Sydney Grammar School
College Street
Darlinghurst NSW 2010
Tuesday 24th June 2008

Dr John Bennett
General Manager of the Board of Studies
GPO Box 5300
Sydney NSW 2001

Dear Dr Bennett,

<u>Confusion in the Draft Calculus Syllabuses</u>

As you know, the Board has asked me over many years to help in an advisory way with its mathematics syllabuses. I am therefore sending a copy of my submission to you.

I am also sending it to you because it appears that four issues, and perhaps more, arising from the recent drafts can only settled at a level higher than the mathematics inspectorate:

- I have written at some length in this submission, and previously, on the need for the calculus syllabuses to be written under the supervision of the leading university mathematicians, in cooperation with experienced and well-qualified school teachers. Professor Gus Lehrer and I discussed this with Professor Stanley several years ago when the review of the calculus courses was first being considered. The confused texts of the drafts, however, with both poor mathematics and poor classroom practice, makes it clear that this has not been done.
- It appears that the Board is imposing on mathematics some general examination specifications that require multiple-choice questions. This decision threatens to undermine seriously the excellence and rigour that the calculus examinations have had now for many decades.
- The Board is also imposing on mathematics a general requirement that mathematics syllabuses be written in dotpoints. Such a dotpoint structure has proven, once again, to be inadequate for the task of expounding the mathematics and defining the syllabus.
- The insistence that any review of Stage 5 material be removed from Advanced has made the Advanced course too hard, and will prevent most Stage 5.2 students from attempting it.

Our current calculus courses are well known to be excellent in their imagination, their rigour, their usefulness, their appeal to students, and their assessment procedures. They are the envy of our colleagues in other disciplines and a great credit to the NSW Board of Studies.

In contrast, the poor quality of these new draft syllabuses, and the lack of balance in their content, pose a serious threat to our current excellence. I appeal to you not to allow any version of these syllabuses to be implemented until they are a clear improvement on what we already have.

I would hope that the Board will rewrite these documents and put them out again for comment by the mathematics community, many of whom I know from meetings and discussions are equally concerned. There is so much that we cannot assess effectively at the moment.

My apologies for what is unavoidably a rather technical submission.

Yours sincerely,

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Sydney Grammar School

Cc: John O'Brien
Director of Curriculum

Peter Osland and Margaret Bigelow
Inspectors of Mathematics

As I remarked in my previous submission, it is a relief that the basic outlines of calculus have been retained in the three courses, due largely to the decisions to retain the calculus of e^x and $\sin x$ in the 2 Unit course, and to retain the structure of the three courses. The decision to return to two non-calculus HSC courses will also be welcomed in schools as a practical and sensible measure.

I'm afraid, however, that despite these good decisions having been made, the texts of the three calculus syllabuses are at this stage in a depressingly poor and confused state.

- There are wrong mathematics, contradictory and inappropriate terminology, and confusing notation throughout the documents. The new material is particularly badly written.
- There are many small-scale and large-scale misjudgements about the abilities of students, with the 2 Unit Preliminary course particularly being too long and difficult.
- Topics unsuitable for school have been included — in particular the logistic equation and second-order differential equations — with important and useful topics removed. This has resulted in far too much calculus in Extension 2, at the expense of geometry and algebra. No reasonable case has been made for making any change to the Extension 1 and 2 courses.
- Calculus seems to have been cut adrift from its foundation in Stage 5 content.
- There are an astonishing number of ambiguities in documents whose main task is to define precisely what content teachers present in their classes.
- The division between Preliminary and HSC will prove unsuitable for many classes and abilities, and the regulations need to be softened to allow teachers and schools to make adjustments.
- The 'mean and tricky' approach to examinations, using multiple-choice questions, will seriously undermine the Board's longstanding tradition of excellence in HSC examinations.

The problems in these texts are serious, and will lead to a significant deterioration of mathematics teaching standards in the State. They cannot be addressed by the quick cut-and-paste editing that the Board presently seems to be envisaging — the texts need to be completely redrafted in a reconstituted writing process as described below.

It would be unacceptable for the Board to introduce new syllabuses until the mathematics teaching community is convinced that the new courses are better than the excellent calculus syllabuses that we now have — a high demand indeed. The texts, after full redrafting, should be resubmitted to the mathematics community. There is no urgency whatsoever to change syllabuses.

1. Syllabus writing needs academic leadership and teaching experience

I, and many others, have been recommending strongly to the Board for many years that the texts of their mathematics syllabuses need to be written by, or under the close supervision of, the leading professors of the day, together with experienced and well-qualified classroom teachers. It is clear from the documents that once again, this simple and obvious recommendation has not been acted upon as far as the texts of the three calculus syllabuses are concerned. This is a great disappointment, and will need to be remedied before any real progress can be made.

The present state of the documents gives the impression of pages having been worked over by successive writers without consultation with previous writers or reference to other pages, as if comments have been pooled and thrown together in a jumble with little coordination, and with little overall idea of the mathematics involved or of the unity of the courses. It is therefore difficult and time-consuming to make any coherent critique of the documents.

Part of the problem lies in the Board's insistence on writing its syllabuses in dotpoints, with facing pages having some incomprehensible and inconsistent relationship between them. Coherent mathematics cannot be written like this, as we have been arguing for years, and as these documents confirm once again. The best syllabus text so far written under these constraints was the very first — the previous 4 Unit syllabus. It should be consulted as a reasonable example of a poor model.

Teachers deserve better than this, as was clear from this month's MANSW meeting, where severe and detailed criticisms of the drafts came from a large number of individual teachers who clearly knew their mathematics and their classrooms very well. More importantly, our State's mathematics students deserve better — we cannot present a coherent course to them based on these drafts.

I give below four detailed examples, followed by notes on some further scattered examples from the Advanced syllabus. The problems, however, are legion, and the Board will need considerable time and effort to sort it all out.

Mathematical induction (Extension 1): Pages 50–53 provide good examples of what is wrong with the written text. First, dotpoint 2 on page 51 contains the sentence, 'We let S denote the set of positive integers n for which $S(n)$ is true.' No mathematician, and no teacher, would use the same letter to denote two quite different things — such notation is misleading and unusable. This confusion in notation continues for the rest of the page.

Secondly, the language of sets has been used, making the discussion unnecessarily technical and difficult, particularly since set notation has been removed from earlier syllabuses. It also uses notations from formal logic, which again is not otherwise in this or any earlier course. Is this intended to constrain teachers to use set language and formal logic notation for proofs by induction? Our School presently teaches induction using successive sentences to explain the logic — is the Syllabus instructing us to abandon this practice?

Thirdly, so-called 'strong' induction has been prescribed for Extension 1. Most teachers would judge that ordinary induction is quite difficult enough at this level. Extension 1 should specify the study of 'ordinary' induction, and only Extension 2 should study 'strong' induction.

Fourthly, if 'strong' induction is being prescribed, why are no examples given that require it? Is it perhaps the intention that results associated with second-order difference equations be proven by induction? Even this is nothing like a full use of 'strong induction'?

Fifthly, induction in Extension 2 should not be limited just to series and divisibility.

Sixthly, why does dotpoint 3 on page 51 restrict M to a positive integer — many examples of induction start at $n = 0$, and others start at a negative number. Similarly, is it intended in the last example on page 53 (a divisibility proof for odd integers) that induction advance by 2 at each step, or that some substitution be made? Extension 1 students do not find either method easy.

Polynomials (Extension 1): Similar mathematical misunderstanding, poor notation, imprecisions in content specifications, and lack of thought about the classroom occur throughout pages 44–47. First, the last dotpoint on page 47 remarks that 'Since all coefficients and roots will be integers in this topic, ...'. This is false for a non-monic polynomial like $(2x - 1)(3x - 1)$ — all six theorems in dotpoint 5 above are clearly intended to involve non-monic polynomials. Even for monic polynomials, the example of $x^2 - 2$ shows that such a remark only applies to rational roots.

Secondly, the notation is confused. The letters a and p are constantly interchanged, each being used both for coefficients and for roots in different places (p 45 dotpoints 2 & 6 and p 47 dotpoints 4, 5(1), 5(2), 5(5)). The text should use a standard notation with Latin letters a_i or a, b and c for coefficients, and Greek letters α_i or α, β and γ for roots.

Thirdly, dotpoint 6 on page 45 reads 'For very large $|x|$, $P(x) \approx p_n x^n$.' Does this mean that some general theory about the symbol $\approx$ is to be taught? Are we to explain that $\lim\limits_{|x|\to\infty} \dfrac{P(x)}{p_n x^n} \to 1$, and to contrast this with the fact that $P(x) - p_n x^n$ probably diverges? There is no mention of this matter in the graphing section. Such imprecision is characteristic of the texts, and would not have been left unresolved by any experienced schoolteacher, who would recognise immediately the classroom difficulties involved here.

The symbol $\approx$ is used elsewhere in the Advanced syllabus (pp 92–93 on the trapezoidal rule) to mean 'approximately equals'. Are students to cope with two meanings of one symbol? Is the symbol $\doteqdot$, standard in NSW for 'approximately equals', now rendered obsolete?

Fourthly, in dotpoint 8, is calculus to be the basis of the explanation as to why a polynomial graph touches the x-axis at a multiple root, but cuts it at a single root? This needs to be made precise.

Fifthly, in dotpoint 1 on page 46, the division algorithm for integers has been mis-stated, there being no mention of the fact that the remainder is non-negative and less than the divisor.

Increasing functions (Advanced course): Dotpoint 5 on page 42 of the Advanced syllabus involves pre-calculus sketching of graphs, but specifies 'stating intervals for which a function is increasing or decreasing'. Our past syllabuses, and the present drafts in other places, define 'increasing at $x = a$' to mean $f'(a) > 0$, and define 'stationary at $x = a$' to mean $f'(a) = 0$, both definitions occurring in the context of stationary points, turning points and inflexions.

The definition required here, however, is presumably that '$f(x)$ is increasing in the closed interval $a \le x \le b$ if $f(x_1) < f(x_2)$ whenever $a \le x_1 < x_2 \le b$'. The mean value theorem is, thankfully, not part of our armoury at school, and we tacitly assume from naive geometry the necessary theorems, except that with very able students, we may challenge them until some version of the mean value theorem emerges.

This is a well-known difficulty with high-school calculus, and I am most surprised to see the dotpoint, which must be removed. To put things simply, if this dotpoint remains, $y = x^3$ will be stationary at $x = 0$, while at the same time being increasing in the interval $-1 \le x \le 1$.

Recursively defined sequences (Extension 1): I would far prefer that the parametric treatment of the parabola be retained, allowing students to learn in a reasonably straightforward situation how to do calculus and geometry with parameters. If, however, this new topic is to replace it, then the terms must be properly defined, the terminology must be stable, and there must be clarity as to what is to be taught.

First, the language and notation has not been thought out.

- Students will be hard pressed to cope with three names — 'difference equation', recurrence relation' and 'recursively defined sequence'. They have been using the word 'sequence' in APs and GPs, and the topic will make better sense to them if this same word continues to dominate, leading me to opt for retaining 'recursively defined sequence' and dropping 'difference equation' entirely. (Moreover, the term 'difference relation' is often used in a more restricted sense.)
- This situation is made more difficult by confusions in the text, which occur contrary to the attempt to distinguish 'difference equation' and 'sequence' in dotpoint 1 of page 57. Immediately after this, dotpoint 2 on page 57 introduces the hybrid term 'difference relation'. Dotpoint 3 on page 57 again identifies the equation and the sequence. Dotpoint 4 on page 57 should strictly be talking about the associated recurrence relation, not the difference equation. Dotpoints 5–6 on page 58, and dotpoint 5 on page 59 seem to identify the difference equation and the recurrence relation. Indeed dotpoint 3 on page 60 actually defines the logistic difference equation to be a recurrence relation, although this is contradicted by the language of dotpoint 4 on page 61. So much confusion in so few words!
- The word 'argument' (p 57 dotpoint 4) is used nowhere else in the syllabus to characterise a variable, and should be replaced by 'subscript'. (Indeed the whole last sentence is badly expressed and should be omitted.)
- Why is the phrase 'closed-form' used in dotpoint 2 on page 58, when with APs and GPs we have been talking about a 'formula for the nth term'? The term should be dropped. Similarly, the term 'integer function' of dotpoint 1 on page 58 is normally used in computing for a function that *returns* an integer value — its *domain* may be the set of all real numbers.
- Why does the text keep alternating between T_n and x_n for the nth term of the sequence? Why does it then introduce a third symbol P_n for the terms of the logistic sequence? We should continue to use the notation T_n introduced for APs and GPs.
- In dotpoint 4 on page 61, the text swaps from P_n and x_n within the one formula, making it meaningless. Even allowing for this, the formula given here is not a logistic recurrence

equation according to definition given in dotpoint 3 on page 60 — or is the syllabus' intention that we teach some approach to scaling factors here?

Secondly, there is incorrect mathematics.

- According to the definition in dotpoint 5 on page 57, 'A linear difference equation is one in which x_n is defined by previous terms, which are all to the power of one ...'. This would make $x_n = x_{n-1} \times x_{n-2}$ linear. (Moreover, it is presumably the recurrence relation that is intended.)
- Dotpoint 2 on page 59 claims that difference equations can be solved by mathematical induction, which can't be done.
- How does a teacher of Extension 1 prove to a school student that a difference equation does not have a closed-form solution (p 58, dotpoint 2)?

Thirdly, the level and manner of treatment in the classroom is unclear, and seems not to have been properly thought out.

- Do we conclude from dotpoint 4 on page 59 that a student is to memorise the solution of a general first-order difference equation, much as the various formulae for APs and GPs are to be memorised?
- In dotpoint 5 on page 59, we are asked to assume that $x_n = \lambda^n$ is a solution of a difference equation, when in most cases it is not a solution. If the initial values are being ignored, this must be stated and explained clearly. Why not $x_n = A\lambda^n$? Is it intended that some notion of linear combination be taught? Are we to talk about homogeneous equations?
- The given recurrence relation in this dotpoint should read $x_n - bx_{n-1} - cx_{n-2} = 0$ or $x_n + bx_{n-1} + cx_{n-2} = 0$, with 1 as the coefficient of x_n, because of the way recurrence relations have been defined.
- Second-order difference equations throw up three cases: distinct real roots, distinct unreal roots, and repeated roots. No mention has been made of the repeated root case — has it been considered? (I note that the DEs topic in Extension 2 contains no method for dealing with this case). Neither has any mention been made of the unreal roots case. Has it been considered, and is the plan to ask questions about complex-valued sequences in the Extension 2 paper? If so, we teachers had better be told about it, because it will take some teaching on our part!
- This is a new topic, but whereas the syllabuses readily give examples of topics that have all been taught before, there are no examples given on page 59.

Some further examples of errors, confusion or poor classroom practice: This list is by no means complete, but is simply offered to confirm the remarks made at the start of this section.

- Pages 32–35 (Advanced) seem to use the words 'results' and 'events' to mean the same thing, with the word 'outcome' occasionally also used in the same way (p 33 dotpoint 3). The text should stick with 'event', and carefully define 'outcome' to mean one of the 'equally likely possible outcomes'.
- Dotpoint 3 on page 33 (Advanced) claims that 'mutually exclusive outcomes do not occur together', which is misleading, since 'outcomes' are just points in the sample space.

  Here are two further problems in counting and probability. First, the easier situation of counting with replacement needs to be clearly placed in the Syllabus. Secondly, the product rule for probability needs to be stated before probability tree diagrams are expounded.

- Dotpoints 1–2 on page 38 (Advanced) use the terms 'inequality' and 'inequation' interchangeably. I personally am happy with the standard NSW practice of 'solving inequations' and 'proving inequalities', but whatever is done, the words can't be confused.
- Dotpoint 4 on page 38 (Advanced) talks about 'rationalising of denominators that are the sums or differences of two surds', which excludes the example of $\dfrac{4}{2+\sqrt{5}}$ on the facing page.

- Dotpoints 2 on page 39 (Advanced) claims that 'The absolute value of x can also be defined as the positive square root of the square of x'. This excludes $|0| = 0$.

  I would also recommend that the notation . for $\times$ be discarded on page 39. We have already seen in past Extension 2 papers the confusion that this can cause with the decimal point. Also, the drafts seem to have abandoned standard NSW practice of a raised decimal point, as in 3·14.

- Dotpoint 7 on page 41 (Advanced) claims that 'A property of a graph of a function is that no two points will have the same ordinate', thus confusing the vertical line test with the horizontal line test. It then claims that this *property* is a *test*, thus confusing antecedent and consequent.

  I would also recommend that the term 'natural domain' be used in dotpoint 4 on this page.

- Dotpoints 1–3 on page 54 (Advanced) seem to use the word 'relation' to mean 'identity'. Then dotpoint 4 correctly uses the term 'identity'.

- Dotpoint 3 on page 55 (Advanced) on graphical solutions should read 'to solve simple equations *approximately*'.

- Why has the term 'partial sum' (pp 82–83, Advanced) not been placed where it belongs, in the previous section on the sum of the first n terms of a series?

- There are two separate mistakes in the clumsy and unnecessary formula for the trapezoidal rule on page 92 (Advanced). The initial factor should be $\dfrac{b-a}{2n}$, not $\dfrac{b-a}{n}$, and the last term in the series should be $f(x_{n-1})$, not $f(x_n)$. Also, it is confusing to place $f(b)$ second rather than last, and the last example on page 93 uses a different formula.

  This formula should not be in the course, being far too difficult to remember and to use. (If the writers cannot transcribe it correctly, what hope have 2 Unit students?) The version appropriate for school is $\dfrac{b-a}{2}\Big(f(a) + f(b)\Big)$, which can be applied repeatedly if necessary. This far more manageable formula follows immediately from the area of a trapezium once the curve is approximated by a chord, giving the unified geometric–algebraic view that we are constantly seeking. It also makes it clear from the concavity whether the approximation overestimates or underestimates the integral — the writers have ignored this matter.

- Pages 94–95 (Advanced) on finding areas by integration thankfully makes no mention of the formula $\displaystyle\int_a^b |f(x)|\,dx$, which most of us would regard as far too complicated for 2 Unit students, and not particularly useful in Extensions 1 and 2. Yet question 4 of the sample Advanced examination paper requires a detailed understanding of the notation.

  Apparently left-hand pages, right-hand pages and specimen papers all define different syllabus fragments, chosen more or less at random.

- Dotpoint 3 on page 99 (Advanced) identifies a 'turning point' on a curve with a local maximum or minimum. This is contrary to standard mathematical practice, and to all previous use in NSW. A 'turning point' should be a *stationary point* that is a local maximum or minimum. We would never want to say that $y = |x|$ has a turning point at the origin.

- The last dotpoint on page 100 (Advanced) talks about 'the behaviour of the function for large and small values of x'. The example of $y = \dfrac{1}{x-2}$ shows that what is probably intended is 'the behaviour for large values of $|x|$, and for values of x near the boundaries of the domain.'

- Page 105 (Advanced) suggests the compound interest limit $\lim\limits_{n\to\infty}\left(1+\frac{1}{n}\right)^n$ as a path to introducing e^x and e. The limits involved here are quite beyond Extension 1, but belong amongst the extremely difficult questions at the end of the Extension 2 paper. They have no place here.

- Also, the result $\dfrac{d}{dx}\, e^x = e^x$ is not a 'mathematical curiosity'. It is the basis of all exponential growth and decay, and thus underpins all mathematics of science and economics.
- The third formula in dotpoint 1 on page 112 (Advanced) is wrongly stated — the variables have been confused. It is also far too hard and abstract for 2 Unit.
- The language of the first dotpoint on page 121 is hopelessly muddled, with four different rates under discussion. In anyone's language, the 'growth rate' of a population is the 'birth rate' minus the 'death rate'. Yet the syllabus calls this difference the 'rate of growth', and reserves the term 'growth rate' for a proportionality constant that is more properly called the 'instantaneous proportional growth rate', but is far better left unnamed. It is a linguistic absurdity to use the terms 'growth rate' and 'rate of growth' for two quite different things.
- Also, the hybrid and confusing notation $N(0)$ should be dropped, here and elsewhere. We should simply talk about 'the value of N when $t = 0$'.
- The last dotpoint on Page 34 (Extension 1) seems to indicate that the word 'turning point' is to replace the usual word 'vertex' of a parabola. I hope that this is an oversight.
- Dotpoint 4 on page 35 (Extension 1) confuses the terms 'function' and 'expression'.
- Dotpoint 4 on page 41 (Extension 1) requires the teaching of transformed graphs of $y = e^x$ in Preliminary Extension 1, when the number e is only introduced in the HSC Advanced course.
- The task envisaged on pages 72–73 (Extension 1) of forming equations whose roots are a function of the roots of a given polynomial is far too hard for Extension 1. I hope that it has just been cut and pasted wrongly.
- The introduction on page 74 (Extension 1) asks students to 'see the connection between particular functions of a real variable through the use of complex numbers'. Complex numbers, however, are only introduced in Extension 2.
- The outcomes on page 74 mention three-dimensional trigonometry, which is a normal topic for Extension 1. But nothing more of it is mentioned in the content summary following, nor in the details on pages 75–79.
- The last dotpoint on page 77 (Extension 1) introduces the symbol $\equiv$ in the context of trigonometric identities, but it was not used at all in the more usual context of polynomials. What is it intended that we teach? I would recommend dropping the symbol entirely at school level.
- The symbols $\equiv$ on page 77 and $\approx$ on page 45 are only mentioned on the *right-hand side* of the double page. Such introduction of new work on the right is common throughout the document, hence my remark about the 'incomprehensible relationship' between the two facing pages.
- The first dotpoint on page 79 (Extension 1) claims that the auxiliary angle is $\alpha = \tan^{-1} \frac{b}{a}$. This is a well-known schoolboy howler, which is false if α is in the second or third quadrant. Secondly, $\tan^{-1}$ is only introduced 10 pages later on page 88. Thirdly, the angle cannot be called the 'phase angle', because $\theta - \alpha$ is the phase angle (in this example). Fourthly, why has there been a trivial name change from 'auxiliary angle' to subsidiary angle'?
- The last dotpoint on page 83 (Extension 1) is poor mathematics, requiring a complicated substitution into an integral to evaluate the area of a quadrant of a circle.
- Pages 88–89 (Extension 1) on inverse trigonometric functions provide several examples of errors, confusions, and poor classroom practice. First, dotpoint 2 on page 89 has confused domain and range in one of the few 'formal definitions' that the text offers. The function $\sin^{-1} x$ is not 'defined on the interval $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$', it is defined on the interval $-1 \le x \le 1$.
- Secondly, a related confusion seems to occur in dotpoint 5 on the facing page 88. The identity $\sin \sin^{-1} x = x$ is true for all x in the domain of the left-hand side, whereas $\sin^{-1} \sin x = x$ is only true for a subset of the domain of the left-hand side.

- Thirdly, the last dotpoint on page 88 gives the wrong restriction on b. The equations $\sin\theta = b$ and $\cos\theta = b$ can be solved for $-1 \le b \le 1$, not just for $-1 < b < 1$. The error is repeated on page 89.

  In addition, the last dotpoint fails to mention equations of the form $\sin\alpha = \sin\beta$, $\cos\alpha = \cos\beta$ and $\tan\alpha = \tan\beta$, which are the forms in which these equations usually arise in applications.

  It is unfortunate that the last dotpoint on page 89 specifies that the solution of $\sin\theta = b$ is $\theta = n\pi + (-1)^n \sin^{-1} b$, which is almost incomprehensible to average Extension 1 students. Far better is '$\theta = 2n\pi + \sin^{-1} b$ or $\theta = 2n\pi + (\pi - \sin^{-1} b)$', which they have some reasonable chance of interpreting on the graph and using in applications.

- Fourthly, the first dotpoint on page 88 has mis-stated the reason for restricting the domain of $\sin x$. We restrict the domain first and foremost so that the inverse is a function — the question of increasing or decreasing (which as remarked above has unfortunately been defined in two ways) arises as a consequence of which branch is selected.

- Dotpoint 3 on page 33 (Extension 2) uses the notation $\sqrt{a + ib}$, which we are always careful to avoid at school, being a 'multi-valued function', which is a contradiction in terms at our level.

- Dotpoint 11 on page 63 (Extension 2) claims that 'The terminal velocity can be calculated from the equation of motion by finding V when $\ddot{x} = 0$' (where I presume the writers intend v). This is false, as the simple motion equation $x = e^t$ shows, where setting $\ddot{x} = 0$ in resulting equation $\ddot{x} = v$ gives a terminal velocity of zero, whereas the particle accelerates to infinite velocity.

- The table on pages 13–20 (Extension 1) does not agree with the text of the syllabuses. For example, Mathematical induction (a Preliminary topic) is listed as an HSC topic, and projectile motion (an HSC topic) is listed as a Preliminary topic.

For those of us whose life work has been maintaining the excellence and rigour of the classroom teaching of mathematics, it is disappointing and depressing to see such a flawed document being issued by the NSW Board of Studies itself.

2. Unsuitable choices of topics included and omitted

The logistic equation: This topic needs to go, or it will cause chaos in NSW classrooms.

Teachers can't say anything sensible about chaos to the students, and nothing particularly can be achieved at Extension 1 level with the given class of functions. Even with a computer, one can find the bands approximately, and draw the pretty pictures in Wikipedia, but there are no sensible algorithms to learn, nor can one prove anything much using high-school mathematics. This is project material, not content to be set for an examination syllabus.

Do the writers really expect teachers to define chaos in the Extension 1 classroom? This topic is notorious for its difficulty and for the confusion that it has caused throughout the history of differential and difference equations. Without any proper experience of well-behaved DEs and difference equations, students will understand nothing. Teachers also will have no background on which to base any understanding.

First-order and second-order difference equations, on the other hand, could be redrafted to give some reasonable algorithms, although the topic would still be too hard for Extension 1, not as useful as parameters and parabolas, and underweight geometry in the course.

Extension 2 Differential Equations: This topic was a mess in the previous Writing Brief, and remains a mess in the draft, with a complete lack of clarity about what methods are to be taught and what equations are to be considered. I will spare you an exposition of the manifold confusions here — I know that you have received them from other sources — and simply make the following recommendations as a means of rescuing what has clearly become a difficult situation:

- Replace the chatty material about first-order DEs by a proper course, presenting the two algorithms of the integrating factor, and the separation of variables. Students will thus learn some solid mathematics, which they can then apply to some interesting practical situations.
- Drop the material about general second-order DEs. There is no satisfactory way to teach students at this stage a proper understanding of the characteristic equation. Nor, in the absence of complex analysis, can we explain properly what is going on with the three cases (distinct real roots, distinct unreal roots, repeated roots). Teachers also will be working in the dark.
- Bring back into Extension 2 the material now in Extension 1 about $\ddot{x} = \dfrac{d}{dx}\left(\tfrac{1}{2}v^2\right)$. This allows a rigorous treatment of at least some classes of general second-order DEs.
- Make it clear that the teaching of $\ddot{x} = \dfrac{d}{dx}(\tfrac{1}{2}v^2)$ is required in SHM to derive v^2. At the moment the text is all quite vague as to how v^2 is to be obtained.
- Bring back the material on circular motion. This is an example of a *second-order vector DE*. Experience with different types of particular DEs is far better in a first course than battling with general second-order DEs, which should only be done at tertiary level after high-school experience with particular DEs. Moreover, the removal of circular motion has taken away one of only three uses of vectors in the courses (the others are complex numbers and projectile motion), and it has taken away one of the essential understandings of what the trigonometric functions are. It has also removed the understanding of orbits, which have now become part of our daily lives, and which are essential for atomic physics and chemistry.

  It should be made clear, however, that resolution of vectors is the point here, and that the cumbersome derivations of acceleration that we saw some years ago in the HSC are not required.

Alternatively, retain conics and leave the Extension 1 course as it is. That would be my choice.

Direction fields: The inclusion of this topic is also a mistake. We can't do anything sensible except draw pretty pictures. When introducing the primitive and the family of curves that arise from it, I actually make a practice in my classes of explaining the beginning of direction fields. But with general DEs, there are very serious problems of existence and uniqueness that the pictures do not address. Direction fields should be omitted.

Locus, parameters and implicit differentiation: Locus and parameters are key ideas in calculus, yet locus has been completely omitted. Parameters are mentioned in passing in pre-calculus work in the Advanced syllabus of the parametric equation of the circle, and of the parametric definition of the Lissajous figures (p 55, dotpoints 4–5). It is mentioned with projectile motion in Extension 1, but this is normally regarded as resolution of vectors rather than as parametric differentiation. Has parametric differentiation been dropped?

Implicit differentiation is mentioned explicitly in Extension 1 in relation to related rates, as if it had already been covered (page 97), even though this is more an application of the chain rule. Should one presume that the formula $\dfrac{dx}{dy} = \left(\dfrac{dy}{dx}\right)^{-1}$ in inverse functions is obtained by implicit differentiation? All this needs to be made explicit.

There is no room in Advanced for parameters, nor for parametric differentiation. The unfortunate dotpoint about the Lissajous figure should be dropped.

The dropping of the parametric treatment of the parabola from Extension 1, however, is a mistake, given its importance in applications of calculus, and given the playground that it provides for locus and for parameters — it should be restored, being far more important than difference equations.

If the unfortunate decision to omit conics from Extension 2 is final, then at least if parametric and implicit differentiation are placed explicitly back into Extension 1, they can be applied to circles, which have a natural parametrisation, and extended quickly to ellipses and hyperbolas, but without any mention of their focus–directix geometry.

Complex numbers in Extension 2 would be difficult without ellipses and hyperbolas, because they describe many simple loci in the complex plane. Dotpoint 3 on page 33 explicitly requires them.

Ratio division and perpendicular distance in Extension 1: There was no good reason to omit them, and they support other material in coordinate geometry and calculus in useful and illuminating ways.

Too much calculus in the Extension 2 course: The Extension 2 course has always been heavy on calculus. It now has too much of a good thing, and other aspects of mathematics — particularly the geometrical–algebraic relationships of conics — have gone. There is an unfortunate imbalance here that will not give students the imaginative feel for mathematics as a whole.

I confess that I now regret arguing against vector geometry. A well-written and imaginative course in vector geometry — whichever of several directions it took — would be far preferable to a technical and fiddly grinding away with first and second-order differential equations.

3. Softening the division between Preliminary and HSC

We urgently need explicit flexibility on this matter from the Board. There are so many different situations in school, so many levels achieved at the end of Year 10, so many different abilities in the students we are teaching, and so many different equally valid approaches by teachers and schools.

In particular, it is generally agreed that the Preliminary Advanced course is too long to be completed in three terms. Some schools will want students to change courses at the start of Term IV in Year 11, and if a student drops calculus, there could be perceived problems with the Board's 'rules'.

There is great fear out here in classroom-land that if one topic is not completed, or not taught in the specified order, or with the specified level of detail, then the whole School will be threatened with deregistration and closure. I am not exaggerating here, although I am very aware that such dogmatic warnings are coming from 'consultants' rather than from Board inspectors.

What is needed is an explicit statement that, as regards the Preliminary–HSC split, details of order and level of treatment are to some extent advisory, and small-scale deviations from them for sound educational reasons are quite acceptable to the Board. Here are some examples of the sort of problems that too much rigidity will cause, but there are many more.

- Can primitives be delayed until the HSC course?

With a weak 2 Unit class, I would never teach primitives in the Preliminary course in the same topic as differentiation. Reversing differentiation just after they have learnt to go forward would, I believe, be unwise. Other teachers, however, will disagree with me, and with more able students it really doesn't matter.

- Can radian measure be delayed until Year 12?

Many of us are concerned that the treatment of the general angle is confusing enough for one topic, and that this material needs time to settle before radian measure is introduced as a lead-in to the calculus of the trigonometric functions.

- Can counting be delayed until Year 12?

It is quite reasonable for the Board to suggest starting Year 11 with something dramatically new. For less able 2 Unit classes, however, most teachers will judge that this is not really possible, and should be delayed until students have more confidence and maturity in Year 12. Other teachers may just want to get straight into calculus.

- Can statistics be delayed until Year 12?

A teacher's judgement may well be that sorting out the introduction to calculus is the essential task of the first three terms of Year 11.

- Can Extension 1 geometry be delayed until the HSC course?

In our School, with its classical traditions, geometry is stressed strongly in Years 7–10. We therefore leave Stage 6 geometry until Year 12, preferring to develop calculus as quickly as possible in Year 11. We then easily complete geometry in Year 12.

- Can the Preliminary Extension 1 polynomial work be delayed until all the polynomial content is treated together in Year 12?

This is relevant for more able students, perhaps preparing for Extension 2, who have covered the Preliminary polynomial work already in Stage 5 during Year 10. There would be no point in their breaking off calculus for such a small piece of work.

4. Assumed knowledge in the Advanced and Extension 1 courses

The Board has created a serious problem in the draft Advanced and Extension 1 syllabuses by insisting that *all* Stage 5 material be omitted from the Stage 6 syllabuses. Calculus must be firmly based at school on a secure knowledge of algebra, coordinate geometry, trigonometry, and plane geometry.

The clear impression is given in these documents, however, that teachers are expected to teach calculus without first making sure that this foundation has been properly laid, and that all that is needed in the Year 11 classroom are occasional digressions to review Stage 5 topics. Such an approach will not work — 2 Unit students particularly need thorough review of this material at a higher level than was demanded in Year 10 if they are to succeed in calculus. The Board needs to give leadership to teachers in constructing this systematic treatment, and needs to allocate realistic time for it to happen.

Moreover, there is no defined 'Stage 5'. Students may enter Year 11 calculus from Stage 5.2, Stage 5.3, and Stage 5.3§. Many enter with knowledge intermediate between these reference points. For Stage 5.2 pupils, most of the omitted 'review' is completely new work. Is the Board intending in the future to exclude Stage 5.2 students from calculus?

The draft syllabuses are confusingly inconsistent in their treatment of Stage 5 material. Preliminary probability, for example, is covered in Stages 4–5, yet is recapitulated in its entirety in the Advanced syllabus, despite the fact that in school courses it has almost no relationship with calculus. The sine and cosine rules, on the other hand, which are essential for the development of the calculus of the trigonometric functions, are entirely omitted from the draft syllabuses, and logarithms, which underpin exponential functions, get just a one-line mention (p 38, dotpoint 5). Are we to conclude that the Board intends to avoid these topics in the HSC examinations? Similarly in Extension 1, polynomials are recapitulated in detail, but congruence, similarity, special triangles and quadrilaterals are completely omitted.

Here are some details of problems with 'assumed knowledge' topics. The significant question about each topic is, 'Is it possible that the calculus examination papers will contain questions where the topic is specifically required (probably as a lead-in or some other part of a longer question)?'

Trigonometry:

- Practical right-angled trigonometric problems, including two-step problems.
- Angles of elevation and depression, bearings (like 210°T) and compass readings (like N25°E).
- The sine, cosine and area formulae, including practical problems.
- The general angle in degrees.

All this trigonometry should be taught or re-taught in Year 11, unless of course the teacher is completely convinced that the student knows it well. It should also be routinely examined separately and as part of larger problems in the HSC.

Even Stage 5.3§ students will only have covered obtuse-angles trigonometry, not the trigonometry of the general angle. Since most of us believe that dealing with the general angle for the first time in radians would be a disaster, and since the Board is apparently not wanting it to be taught that way, why is it restructuring its syllabuses like this?

Coordinate geometry:

- Distance and midpoint formulae in coordinate geometry.
- Pythagoras' theorem, in the coordinate plane and in applications.
- Various forms for the equation of a straight line.

We all know that it is pointless to begin calculus before thoroughly reviewing this material.

Quadratics:

- Factoring non-monic quadratics and factoring by grouping.
- Completing the square in monic and non-monic quadratics. The Extension 1 course seems to discuss completing the square as if it is new work.
- Finding the equation of a quadratic given:
  - the vertex and another point,
  - the x-intercepts and another point,
  - three points on the curve.

These matters need precise clarification in the syllabuses. Factoring non-monic quadratics, and factoring by grouping, are necessary in 2 Unit because many applications throw them up. Completing the square in a monic quadratic is also important 2 Unit material.

Completing the square in non-monic quadratics, however, sinks the boat for most 2 Unit students. I would recommend it be included in Extension 1, but specifically excluded from 2 Unit.

Finding the equation of a quadratic given certain points on it is difficult and can hardly be classified as 'Stage 5 review', yet the Advanced syllabus seems to assume it on page 45. The topic is important in applications, and needs specific exposition in the Advanced syllabus.

Simple and compound interest formulae: This can easily be specified in the sequences topic, where simple and compound interest become examples of APs and GPs respectively.

Assumed knowledge of geometry in Extensions 1 and 2: The circle geometry of Extension 1 requires the knowledge of congruence, similarity, special triangles and special quadrilaterals. This is generally poorly taught and understood in Years 7–10, and students know very little at the start of Year 11. What is the Board expecting Extension 1 candidates to know in the HSC Examinations?

The complex number work of Extension 2 also requires such knowledge, particularly of special quadrilaterals. What is the Board intending Extension 2 candidates to know?

Since intercepts cut off by parallel lines have been omitted, there is indeed no new work here. Nevertheless, just about all calculus students need a review of it, and the Extension 1 syllabus must mention this far more explicitly. Geometry is a part of the foundation of calculus, and few schools now have the time to teach it coherently in Years 7–10.

5. Some further points needing review or clarification

The following list of questions is by no means exhaustive. Many of these details are due to the difficulty of writing coherent mathematics in outcomes, or to the fact that most concepts have been left undefined, or to problems in prose when definitions are given.

Harder questions:

- Does the Extension 1 course include 'Harder questions on the Advanced course'?
- Does the Extension 2 course include 'Harder questions on the Advanced and Extension 1 courses'?

This needs to be clarified in the texts of the syllabuses. For example, circle geometry is surely to be examined at a higher level in the Extension 2 paper as it always has been, and proofs of inequalities often require deeper understanding of the fundamental theorem of calculus.

For the new topics in Advanced and Extension 1, however, there is as yet no tradition of examination papers. I've raised above the possibility of definite quadratics in difference equations, leading to complex-valued sequences. The normal distribution function yields a little to the integration techniques of Extension 2, in that the calculation of $\int_{-\infty}^{\infty} e^{-\frac{1}{2}x^2}\,dx$ is just in range of a very hard structured Extension 2 question.

I would recommend a simple statement in the Extension 1 and Extension 2 syllabuses saying that harder questions respectively on the Advanced course, or on the Advanced and Extension 1 courses, are possible.

Will Extension 2 students ever be examined in statistics, counting or probability? This is a particular case of the previous item. Since there is no statistics in the Extension 1 or Extension 2 syllabuses, and since Extension 2 students do not sit for the Advanced paper, can Extension 2 students be confident that they will never again need to look over this material once their Preliminary course is over?

Trigonometry:

- 'Given that $\sin\theta = \frac{3}{4}$ and θ is acute, find $\cos\theta$'.
- 'Given that $\sin\theta = \frac{3}{4}$ and $\tan\theta < 0$, find $\cos\theta$'.

These things seem to have been forgotten. They need to be in all three courses.

- Are Advanced students not in Extension 1 really supposed to know the secant, cosecant and cotangent graphs? I don't know of any school applications that use them.

I see little point in 2 Unit students sketching secant, cosecant and cotangent, and I would recommend that these three graphs be left to Extension 1.

- Is the primitive of $\sec^2 x$ in the Advanced course?

Since we are teaching the derivatives of $\sin x$, $\cos x$ and $\tan x$, and since we are teaching that the primitive is the derivative reversed, this standard form should be included in Advanced. It should then be extended, as with sine and cosine, to $\displaystyle\int \sec^2(ax+b)\,dx = \frac{1}{a}\tan(ax+b) + C$.

There are further problems with primitives, as detailed below.

- Is the formula for $\sin 2\theta$ fair game in a question? Dotpoint 1 on page 112 (a left-hand page!) contains the formula for $\sin(x+h)$.

The point needs to be clarified; $\sin 2\theta = 2\sin\theta\cos\theta$ is a useful formula, but this in turn raises the question of $\cos 2\theta$ and $\tan 2\theta$.

Graphing:

- Why is there so much emphasis on $y = |x|$ in the Advanced course?

These pupils need a straightforward course in differentiable functions. In particular, the beginning of calculus is no place to be discussing distractions like $\dfrac{|x|}{x}$ (pp 58–59).

- Are Advanced students really supposed to cope with a sequence of three compound transformations? Dotpoint 6 on page 53 mentions $y = 1 - \cos\frac{1}{2}x$. Dotpoint 5 on page 41 of Extension 1, however, restricts to a sequence of just two transformations.

The general case of sketching of $y = f(bx)$ has rightly been omitted from 2 Unit, but should, of course, be in Extension 1. With trigonometric graphs, however, stretching is needed in 2 Unit as well, because they must be able to sketch, for example, $y = \cos\frac{1}{2}x$ and $y = -\cos\frac{1}{2}x$.

I would recommend, however, that composition of transformations be restricted in 2 Unit to no more than two transformations, but that such restrictions be lifted in Extensions 1 and 2.

- Is the graphing of the sum and difference of two graphed functions required in Advanced? The topic is only mentioned in applications of trigonometric functions (p 53, dotpoint 8), but then gets further treatment in Extension 2.
- Is the graphing of the quotient of two graphed functions required in Advanced? The instructions for graphing $y = \tan x$ and the reciprocal functions seem to indicate that it is (p 53, dotpoints 3 & 4).

I would recommend that sum, difference, product and quotient be omitted entirely from Advanced, that sum and difference be explicitly placed in Extension 1.

- Are unions as well as intersections of regions intended?

I would recommend that unions of regions be explicitly omitted from all three courses. They rarely occur in application, and in the absence of any formal training in logic, they cause confusion about 'and' and 'or' without really helping in the exposition of calculus.

- Is the graphing of a factored cubic part of the Advanced course?

It should be, but the necessary analysis of the zeroes and sign is best delayed until it has a natural context with turning points and inflexions. At the moment, the Advanced syllabus prescribes a careful analysis of the zeroes of the first and second derivatives, but does not deal with the analogous analysis of the zeroes of the function itself.

Inequalities:

- What do we teach about the triangle inequality $|a+b| \le |a| + |b|$?

I would recommend that it be dropped from 2 Unit, and from Extension 1. It is, however, vital for the complex numbers in Extension 2, where it finally has some context and where it makes sense to call it 'the triangle inequality'. At this level, it should be extended to $|a| - |b| \le |a+b| \le |a| + |b|$.

Calculus:

- Is the use of $f'(c) = \lim\limits_{x \to c} \dfrac{f(x) - f(c)}{x - c}$ an alternative to $f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$, or must both be taught? May we use $f'(x) = \lim\limits_{u \to x} \dfrac{f(u) - f(x)}{u - x}$, which avoids the final confusing replacement of c by x? Must we also teach Δx and Δy, and why the change from δx and δy?

This material on page 61 of the Advanced syllabus should be discarded almost entirely. It is confusing, and it is quite limiting to have to present it like this. In particular, the formula $f'(c) = \lim\limits_{x \to c} \dfrac{f(x) - f(c)}{x - c}$ is difficult to use in any class, because students need to replace c by x at the end, and they get bamboozled. This version of the derivative should certainly go.

The best formula to use in 2 Unit is $f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$, because 2 Unit pupils cannot factor anything higher than a quadratic, and there is thus little point in giving them anything else. The best formula for much theoretical work is $f'(x) = \lim\limits_{u \to x} \dfrac{f(u) - f(x)}{u - x}$, and Extension 1 pupils need it in several places. But it would be a great relief if the syllabus would just drop the whole turgid page and just prescribe first-principles differentiation.

I would recommend similarly that the treatise about the fundamental theorem of calculus be transferred to the support document, and likewise with the treatise about volumes of revolution.

- Is the integration of x^n to be restricted to n an integer (p 67, dotpoint 3)? Integrating $\sqrt{x}$ also appears in the Advanced Syllabus (p 95, dotpoint 1(iii)).

What is needed in the Advanced course are the two formulae

$$\frac{d}{dx}(ax+b)^n = an(ax+b)^{n-1} \qquad \text{and} \qquad \int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C, \ \text{ where } n \ne -1,$$

where n is any rational number. There is no reason to restrict 2 Unit students by giving them a weaker form than this, and in preparation for the calculus of the exponential function, the comment should be made that the formula applies for real n as well, although a full proof of this is impossible at school. The initial treatment of primitives on page 67, however, could well be restricted to expanded polynomials, but such restriction should be advisory only, as should its placement in the differentiation section.

Exponential functions:

- What treatment is expected of $\lim\limits_{x \to -\infty} xe^x$ in the Advanced course and in Extension 1?

This is a tricky and irritating classroom and examination problem, and needs to be clearly defined so that it does not cause difficulties. The general results concern limits of $x^k e^x$ and $x^k \log x$. I would recommend that:

– Advanced, and probably Extension 1 students, should be given any such limits in a question,
– Extension 2 pupils should know the limits, and be able to reconstruct proofs of them in the context of a structured question leading them through the steps.

- We are apparently using $y = a^x$ to model applications. Are the derivative and integral of a^x part of the Advanced course or the Extension 1 course?

This material is far more difficult to teach than may appear at first sight. I would recommend:

– The matter should be explicitly included in Extension 1, because it is important for applications and for further work in calculus.
– It should be specifically excluded from 2 Unit, because it is too hard for 2 Unit students, who as we all know are flat out just understanding the calculus of $A\,e^{kx}$ and $\log(ax + b)$.

On the other hand, a simple application of the change-of-base formula allows even 2 Unit students to differentiate $\log_a x$.

6. Statistics

I am not a statistician, nor have I had the opportunity to consult a statistician in any depth, but this part of the syllabus looks poorly written, and doesn't make sense to me. The material is new for teachers, and suffers from the same flaws as the other pieces of new work in the syllabus — incoherent presentation, inconsistent notation, lack of clarity about what is to be taught and examined, unrealistic expectations of the classroom, and the absence of illuminating examples.

We teachers cannot satisfactorily assess such a confused text. I hope that any misunderstanding by me in the following comments on pages 70–75 and 128–129 of the Advanced syllabus will help an expert in mathematical statistics sort out a coherent course, with suitable guidance from teachers.

- Why are S, s and σ all used for the standard deviation on different pages (pp 71–74 & 129)? If there is a code here, it should be explained — sample and population seems not to be the reason. In this context, using S for the sum of squares on page 129 seems best avoided.
- Why is $\overline{x}$ used for the mean on every occasion except the second-last dotpoint on page 129, where μ is used?
- Why do dotpoints 5 & 7 on page 129 use three different letters x, z and X for the variable? Is it intended that we introduce the notion of a random variable, and if so, how do we define it?
- Why does dotpoint 8 on page 70 specify in one clause 'comparing related sets of data using the concepts of location, variance and skewness', when the course has well-defined measures of location and variance, but not of skewness? Why variance rather than standard deviation?
- What are 'the full reasons' that teachers should understand for dividing by $n - 1$ and not n (p 71, dotpoint 4)? What, realistically, is going to be said about this in the State's classrooms?
- How can dotpoints 3 & 4 on page 71 give two different formulae for the same thing? There must surely be a problem with notation here.
- Nobody can 'describe the difference between correlation and causality' — it is an intractable philosophical problem, and teachers chatting about it will do nothing but cause confusion. On the other hand, a bowl-shaped scatter graph may have zero correlation, but there may be clear causation — is this what is intended by the remark?
- The formula in dotpoint 4 on page 73 is truly scary and beyond 2 Unit. Two implications drawn from it below seem wrong:
  – The inequality should read $-1 \le r \le 1$, not $-1 < r < 1$. (And the proof seems too hard.)
  – The correlation coefficient surely does not 'vary inversely with quantities related to the standard deviation of both variables', because the numerator will also change.

- What do we do with the line of best fit once it has been obtained? This is quite a serious problem, which extends to correlation generally. I would leave line-of-best-fit and correlation out of the syllabus, but if they are to be in, the nature of the classroom treatment and the nature of the HSC questions both need very careful specification by statistics experts.
- A question asked out of ignorance — do the first dotpoints on pages 128–129, in their discussion of vertical distances and the line of best fit, envisage that x is a 'controlled variable' in the experiment, with negligible error? Is this to be explained, and how? Also, is some assumption being made that one or both variables are normally distributed?
- Dotpoints 4, 5 & 7 on page 74 should surely be referring to the *relative* frequency histogram. The frequency histogram will tend to infinity at all points on the curve.
- The numbers in dotpoint 2 on page 75 seem wrong. The mean should surely be 1·06 and the standard deviation 0·03.
- Dotpoint 3 on page 75 makes no sense, because no conclusion can ever be drawn from a sample of size 1. (Was this perhaps the intended answer?)
- Dotpoints 4 & 5 on page 75 are impossible in the absence of a test for the normal distribution. Most students quite reasonably think that $y = \dfrac{1}{1+x^2}$ must be a 'bell-shaped curve'.
- What is going to be said in this course about assuming that a distribution, or a sampling mean, is normal? By the way, we are all most relieved that chatter about the central limit theorem has now been removed.
- Why does dotpoint 1 on page 75 ask one to 'determine the proportion of individuals in a normal population that lies within one standard deviation of the mean'? One just 'looks it up' in a table, or presses a calculator button. We certainly can't evaluate the integral.
- Are candidates to be provided with a table of values of the normal distribution? What form will the table take? Will they have printed as well as calculator forms of the table? I would recommend that they be given a printed table, not just in a textbook, but in the HSC itself. The whole idea of a probability distribution is so new that it is better for them to see the figures gathered together on one page, with the shaded integral above showing just what the figures represent (there are several versions of the table). Numbers coming out of a calculator screen are most useful, but will not bring it to life. It's a very challenging idea, and the more visual we can make it, the better.

  The teaching of the normal distribution is the central idea in the topic. Without any indication as to how its values are to be obtained, we can hardly give considered comment about its reception in the classroom.

Statistics replaces geometry – one of the foundations of calculus — which is not a satisfactory situation. If after so much discussion and advice, the Board cannot write a simple, straightforward course, it should abandon statistics and return to its previous clear course in Euclidean geometry.

7. The undermining of the Board's excellent examination papers in calculus

'Mean and tricky' is the best description that I have heard so far of the Board's specimen papers, with their multiple-choice and multiple-correct-multiple-choice questions.

The tradition of HSC calculus examinations in NSW is one of the best assets we have in our schools. I am quite dismayed that the Board should see fit to undermine a tradition that has so firmly underpinned the whole of NSW school mathematics for so many decades. Our current papers are excellent. They provide for all of us a global understanding of all the mathematics that we teach throughout school, and a constant inspiration to develop our ideas more elegantly and more generally.

They also provide a wonderful example of leading academics and leading school teachers working together and learning from each other to produce excellence in the classroom — precisely the cooperative effort so glaringly absent from the writing of these syllabus texts.

Our examination papers begin simply and easily, testing basic ideas. They then proceed systematically, testing simple and then more complicated algorithms, until in Extensions 1 and 2 they end with unpredictable problems, that nevertheless are structured into part-questions that lead capable students through the mathematics they require. In all aspects of the examination, the examiners and the markers are clearly on the student's side, trying to award marks for things that are well done.

These multiple-choice questions on the specimen paper, on the other hand, are designed to trick, to cause students to stumble and make mistakes. Instead of testing whether a student can apply an algorithm, they invite the student instead to guess, to eliminate some possibilities by working backwards as well as forwards, to jump at answers instead of reasoning them out.

The approach that these papers are teaching is contrary to the nature of mathematics, to the calm and ordered working out of the algorithm to demonstrate one's mastery of the discipline. These papers will force us to teach examination 'tricks', to teach devices that will eliminate possibilities, to spend time teaching students to guess, and to work problems through only to the point where other possibilities can be seen to be wrong. We need leadership from the Board — it is disappointing to see the Board acting as an obstacle to teaching mathematics.

I stated in my symposium submission 'a basic principle of all examining — diligent students of adequate ability should come out of the examination room feeling that they have given a good account of themselves', and complained about the difficulty of some recent 2 Unit papers. These multiple-choice and multiple-correct-multiple-choice questions will make the situation much worse. They will upset the students, make them edgy and unconfident, and set up right at the start the feeling that the examiners are 'out to get them'.

Finally, multiple choice papers do not assess effectively. Our experience, and that of all other schools I talk to, is that the rank order on multiple-choice papers is variable and unreliable. The strongest usually come to the top, but not always. There are always a few weaker students who suddenly get lucky and are awarded a fine grade, but our great concern is for the diligent and reasonably able student who just can't cope with having to do mathematics in precisely the opposite direction from what has been taught. If effective assessment is what the Board is seeking, this is no way to go.

I advise in the strongest terms to abandon this course of action and remain with the excellence that the Board has always supported.

I will give a few examples of the extreme difficulty, and the ubiquitous errors and confusions in the papers, and also illustrate the rigidities of thought that are inevitable with this style of question.

The Advanced specimen paper:

- Question 1 is a difficult curve sketching question. The function $y = |x|$, which I have argued above should not be unduly emphasised in Advanced, is further transformed by a sequence of two transformations. Sketching this alone would be worth 2 or 3 marks, yet the whole question, with $y = x^2 - 1$ as well, and the shading, is worth only 1 mark.
- Question 2 is very difficult verbally for 2 Unit students, requiring simultaneous reading of gradient and concavity, both interpreted in terms of motion. Yet it is only worth 1 mark.
- Question 3 requires three successive transformations of a trigonometric function — beyond the two I recommended above, yet is only worth 1 mark.
- Question 4 would unfortunately be a complete surprise to any of my Advanced, Extension 1 or Extension 2 students, because as mentioned above, I never find areas using absolute value, and the syllabus does not even mention the possibility of doing so. Why is there this obsession with absolute value — 2 marks out of the first 5 marks?
- Question 5 requires simplification of $\cos(\theta - \frac{\pi}{2})$, which is beyond practically all my past 2 Unit students. They have no compound angle formulae, and at this level would need careful prompting to obtain the result by shifting the graph.

The Extension 1 specimen paper:

- In question 3, the first graph looks as if it has two points where the curve is not differentiable, although on closer examination, perhaps the curve was intended to be smooth everywhere. Is it intended to have vertical asymptotes?
- In question 4, the inverse cosine graph is drawn with different scales on the axes so that it looks almost flat at $(0, \frac{\pi}{2})$ instead of having gradient -1 there. This is a poor example for school students, and continues a misunderstanding promulgated in many textbooks.

The Extension 2 specimen paper:

- Question 1, besides all the difficult reading, also introduces a complete red herring by mentioning the mass of the particle. This is designed to trick and cause confusion — all for one mark.

  Besides that, units are given for the velocity v, but not for the mass m or for g, which is inconsistent. The acceleration is not alone on the left as in Newton's second law, which is probably intended to be confusing. The variable y is not defined. It uses the unwelcome notation $y(0)$, which I have objected to above as a hybrid of variable and function notation, and which regularly causes real trouble for students.
- Question 2 is a real puzzle. Graphs (B) and (C) look exactly the same, given the constraints of the diagrams we draw. Graph (A) may (or may not) have a horizontal asymptote at $y = \frac{1}{2}$, which does not seem to be a property that the original graph would induce in its reciprocal. The graphs are so ambiguous that one cannot be certain what is intended.

  I believe that someone is playing word-games with 'the most accurate representation'. This is an outrageous way to assess students who have worked so hard at mathematics for 13 years of school.
- Question 4 is a wonderful example of how difficult and jumpy these questions can be. For just two marks, the students has to slice a solid by cylindrical shells, and then slice it again by horizontal planes, and then realise that this solid consists of a hollow cylinder, whose volume must be calculated, with another more complicated solid on top.

  Volumes by slicing is a cumbersome procedure to carry out, and difficult to visualise satisfactorily. We all go to great pains to teach careful setting out and systematic work in this topic. Yet here what is required is two slicings, partial working, and a jerky realisation that a hollow cylinder is involved. The assessment is contrary to everything we are teaching students about how to think about their work and how to set it out.
- Question 5 and 6 are presumably multiple-correct-incorrect, but that heading is not on the page, and I didn't realise for a while. Pity the poor students swapping backwards and forwards from single-correct to multiple-incorrect.

I am not as concerned with the 'constrained response' Advanced question 6, although I dislike it because it encourages writing the answer down without working. Why have it at all? The 'free-response' questions have the possibility of disaster if the student does not 'spot the issue', or the examiner overreaches, but although I dislike them, they do not concern me quite as much as the multiple choice tasks.

The proposal has been made to mark out of 100 instead of 120. All this will do is make the marks harder to get. The present situation of easy, straightforward questions at the start will entirely disappear under the overwhelming necessity to 'separate the top' for the UAI scaling.

Can the Board be a little more on the candidate's side in these very difficult calculus examination papers, and simply maintain the excellence that we presently enjoy in our HSC examinations. There is no reason to change, and no argument has been given. One suspects that cost is the only issue.

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