentiating an expression for velocity, given the position at a particular time
- determining velocity by antidifferentiating an expression for acceleration, given the velocity at a particular time.

## Unit 3

### Mathematical induction

Mathematical induction is an iterative method of proof. It is directly analogous to the concept of iterative definition of sequences. We use mathematical induction when we want to prove an entire sequence of propositions, typically one for each positive integer. The idea is that we prove the first or first few, propositions directly and then show that any one proposition is true if the ones before it are all true. The principle of induction allows us to conclude the truth of all the propositions. Proofs by induction will also be used to establish results in other topics in this course. Students will be taught to understand concepts, acquire skills and solve problems related to:

The nature of inductive proof. This includes:

- the initial step
- the inductive step.

Examples of proofs by induction. This includes:

• proving results for sums such as $1+4+9+\ldots n^2=\frac{n(n+1)(2n+1)}{6}$ for any positive integer $n$

• proving results on divisibility such as $3^{2n+4}-2^{2n}$ is divisible by 5 for any positive integer $n$

• proving results by induction of results which occur in Units 3 and 4 of Course C (Mathematical Methods) or in this course.

## Vectors

Vectors have many applications in both mathematics and physics. They are used in one- two- and three-dimensional situations including the representation of lines and curves in space, forces and other quantities with both magnitude and direction. They are also used to describe the position of a particle in the plane or in three-dimensional space and to prove geometric results. Computers and calculators can be used to plot the graphs of vector functions and to manipulate and study vectors in two and three dimensions. Assumed knowledge: geometry and coordinate geometry from Year 10. Students will be taught to understand concepts, acquire skills and solve problems related to:

The algebra of vectors. This includes:

• addition and subtraction of vectors and their multiplication by a scalar

• magnitude of a vector, unit vector and the perpendicular unit vectors $i, j, k$

• scalar (dot) product, $a.b=|a||b|\cos(\theta)$ where $\theta$ is the angle 'between' $a$ and $b$ and multiplication defined through components when the vectors are expressed in $i, j, k$ notation, $a=a_1i+a_2j+a_3k$ and $b=b_1i+b_2j+b_3k$ then $a.b=a_1b_1+a_2b_2+a_3b_3$

• parallel and perpendicular vectors

• resolution (projections) of vectors.

Vector equations. This includes:

• vector equations of curves in two or three dimensions and determination of the 'corresponding' Cartesian equation

• determining the vector equation of a straight line, given the position of two points or equivalent information, in both two and three dimensions

• concurrence of lines

• the position of two particles each described as vector function of time and determining if their paths cross or if the particles meet.

Geometric proofs using vectors. This includes proofs in Euclidean geometry such as:

• the diagonals of a parallelogram meet at right angles if and only if it is a rhombus

• midpoints of the sides of a quadrilateral join to form a parallelogram

• the sum of the squares of the lengths of the diagonals of a parallelogram is equal to the sum of the squares of the lengths of the sides

• in a regular tetrahedron the lines from the circumcentre of each face to the opposite vertex are concurrent.

## Differential calculus

Differential calculus is introduced in Course C (Mathematical Methods) and its applications to kinematics are introduced in Semester 2 of this course. In this semester further methods of differentiation are studied and these new techniques and previously introduced techniques are used to knowledge: differential calculus from Course C (Mathematical Methods). Students will be taught to understand concepts, acquire skills and solve problems related to:

Implicit differentiation.

Sketching graphs. This includes:

• finding maximum and minimum values, stationary points, points of inflexion, axes intercepts, and determining convexity

• sketching the graphs of inverses of one-to-one functions

• the graphs of functions of the form $y=\frac{1}{f(x)}$, $y=|f(x)|$ where $f$ is a function introduced in Course C (Mathematical Methods) or this course

• the graphs of functions of the form $y=f(x)+g(x)$ where $f$ and $g$ are functions introduced in Course C (Mathematical Methods) or this course

• sketching the graphs of simple rational functions where the numerator and denominator are polynomials of degree $\leq 2$

• applying translations, reflections in the axes and dilations from either axis, and combinations of these, to graphs.

## Complex numbers

This topic reviews and continues the study of complex numbers. In Unit 1 the focus was on complex numbers as solutions of quadratic equations and on complex arithmetic. In this unit the geometric properties of complex numbers are explored, using their representation as points in the complex plane. Some important and elegant links with trigonometry are established, and the unit concludes with satisfying revelations about the location of square, cube and higher order roots of numbers. Technology can be used throughout this topic. For example, calculators with complex number capabilities can be used to illustrate the rules for complex number arithmetic. Students will be taught to understand concepts, acquire skills and solve problems related to:

Rectangular forms. This includes a review of:

• real and imaginary parts $Re(z)$ and $Im(z)$ of a complex number $z$

• rectangular form: $z=x+iy$ where $x=Re(z)$ and $y=Im(z)$

• complex arithmetic using rectangular forms.

Complex arithmetic using rectangular forms and polar forms. This includes:

• the modulus $|z|$ and for $z\neq 0$, argument, $\arg(z)$ with $-\pi<\arg(z)\leq\pi$, of a complex number $z$

• $|z|$ and $\arg(z)$ as the polar coordinates of the number $z$ in the complex plane

• polar forms: $z=r(\cos\theta+i\sin\theta)$ and $z=re^{i\theta}$ where $r=|z|$ and $\theta=\arg(z)$

• conversions between rectangular and polar forms: $x=r\cos\theta$ and $y=r\sin\theta$

• multiplication, division and powers of complex numbers in polar form and the geometric interpretation of these

• de Moivre's theorem $(\cos\theta+i\sin\theta)^n=\cos n\theta+i\sin n\theta$.

The complex plane. This includes:

• addition of complex numbers as vector addition in the complex plane

• multiplication by $i$ as rotation through $90^\circ$ in the complex plane

• identification of subsets of the complex plane determined by relations such as $|z-3i|\le 4$, $\frac{\pi}{4}\le \arg(z)\le \frac{3\pi}{4}$, $Re(z)>Im(z)$ and $|z-1|=2|z-i|$.

Roots of complex numbers. This includes:

• the $n^{th}$ roots of unity and their location on the unit circle

• the $n^{th}$ roots of non-zero complex numbers and their location in the complex plane.

Factorisation of polynomials with real coefficients. This includes:

• conjugate roots

• solution of simple polynomial equations.

## Unit 4

### Integral calculus and applications of calculus

Integration is introduced in Course C (Mathematical Methods) where integral calculus is used to find the area of a region. In this topic, a number of techniques for integration are developed and integration is applied to kinematics problems to find the area between curves and to find volumes of solids of revolution. Differential equations play a prominent role in engineering, physics, economics and other disciplines. In this course, simple first- and second-order differential equations are formulated and solved. Two iterative methods for finding approximate solutions are introduced: Newton's method for finding approximate solutions to equations and Euler's method for finding numerical solutions to first-order differential equations. Calculators and computers can be used with all aspects of this work, including graphing and the solution of equations by a number of techniques. Students will be taught to understand concepts, acquire skills and solve problems related to:

Integration techniques. This includes:

• using partial fractions to integrate rational functions which have two linear factors in the denominator

• integration using the trigonometric identities $\sin^2(x)=\frac{1}{2}(1-\cos(2x))$, $\cos^2(x)=\frac{1}{2}(1+\cos(2x))1+\tan^2(x)=\sec^2(x)$

• using substitution $u=g(x)$ to integrate expressions of the form $f(g(x))g'(x)$

• inverse trigonometric functions arcsine, arccosine and arctangent

• the derivative of the inverse trigonometric functions arcsine, arccosine and arctangent

• integrating expressions of the form $\frac{1}{\sqrt{a^2-x^2}}$ and $\frac{a}{a^2+x^2}$ by recognising that they are the derivatives of corresponding inverse trigonometric functions. For example: $\frac{d}{dx}\left(\arcsin\left(\frac{x}{a}\right)\right)=\frac{1}{\sqrt{a^2-x^2}}$.

Applications of integral calculus. This includes:

• the calculation of displacement of a particle by considering velocity-time graphs

• the calculation of areas between curves determined by functions studied in this course

• volumes of solids of revolution about either axis.

Using Newton's method to find approximate solutions to equations of the form <img src="/Image/482eae86-2acc-44fb-bbf2-bc2e9c6bfcb1?h=20" class="formula" alt="" /> where <img src="/Image/997f9ea6-4dae-4862-b4fc-fe5fc9ca6d60?h=20" class="formula" alt="" /> is a differentiable function.

Differential equations. This includes:

• solutions of simple first-order differential equations of the form $\frac{dy}{dx} = f(x)$, differential equations of the form $\frac{dy}{dx} = g(y)$ and in general differential equations of the form $\frac{dy}{dx} = f(x)g(y)$, using separation of variables. (Care must be taken with zeros of $g(y)$.)

• slope (direction or gradient) fields of a first-order differential equation

• Euler's method for finding approximate solutions of first-order differential equations

• formulation of differential equations that will arise in the physical and biological sciences and economics in situations where rates are involved.

Simple harmonic motion. This includes:

• the different forms for acceleration, $\frac{dv}{dt}$, $v\frac{dv}{dx}$ and $\frac{d(\frac{1}{2}v^2)}{dx}$

• determining and using equations for displacement, velocity and acceleration

• using relevant formulas and graphs to solve problems.

## Option 1: Statistical inference

This option uses past work on statistical description and probability to move in a new direction – statistical inference. Two problems of statistical inference are considered in this topic. The first uses the concept of confidence intervals introduced in Course C (Mathematical Methods) to investigate the problem of estimating means. The second is concerned with estimation in regression. The use of technology to support the graphical and computational aspects of this topic is assumed. Assumed knowledge: statistics and probability in 10A. Students will be taught to understand concepts, acquire skills and solve problems related to:

The sampling distribution of the sample mean for samples from a normal distribution. This includes:

• parameters and sample statistics

• understanding simple random sampling (SRS)

• defining the sampling distribution of the sample mean for samples of size $n$ drawn from a normal distribution with mean $\mu$ and variance $\sigma^2$

• using simulation to explore the distribution of sample means for samples drawn from a normal distribution

• understanding that for samples of size $n$ drawn from a normal distribution with mean $\mu$ and variance $\sigma^2$, the sampling distribution of the sample mean is also normal, with mean $\mu$ and variance $\frac{\sigma^2}{n}$

• examples in working with the sampling distribution of the sample mean.

Estimating the unknown mean of a normal distribution, assuming the variance is known. This includes:

• understanding in context the concepts: point estimator, mean and standard error of a point estimator, unbiased point estimator, interval estimator (= confidence interval)

• informal development of the formula for constructing an interval estimate of the theoretical mean, $\mu$, at a specified level of confidence

• proof of the confidence interval result

• interpretation, via simulation, of the confidence level in the confidence interval formula

• interpretation of the confidence interval evaluated from the data of a single sample

• factors governing the width of the confidence interval

• determining the sample size required for a confidence interval with a specified margin of error

• extension: estimating the mean when the variance is unknown and the sample is large

• applications to interval estimation of the mean in such contexts as human height and limb length, and departure from technical specifications in the routine performance of machinery.

Interval estimation of the mean of a continuous non-normal distribution when the variance is unknown and the sample is large. This includes:

• using simulation to sample from various continuous non-normal distributions to discover the effect of increasing sample size on the sampling distribution of the mean

• informal statement of the central limit theorem (CLT)

• the confidence interval formula, based on the CLT, and its interpretation

• applications to interval estimation of the mean in such contexts as average spectator attendance at sporting events, TV viewer numbers, suburban crime rates, unemployment rates, retail cost of groceries across different types of stores, price of petrol across capital cities.

Fitting a straight line to data in a scatterplot by the method of least squares. This includes:

• deciding which is the explanatory variable and which is the response variable

• the method of least squares for fitting a line, showing which 'squares' are made 'least'

• formulas for the slope and intercept of the fitted line and their derivation

• residual plots and their use in assessing the appropriateness of fitting a linear model and identifying outliers

• how outliers may influence the direction of the fitted line and its strength of fit to the scatter

• using the least squares line to make predictions; interpolation and extrapolation and the dangers of extrapolation

• understanding that the slope and intercept of the fitted line are (point) estimates of parameters in a model. This model assumes that the mean of the response variable is a straight line in the explanatory variable

• interpreting the estimated slope in terms of changes in the expected value of the response variable as the explanatory variable changes

• interpreting confidence intervals for the expected value of the response variable at selected values of the explanatory variable (confidence intervals either given or obtained through technology)

• interpreting prediction intervals for individual values of the response variable at selected values of the explanatory variable (prediction intervals either given or obtained through technology)

• reporting on fitted lines, residual plots, confidence and prediction intervals in context in a systematic and concise manner.

## Option 2: Vectors and dynamics

This option provides the opportunity for the student to use their knowledge of calculus and vectors to solve motion of a particle problems in both two and three dimensions. Newton's laws are introduced and applied with both constant and variable force problems allowing a diversity of physical situations to be considered. Calculators and computers can be used with all aspects of this work including graphing and the solution of equations by a number of techniques. Assumed knowledge: the algebra, calculus and functions from this course and Course C (Mathematical Methods) and the vectors topic from this course. Students will be taught to understand concepts, acquire skills and solve problems related to:

Kinematics. This includes:

• motion in a straight line with both constant and non-constant acceleration

• using the different forms for acceleration, $\frac{dv}{dt}$, $v\frac{dv}{dx}$ and $\frac{d(\frac{1}{2}v^2)}{dx}$.

Vector calculus. This includes:

• position vector as a function of time

• deriving the Cartesian equation of a path given as a vector equation and sketching the path

• differentiation and antidifferentiation of a vector function with respect to time

• application of vector calculus to motion in two and three dimensions

• projectile motion.

Newton's laws. This includes:

• momentum, force, resultant force, action and reaction

• motion of a particle on an inclined plane

• systems of connected particles in two dimensions

• non-constant force

• motion of a body under concurrent forces

• frictional forces: sliding friction and the coefficient of friction

• motion under gravity with restrictive forces.

## Option 3: Further calculus techniques and inequalities

This option introduces further techniques of integration that give a greater diversity of functions that can be worked with in the applications discussed previously in the course. Further applications of integration such as length of a section of a plane curve are studied. A study of the use of inequalities is also included. In mathematics many important results are obtained through the use of inequalities. Their uses include finding estimates of accuracy in the field of numerical analysis. Calculators and computers can be used with all aspects of this work including graphing and the solution of equations by a number of techniques. Assumed knowledge: the algebra, calculus and functions topics from this course and Course C (Mathematical Methods). Students will be taught to understand concepts, acquire skills and solve problems related to:

Further differentiation and integration techniques. This includes:

• partial fractions with both an irreducible quadratic and a linear factor in the denominator

• integration by parts

• reduction formulas

• further integration using simple trigonometric substitutions.

Applications of integration using the techniques developed. This includes:

• kinematics, motion in a straight line with both constant and non-constant acceleration. Using the different forms for acceleration, $\frac{dv}{dt}$, $v\frac{dv}{dx}$ and $\frac{d(\frac{1}{2}v^2)}{dx}$.

• volumes of solids of revolution

• arc length, lengths of a segment of a plane curve

• finding the volume of a solid which has similar cross-sections.

Further inequalities. This includes:

• proving inequalities by a variety of methods, including algebra, calculus and induction. For example, the AM ? GM (arithmetic mean ? geometric mean inequality)

• proving further results involving inequalities by logical use of previously obtained inequalities.